2/3 (4x + 3) = 1/4 (3x - 15)

Understanding the Equation: 2/3 (4x + 3) = 1/4 (3x - 15)

Mathematics often presents us with equations that challenge our problem-solving skills and deepen our understanding of algebraic principles. One such equation is:

2/3 (4x + 3) = 1/4 (3x - 15)

This equation involves fractions, linear expressions, and variables, making it an excellent candidate for practice in algebraic manipulation and solving linear equations. Whether you're a student aiming to improve your algebra skills or someone interested in understanding how to solve such equations, this article will guide you through the process step-by-step, providing clarity and insights along the way.

In this comprehensive guide, we will explore the following:

- The importance of understanding fractional equations
- Breaking down the equation into manageable parts
- Step-by-step solution process
- Common mistakes to avoid
- Real-world applications of solving similar equations
- Tips for mastering similar algebraic problems

Let's begin by understanding the context and significance of solving equations like 2/3 (4x + 3) = 1/4 (3x - 15).

The Significance of Fractional Equations in Mathematics

Fractional equations are prevalent in many areas of mathematics and science, including physics, engineering, economics, and statistics. They often appear when dealing with ratios, proportions, rates, and probabilities.

Why are fractional equations important?

- They help model real-world problems where quantities are not whole numbers.
- They enhance understanding of algebraic properties involving fractions.
- They develop skills in algebraic manipulation, such as clearing denominators and isolating variables.
- They serve as foundational exercises for advanced topics like calculus and

differential equations.

Understanding how to manipulate and solve equations with fractions is crucial for building a strong mathematical foundation.

Breaking Down the Equation: 2/3 (4x + 3) = 1/4 (3x - 15)

Before diving into the solution, let's analyze the structure of the equation.

The given equation involves:

- Two fractions: 2/3 and 1/4
- Two linear expressions inside parentheses: (4x + 3) and (3x - 15)

The goal is to find the value of x that satisfies this equation.

Key challenges:

- Dealing with fractions requires careful manipulation to avoid errors.
- Simplifying the expressions to isolate x.
- Ensuring all steps maintain the equality.

To effectively solve the equation, we need to eliminate the fractions by clearing denominators, which simplifies the process.

Step-by-Step Solution Process

Let's now walk through the method to solve for x systematically.

Step 1: Write down the original equation

$$\frac{2}{3}(4x + 3) = \frac{1}{4}(3x - 15)$$

Step 2: Multiply both sides by the Least Common Denominator (LCD)

The denominators are 3 and 4, so the LCD is 12.

Multiply every term in the equation by 12 to clear fractions:

$$12 \ [(2/3)(4x + 3)] = 12 \ [(1/4)(3x - 15)]$$

Simplify each side:

- Left side: 12 (2/3) (4x + 3) = (12 2/3) (4x + 3) = (8) (4x + 3)
- Right side: 12 (1/4) (3x - 15) = (12 1/4) (3x - 15) = (3) (3x - 15)

The equation now becomes:

$$8(4x + 3) = 3(3x - 15)$$

Step 3: Expand both sides

Distribute the coefficients:

- Left: 8 4x + 8 3 = 32x + 24
- Right: 3 3x - 3 15 = 9x - 45

The simplified equation:

$$32x + 24 = 9x - 45$$

Step 4: Isolate the variable x

Bring all x terms to one side and constants to the other:

Subtract 9x from both sides:

$$32x - 9x + 24 = -45$$

Simplify:

$$23x + 24 = -45$$

Subtract 24 from both sides:

$$23x = -45 - 24$$

Simplify:

23x = -69

Step 5: Solve for x

Divide both sides by 23:

x = -69 / 23

Simplify the fraction:

x = -3

Final answer: x = -3

Verification of the Solution

To ensure the solution is correct, substitute x = -3 back into the original equation:

Original equation:

2/3 (4x + 3) = 1/4 (3x - 15)

Substitute x = -3:

- Left side:

2/3 [4 (-3) + 3] = 2/3 [-12 + 3] = 2/3 (-9) = (-18)/3 = -6

- Right side:

1/4 [3 (-3) - 15] = 1/4 [-9 - 15] = 1/4 (-24) = -6

Both sides equal -6, confirming that x = -3 is the correct solution.

Common Mistakes to Avoid When Solving Fractional Equations

While solving equations like 2/3 (4x + 3) = 1/4 (3x - 15), learners often encounter pitfalls. Being aware of these can improve accuracy:

- Forgetting to multiply both sides by the LCD: This step is crucial for eliminating fractions.
- Incorrect distribution: Always double-check the distributive property applications.
- Sign errors: Pay attention to negative signs, especially during substitution and distribution.
- Dividing by zero: Ensure that the variables do not lead to undefined expressions.
- Incorrect simplification: Always simplify fractions to their lowest terms.

