14. Log 4 + Log (c-9) = 5

14. Log 4 + Log (c-9) = 5 is a fascinating logarithmic equation that offers a great opportunity to explore the properties of logarithms, solve for the variable c, and understand how logarithmic functions work in algebra. This type of equation often appears in algebra courses and standardized tests, making it essential for students and enthusiasts to grasp the methods for solving it efficiently. In this article, we will delve into the meaning of the equation, the properties of logarithms involved, step-by-step solution techniques, and practical tips to master similar problems.

Understanding the Equation: Log 4 + Log (c-9) = 5

Before solving the equation, it’s important to understand what it represents and the key properties of logarithms involved.

What is a Logarithm?

A logarithm is the inverse operation of exponentiation. The logarithm $\log_b(x)$ answers the question: to what power must the base b be raised to obtain x? Mathematically:

- $\log_b(x) = y$ means $b^y = x$

For example, $\log_2(8) = 3$ because $2^3 = 8$.

Properties of Logarithms

Several properties of logarithms are useful when manipulating equations:

- **Product Property:** $\log_b(m) + \log_b(n) = \log_b(m \times n)$
- **Quotient Property:** $\log_b(m) - \log_b(n) = \log_b(m / n)$
- **Change of Base:** $\log_b(a) = \log_k(a) / \log_k(b)$ (useful in calculations)

In our equation, the sum of two logs suggests the application of the product property.

Step-by-Step Solution of the Equation

Let’s analyze and solve the given equation: **Log 4 + Log (c-9) = 5**. For simplicity, assume the logarithm base is 10 (common logarithm), unless specified otherwise.

Step 1: Combine the Logarithms Using the Product Property

Since the equation involves the sum of two logs with the same base:

- Log 4 + Log (c-9) = Log [4 × (c-9)]

Thus, the equation becomes:

```
Log [4 × (c-9)] = 5
```

Step 2: Rewrite the Logarithmic Equation in Exponential Form

Recall that:

```
Log_b(x) = y  ⇒  x = b^y
```

Since the base is 10:

```
4 × (c-9) = 10^5
```

Calculating 10^5:

```
4 × (c-9) = 100000
```

Step 3: Solve for c

Divide both sides by 4:

```
(c-9) = 100000 / 4 = 25000
```

Add 9 to both sides:

```
c = 25000 + 9 = 25009
```

Verifying the Solution and Domain Considerations

When solving logarithmic equations, it's crucial to verify solutions within the domain of the original logarithmic functions.

Domain Restrictions

- The argument of any logarithm must be positive.
- For Log 4, the argument is 4, which is always positive.
- For Log (c-9), the argument c-9 must be greater than 0:

- c - 9 > 0 ⇒ c > 9

Our solution c = 25009 satisfies c > 9, so it's within the domain.

Verification

Plug c = 25009 into the original equation:

- Log 4 + Log (25009 - 9) = Log 4 + Log 25000

  - Log 4 ≈ 0.6021
  - Log 25000 ≈ 4.3979
  - Sum ≈ 0.6021 + 4.3979 ≈ 5

  which matches the right side of the original equation, confirming the solution is correct.

Practical Applications of Logarithmic Equations

Understanding how to solve equations like Log 4 + Log (c-9) = 5 has real-world applications across various fields.

Finance and Compound Interest

Logarithms are used to calculate the time required for investments to grow or decay exponentially.

Engineering and Signal Processing

Logarithmic relationships model sound intensity, decibel scales, and signal attenuation.

Scientific Data Analysis

Many natural phenomena, such as radioactive decay or population growth, are modeled using logarithmic functions.

Tips for Solving Logarithmic Equations

To efficiently solve equations involving logarithms, keep these tips in mind:

- **Always check the domain:** Ensure the arguments of logarithms are positive.
- **Use logarithm properties:** Combine or split logs to simplify the equation.
- **Convert to exponential form:** Rewrite logarithmic equations as exponential equations for straightforward solutions.
- **Verify solutions:** Plug solutions back into the original equation to confirm their validity within the domain.
- **Practice with different bases:** Though this example uses base 10, equations with different bases require understanding change of base formulas.

Conclusion

The equation **14. Log 4 + Log (c-9) = 5** exemplifies the core principles of logarithmic algebra, particularly the product property. By combining logs, converting to exponential form, and respecting the domain restrictions, we found that c = 25009 is the solution. Mastering these techniques not only helps solve similar equations efficiently but also deepens understanding of logarithmic functions' role across science, engineering, finance, and mathematics. Whether you're preparing for exams or applying these concepts in real-world scenarios, a strong grasp of logarithmic equations will significantly enhance your problem-solving toolkit.

Frequently Asked Questions

How do you solve the logarithmic equation Log 4 + Log (c - 9) = 5?

Use the logarithm property: Log a + Log b = Log (a b). So, rewrite the equation as Log (4 (c - 9)) = 5. Then, convert from logarithmic to exponential form: 4 (c - 9) = 10^5. Solve for c: c - 9 = 10^5 / 4, then c = (10^5 / 4) + 9.

What is the first step to simplify the equation Log 4 + Log (c - 9) = 5?

Apply the logarithm property to combine the logs: Log (4 (c - 9)) = 5.

How do you verify the solution for c in the equation Log 4 + Log (c - 9) = 5?

