1/9 (2m - 16) = 1/3 (2m + 4)

1/9 (2m - 16) = 1/3 (2m + 4) is a fundamental algebraic equation that often appears in math exercises, tests, and real-world problem-solving scenarios. Understanding how to solve such equations is essential for students and anyone looking to strengthen their algebra skills. This article provides a comprehensive guide to solving the equation step-by-step, delves into the underlying concepts, offers tips for mastering similar problems, and explores related topics to enhance your mathematical proficiency.

Understanding the Equation

Before delving into solving the equation, it’s important to understand its structure and what each component represents.

The Equation Breakdown

The given equation is:

```
1/9 (2m - 16) = 1/3 (2m + 4)
```

- The terms involve fractions multiplied by linear expressions involving the variable \(m \).
- The goal is to solve for \(m \), which involves isolating \(m \) on one side of the equation.

Key Concepts Involved

- Fractional Equations: Equations that include fractions with variables.
- Distributive Property: Applying multiplication over addition or subtraction inside parentheses.
- Simplification: Combining like terms and reducing fractions.
- Isolation of Variable: Rearranging the equation to get \(m \) alone.

Step-by-Step Solution Process

To solve the equation, follow these systematic steps:

Step 1: Eliminate Fractions

Dealing with fractions can be cumbersome. To simplify calculations, multiply both sides of the equation by the Least Common Denominator (LCD).

- The denominators are 9 and 3.
- The LCD of 9 and 3 is 9.

Multiply both sides of the equation by 9:

```
9 × [1/9 (2m - 16)] = 9 × [1/3 (2m + 4)]
```

This simplifies to:

```
(2m - 16) = 3 × (2m + 4)
```

Step 2: Expand Both Sides

Apply the distributive property:

- Left side: $2m - 16$ (already simplified)
- Right side: $3 \times 2m + 3 \times 4 = 6m + 12$

So, the equation becomes:

```
2m - 16 = 6m + 12
```

Step 3: Collect Like Terms

Bring all terms involving m to one side and constants to the other:

- Subtract $2m$ from both sides:

```
-16 = 6m - 2m + 12
```

- Simplify:

```
-16 = 4m + 12
```

- Subtract 12 from both sides:

```
-16 - 12 = 4m
```

- Simplify:

```
-28 = 4m
```

Step 4: Solve for \(m \)

Divide both sides by 4:

```
m = \frac{-28}{4} = -7
```

Final Answer

\[
\boxed{
m = -7
}
\]

This is the solution to the original equation.

Verification of the Solution

Always verify your solution by substituting \(m = -7 \) back into the original equation.

Original equation:

```
1/9 (2m - 16) = 1/3 (2m + 4)
```

Substitute \(m = -7 \):

- Left side:

```
1/9 (2 \times -7 - 16) = 1/9 (-14 - 16) = 1/9 (-30) = -30/9 = -10/3
```

- Right side:

$$1/3 (2 \times -7 + 4) = 1/3 (-14 + 4) = 1/3 (-10) = -10/3$$

Both sides equal $-10/3$, confirming the solution is correct.

Additional Tips for Solving Fractional Equations

- Always find the LCD first to clear fractions.
- Distribute carefully to avoid mistakes.
- Combine like terms before isolating the variable.
- Check your solution by substituting back into the original equation.
- Practice with different equations involving fractions to gain confidence.

Related Topics and Practice Problems

To deepen your understanding, explore the following:

- **Solving linear equations with fractions**
- **Working with rational expressions**
- **Word problems involving fractions**
- **Applications of algebra in real-life scenarios**

Practice problems:

1. Solve $2/5 (3x - 10) = 1/2 (4x + 6)$
2. Find x if $3/4 (x - 2) = 1/2 (x + 8)$
3. Determine m in $5/6 (6m + 3) = 7/3 (2m - 4)$

Common Mistakes and How to Avoid Them

- Forgetting to multiply both sides by the LCD: Always clear fractions early to avoid complicated calculations.
- Incorrect distribution: Double-check each step when distributing to prevent errors.

- Sign errors: Watch out for negative signs, especially during subtraction.
- Mismanaging constants: Keep constants on one side and variables on the other to simplify solving.