Awareness and careful execution of each step help prevent these common errors.

Real-World Applications of Solving Similar Equations

Equations like 2/3 (4x + 3) = 1/4 (3x - 15) model various real-world scenarios:

- Financial calculations: Determining interest rates or loan payments involving ratios.
- Physics: Calculating rates or velocities where multiple ratios are involved.
- Statistics: Working with proportions and probabilities.
- Engineering: Analyzing systems with proportional relationships.

Mastering fractional equations enhances problem-solving skills applicable across numerous domains.

Tips for Mastering Algebraic Equations Involving Fractions

To excel in solving fractional equations:

- Always identify the least common denominator (LCD) before clearing fractions.
- Distribute carefully and double-check each step.
- Simplify expressions at each stage.

- Verify solutions by substitution.
- Practice a variety of problems to build confidence and familiarity.

Consistent practice and attention to detail are key to becoming proficient.

Conclusion

The equation **2/3 (4x + 3) = 1/4 (3x - 15)** exemplifies the importance of systematic algebraic manipulation, particularly when dealing with fractions. By understanding the process—multiplying through by the LCD, expanding, isolating variables, and verifying solutions—students can confidently solve similar equations.

Remember, mastering such problems enhances logical reasoning, problem-solving skills, and prepares you for more advanced mathematical concepts. Keep practicing, stay attentive to detail, and you'll develop a strong foundation in algebra that will serve you well in academics and real-world applications.

Key Takeaways:

- Always eliminate fractions early by multiplying through by the LCD.
- Carefully distribute and simplify expressions.
- Isolate the variable step-by-step.
- Verify your solutions to ensure accuracy.
- Practice regularly to build confidence.

With these strategies, solving fractional equations like 2/3 (4x + 3) = 1/4 (3x - 15) will become an intuitive part of your mathematical toolkit.

Frequently Asked Questions

How do I solve the equation 2/3 (4x + 3) = 1/4 (3x - 15)?

To solve, first clear the fractions by multiplying both sides by the least common denominator (12). This gives 4(4x + 3) = 3(3x - 15). Expand both sides: 16x + 12 = 9x - 45. Then, isolate x: 16x - 9x = -45 - 12, resulting in 7x = -57. Finally, divide both sides by 7: x = -57/7.

What is the value of x in the equation 2/3 (4x + 3) = 1/4 (3x - 15)?

Solving as shown, x equals -57/7, which simplifies to approximately -8.14.

Can I check my solution for the equation 2/3 (4x + 3) = 1/4 (3x - 15)?

Yes. Substitute x = -57/7 into both sides of the original equation and verify that both sides are equal. This confirms the correctness of the solution.

What steps are involved in solving equations with fractions like 2/3 (4x + 3) = 1/4 (3x - 15)?

The main steps are: eliminate fractions by multiplying both sides by the least common denominator, expand the resulting expressions, gather like terms, and then solve for x.

Is the solution to 2/3 (4x + 3) = 1/4 (3x - 15) unique?

Yes, this linear equation has a unique solution, which is x = -57/7.

How can I simplify solving equations involving multiple fractions like 2/3 (4x + 3) = 1/4 (3x - 15)?

A good approach is to multiply both sides by the least common denominator (12) to clear fractions, then proceed with simplifying and solving for x.

Additional Resources

2/3 (4x + 3) = 1/4 (3x - 15): An In-Depth Investigation into the Algebraic Equation

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Mathematics often presents us with equations that serve as gateways into understanding the relationships between variables. Among these, linear equations like 2/3 (4x + 3) = 1/4 (3x - 15) embody fundamental principles that underpin much of algebraic reasoning. This investigation aims to thoroughly analyze this equation, exploring its structure, solving strategies, and broader implications within the realm of algebra.

Understanding the Equation Structure

Before diving into the solution process, it is crucial to comprehend the components and form of the given equation:

Equation:
(2/3)(4x + 3) = (1/4)(3x - 15)

At first glance, the equation involves fractions multiplied by linear expressions, hinting at the potential complexity due to fractional coefficients. Recognizing the structure, we observe:

- Two products: one involving a coefficient of 2/3 times (4x + 3).
- The other involving 1/4 times (3x - 15).

This configuration is typical in algebra, where distributive multiplication over binomials is used, leading to an equation that can be expanded and simplified.