Substitute the found value of c back into the original equation to ensure both sides are equal. Check that Log 4 + Log (c - 9) equals 5 after substitution.

What restrictions are there on c in the equation Log 4 + Log (c - 9) = 5?

Since logarithms are only defined for positive arguments, $c - 9 > 0$, so $c > 9$.

If Log 4 + Log (c - 9) = 5, what is the approximate value of c?

Calculate c as $c = (10^5 / 4) + 9 \approx (100000 / 4) + 9 = 25000 + 9 = 25009$. So, $c \approx 25009$.

Can the solution c = (10^5 / 4) + 9 be simplified further?

No, it is already in its simplified form. You can leave it as $c = (10^5 / 4) + 9$ or approximate as 25009.

Additional Resources

Understanding and Solving the Equation Log 4 + Log (c - 9) = 5

When it comes to logarithmic equations, they often seem complex at first glance, but with a clear step-by-step approach, they become much more manageable. In particular, the equation Log 4 + Log (c - 9) = 5 offers a great example of how properties of logarithms can simplify the solving process. This guide will delve into the foundational concepts of logarithms, demonstrate how to manipulate them, and walk you through solving this specific equation with detailed explanations at every step.

What is a Logarithm?

Before tackling the equation directly, it’s essential to understand what a logarithm represents.

A logarithm is the inverse operation of exponentiation. Specifically, the logarithm base b of a number x, written as $\log_b(x)$, answers the question:

"To what power must the base b be raised to obtain x?"

Mathematically:

$\log_b(x) = y$ if and only if $b^y = x$

In many contexts, especially in high school algebra, the base is assumed to be 10 (common logarithm), unless otherwise specified. So, log 4 is understood as $\log_{10} 4$.

Core Logarithmic Properties for Simplification

To manipulate and solve logarithmic equations efficiently, certain properties are fundamental:

1. Product Property:
- $\log_b(x) + \log_b(y) = \log_b(x y)$
2. Power Property:
- $k \log_b(x) = \log_b(x^k)$
3. Change of Base (if needed):
- $\log_b(x) = \log_c(x) / \log_c(b)$, for any positive base $c \neq 1$

For this specific equation, the Product Property is the most relevant since the sum of two logs is involved.

Step-by-Step Breakdown of the Equation

Given the equation:

$\text{Log } 4 + \text{Log}(c - 9) = 5$

Step 1: Recognize the Logarithmic Sum as a Product

Using the Product Property, combine the logs:

$\text{Log } 4 + \text{Log}(c - 9) = \text{Log } [4 (c - 9)]$

This transforms the original equation into:

$\text{Log } [4 (c - 9)] = 5$

This is a crucial step because it reduces the problem to solving a single logarithmic equation.

Step 2: Rewrite the Logarithmic Equation in Exponential Form

Since log_b(x) = y implies b^y = x, and assuming the base is 10 (common logarithm), rewrite the equation:

10^5 = 4 (c - 9)

Calculate 10^5:

10^5 = 100,000

So:

100,000 = 4 (c - 9)

Step 3: Solve for c

Isolate c:

c - 9 = 100,000 / 4

Calculate:

c - 9 = 25,000

Add 9 to both sides:

c = 25,000 + 9

c = 25,009

Step 4: Verify the Domain Constraints

Before accepting this solution, verify that it satisfies the domain restrictions of the original logarithmic functions.

Domain considerations:

- The argument of a logarithm must be positive.
- For Log 4, the argument is 4, which is positive.
- For Log(c - 9), the argument is c - 9, which must be > 0.

Check:

c - 9 > 0

Substitute c = 25,009:

25,009 - 9 = 25,000 > 0

Thus, the solution c = 25,009 is valid within the domain.

Final Answer:

c = 25,009

Additional Insights and Tips for Solving Logarithmic Equations

While this example was straightforward, many logarithmic equations involve more complex manipulations. Here are some tips and common pitfalls to watch out for:

1. Always check the domain:

- The arguments of all logarithmic expressions must be positive.
- Ignoring domain restrictions can lead to extraneous solutions.

2. Use logarithmic properties carefully:

- Combine or split logs according to the properties.
- Remember that addition corresponds to multiplication inside the log.

3. Convert to exponential form when necessary:

- This often simplifies the solving process.
- Be cautious with the base; if not specified, assume base 10.

4. Watch for extraneous solutions:

- After solving algebraically, verify solutions satisfy the original equation and domain constraints.

5. Practice with different bases:

- Sometimes, logarithms with different bases appear, requiring the change of base formula.

Practice Problems to Reinforce Concepts

To solidify your understanding, try solving these similar equations:

1. Log 2 + Log (x - 3) = 4
2. Log (x + 5) + Log 7 = 3
3. 2 Log (x) + Log (x - 1) = 5
4. Log 5 + 2 Log (x) = 4

Each problem involves the core properties discussed and offers a chance to develop confidence in handling logarithmic equations.

Conclusion

The equation Log 4 + Log (c - 9) = 5 exemplifies how the properties of logarithms streamline solving for unknown variables. Recognizing that the sum of two logs can be combined into a single log of a product simplifies the problem significantly. Converting the logarithmic equation into its exponential form allows for straightforward algebraic solving, provided the domain restrictions are respected. Mastery of these fundamental steps empowers you to handle more complex logarithmic equations with confidence and clarity.

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