Conclusion

The equation **1/9 (2m - 16) = 1/3 (2m + 4)** exemplifies how fractional equations can be approached systematically. By eliminating fractions through multiplication, expanding expressions, combining like terms, and isolating the variable, you can efficiently find solutions to similar problems. Mastery of these techniques not only enhances your algebra skills but also prepares you for more complex mathematical challenges. Remember to verify your solutions to ensure accuracy and build confidence in solving fractional equations. With consistent practice and attention to detail, you'll find solving such equations becomes intuitive and straightforward.

Frequently Asked Questions

How do I solve the equation 1/9 (2m - 16) = 1/3 (2m + 4)?

First, clear the fractions by multiplying both sides by the least common denominator, which is 9. This gives: (2m - 16) = 3(2m + 4). Then, expand both sides: 2m - 16 = 6m + 12. Rearrange to isolate m: 2m - 6m = 12 + 16, which simplifies to -4m = 28. Finally, divide both sides by -4: m = -7.

What is the value of m in the equation 1/9 (2m - 16) = 1/3 (2m + 4)?

The value of m is -7.

Can I solve 1/9 (2m - 16) = 1/3 (2m + 4) without clearing fractions?

While it's possible by cross-multiplying directly, clearing fractions first by multiplying both sides by 9 simplifies the process and reduces errors.

What is the first step to solve the equation involving fractions: 1/9 (2m - 16) = 1/3 (2m + 4)?

Multiply both sides by 9 to eliminate the denominators, resulting in (2m - 16) = 3(2m + 4).

Is the solution to the equation 1/9 (2m - 16) = 1/3 (2m + 4) unique?

Yes, the equation is linear in m, so it has a unique solution, which is m = -7.

What does the equation 1/9 (2m - 16) = 1/3 (2m + 4) represent?

It's a linear algebraic equation that relates the variable m through fractional expressions, often used to test understanding of solving equations with fractions.

How can I check if m = -7 is the correct solution?

Substitute m = -7 back into the original equation and verify if both sides are equal. Doing so confirms the solution is correct.

Are there any common mistakes to avoid when solving this equation?

Yes, common mistakes include forgetting to distribute correctly, mishandling signs, or incorrectly simplifying fractions. Always double-check each step.

What is the importance of understanding equations like 1/9 (2m - 16) = 1/3 (2m + 4)?

Solving such equations enhances algebraic skills, particularly in handling fractions, distributing, and combining like terms, which are fundamental in mathematics.

Can this type of equation be solved using a calculator?

Yes, you can use a calculator to perform arithmetic steps, but understanding the algebraic process is important for accuracy and conceptual understanding.

Additional Resources

Mathematical Equation Analysis: Solving for m in the Equation \(\frac{1}{9}(2m - 16) = \frac{1}{3}(2m + 4)\)

Introduction: Navigating the World of Algebraic Equations

Mathematics, often heralded as the universal language, provides us with tools to decode complex relationships and solve real-world problems. Among these tools, algebra stands as a fundamental pillar, offering methods to find unknown quantities within equations. Today, we delve into a specific algebraic problem:

$$
\frac{1}{9}(2m - 16) = \frac{1}{3}(2m + 4)
$$

This equation, at first glance, appears straightforward but requires a systematic approach to unravel the value of m. Whether you are a student aiming to sharpen your algebra skills or an educator seeking a comprehensive explanation, understanding this problem in depth is invaluable.

In this article, we will explore the structure of the equation, analyze each component, and guide you through the steps to find the solution. We'll also discuss common pitfalls and best practices in solving such equations, ensuring a thorough grasp of the process.

Understanding the Equation's Components

Before jumping into calculations, it's essential to understand each part of the equation.

The Fractions: $\frac{1}{9}$ and $\frac{1}{3}$

- The coefficients $\frac{1}{9}$ and $\frac{1}{3}$ are scalar multipliers, affecting the expressions inside the parentheses.
- These fractions can be viewed as scaling factors, which, when applied, reduce the magnitude of the expressions.

The Expressions Inside the Parentheses

- $2m - 16$: This expression combines a linear term $2m$ with a constant -16.
- $2m + 4$: Similarly, a linear term $2m$ with a constant $+4$.

The Overall Equation

The equation sets these scaled expressions equal, implying a relationship between m and the constants involved. The goal is to isolate m and find its value that satisfies this balance.

Step-by-Step Approach to Solve the Equation

Let's now dissect the process to solve for m, emphasizing clarity and rigor.

Step 1: Eliminate Fractions by Clearing Denominators

To simplify the equation, one efficient method is to clear the fractions by multiplying both sides by the least common denominator (LCD).