Key Components Breakdown

1. Fractional Coefficients:
- 2/3 and 1/4 serve as multiplicative constants.
2. Linear Expressions:
- (4x + 3)
- (3x - 15)

3. Variables:
- The unknown x appears linearly, meaning solutions should be straightforward once the equation is simplified.

Step-by-Step Solution Strategy

Solving fractional equations requires careful manipulation to eliminate denominators, so as to avoid errors and facilitate straightforward algebra.

Step 1: Eliminate denominators

To clear fractions, identify the least common denominator (LCD):

- Denominators are 3 and 4.
- LCD of 3 and 4 is 12.

Multiply both sides of the equation by 12:

12 [(2/3)(4x + 3)] = 12 [(1/4)(3x - 15)]

This step ensures all fractional coefficients are eliminated:

- Left side: 12 (2/3) = (12/3) 2 = 4 2 = 8
- Right side: 12 (1/4) = (12/4) 1 = 3

Thus, the equation becomes:

8(4x + 3) = 3(3x - 15)

Step 2: Expand both sides

Distribute the constants:

- Left: 8 4x + 8 3 = 32x + 24
- Right: 3 3x - 3 15 = 9x - 45

The equation simplifies to:

32x + 24 = 9x - 45

Step 3: Isolate the variable term

Subtract 9x from both sides:

32x - 9x + 24 = -45

Which simplifies to:

23x + 24 = -45

Subtract 24 from both sides:

23x = -45 - 24

23x = -69

Step 4: Solve for x

Divide both sides by 23:

x = -69 / 23

Simplify:

x = -3

Verification of the Solution

It's crucial to verify the solution by substituting x = -3 back into the original equation:

Original equation:

(2/3)(4x + 3) = (1/4)(3x - 15)

Substitute x = -3:

Left side:

(2/3) [4 (-3) + 3] = (2/3) (-12 + 3) = (2/3) (-9) = (2/3) -9 = -6

Right side:

(1/4) [3 (-3) - 15] = (1/4) (-9 - 15) = (1/4) -24 = -6

Both sides equal -6, confirming that x = -3 is indeed the solution.

Deeper Insights: Algebraic Principles at Play

This problem, while straightforward, encapsulates several key algebraic concepts that are foundational to more advanced mathematics.

Elimination of Fractions

Multiplying through by the LCD is a standard technique for handling equations with fractional coefficients. It simplifies calculations and reduces potential errors. In this case, the LCD of denominators 3 and 4 (which is 12) was used effectively, illustrating the importance of common denominators in equation manipulation.

Distributive Property

Expanding the products using the distributive property is essential to transforming the equation into a linear form. Recognizing when to distribute and how to combine like terms is crucial for solving equations efficiently.

Linear Equation Solving

The process of isolating the variable—subtracting, then dividing—is a fundamental method in algebra. Understanding the properties of equality ensures the solution remains valid throughout the manipulation process.

Potential Pitfalls and Common Misconceptions

Despite the straightforward nature of this problem, several pitfalls can arise:

- Neglecting to multiply both sides by the LCD: This leads to fractional expressions that can be more cumbersome and prone to errors.
- Incorrect distribution: Failing to distribute correctly can lead to incorrect coefficients.
- Dividing by zero: Although not an issue here, in more complex equations, one must be cautious of values that make denominators zero.
- Verification oversight: Always substituting back is a good practice to confirm solutions.

Broader Implications and Applications

While this particular equation is an elementary example, the techniques employed are applicable across various fields:

- Engineering: Solving for unknowns in circuit equations or mechanical systems involving ratios.
- Economics: Modeling relationships where variables are proportionally related.

- Computer Science: Algorithm design often involves solving similar algebraic problems for optimization.

Understanding how to manipulate and solve equations like 2/3 (4x + 3) = 1/4 (3x - 15) fosters critical thinking and problem-solving skills vital for advanced mathematics and real-world applications.

Conclusion

The equation 2/3 (4x + 3) = 1/4 (3x - 15) exemplifies the elegance and systematic approach inherent in algebra. Through strategic elimination of fractions, expansion, and systematic isolation of the variable, we find the unique solution x = -3. This process not only reinforces core algebraic principles but also highlights the importance of careful calculation, verification, and understanding the structure of equations.

Mathematics, at its core, is about uncovering relationships and patterns—equations like this serve as microcosms of that broader pursuit. Mastering these fundamental techniques lays the groundwork for tackling more complex problems, fostering analytical skills that extend far beyond the classroom.

2 3 4x 3 1 4 3x 15

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