- The denominators are 9 and 3.
- The LCD of 9 and 3 is 9.

Multiplying both sides by 9:

$$
9 \times \left[\frac{1}{9}(2m - 16)\right] = 9 \times \left[\frac{1}{3}(2m + 4)\right]
$$

Simplifies to:

$$
(2m - 16) = 3 \times (2m + 4)
$$

Why this step matters:
Clearing denominators simplifies the equation into a form where standard algebraic operations can be applied directly, reducing the chance of algebraic errors.

Step 2: Expand Both Sides

Next, expand the right side:

$$
(2m - 16) = 3 \times (2m + 4)
$$

$$
2m - 16 = 3 \times 2m + 3 \times 4
$$

$$
2m - 16 = 6m + 12
$$

This step involves distributing the 3 across the parentheses.

Step 3: Group Like Terms and Isolate \(m\)

Now, to solve for \(m\), bring all \(m\)-terms to one side and constants to the other:

\[
2m - 6m = 12 + 16
\]

\[
-4m = 28
\]

This involves subtracting \(6m\) from both sides and adding 16 to both sides.

Step 4: Solve for \(m\)

Divide both sides by \(-4\):

\[
m = \frac{28}{-4}
\]

\[
m = -7
\]

Result:
The solution to the equation is \(m = -7\).

Verification: Confirming the Solution

It's crucial not just to find \(m\), but to verify that it satisfies the original equation.

Substitute \(m = -7\) into the original equation:

$$
\frac{1}{9}(2 \times -7 - 16) = \frac{1}{3}(2 \times -7 + 4)
$$

Calculate each side:

Left Side:

$$
2 \times -7 = -14
$$

$$
-14 - 16 = -30
$$

$$
\frac{1}{9} \times -30 = -\frac{30}{9} = -\frac{10}{3}
$$

Right Side:

$$
2 \times -7 = -14
$$

$$
-14 + 4 = -10
$$

$$
\frac{1}{3} \times -10 = -\frac{10}{3}
$$

Both sides equal $-\frac{10}{3}$, confirming that $m = -7$ is indeed the solution.

Common Challenges and Best Practices

When tackling similar algebraic equations, students and practitioners often encounter pitfalls. Let's explore some of these and recommended strategies.

Common Challenges:

- Mismanaging fractions: Failing to clear denominators can lead to complex, error-prone algebra.

- Incorrect distribution: Overlooking the distributive property when expanding expressions.
- Sign errors: Neglecting negative signs during calculations.
- Arithmetic mistakes: Simple multiplication/division errors can derail the solution.

Best Practices:

- Always clear denominators first: Use the LCD to simplify fractions early.
- Double-check expansion steps: Verify distribution to avoid errors.
- Track signs meticulously: Write down each step and check signs.
- Verify solutions: Substitute back into the original equation to confirm correctness.
- Use organized notes: Keep each step clear, especially when dealing with multiple terms.

Real-World Applications and Broader Context

While this problem is purely algebraic, the skills involved are foundational for numerous fields:

- Engineering: Designing systems where proportional relationships matter.
- Economics: Calculating cost functions or profit margins involving linear relationships.
- Science: Analyzing experimental data modeled through linear equations.
- Data Analysis: Fitting models where variables relate linearly.

Understanding how to manipulate and solve equations like $\frac{1}{9}(2m - 16) = \frac{1}{3}(2m + 4)$ enhances problem-solving agility across disciplines.

Conclusion: Mastery of Algebraic Problem Solving

The journey through solving $\frac{1}{9}(2m - 16) = \frac{1}{3}(2m + 4)$ underscores several key lessons:

- Breaking down complex equations into manageable steps is essential.
- Clearing fractions simplifies the algebraic landscape.
- Systematic expansion and grouping lead to straightforward solutions.
- Verification is crucial to confirm the accuracy of solutions.

By mastering these techniques, learners develop a robust foundation for tackling a wide array of algebraic challenges. The solution $m = -7$ not only exemplifies the procedure but also reinforces the importance of careful, methodical reasoning in mathematics. Whether in academic settings or practical applications, such problem-solving skills are invaluable assets in the analytical toolkit.

In essence, this detailed exploration transforms a seemingly simple algebraic equation into a comprehensive learning experience, emphasizing clarity, systematic approach, and verification—cornerstones of mathematical mastery.

1 9 2m 16 1 3 2m 4

1 9 2m 16 1 3 2m 4

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