2, 5, 25/2 ,...Find The 9th Term.

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Introduction to the Problem

Understanding sequences and series is fundamental in mathematics, especially in algebra and arithmetic. In this article, we explore a specific sequence: 2, 5, 25/2, ... and aim to find its 9th term. This requires identifying the pattern or rule governing the sequence, which is crucial in calculating any term in the sequence.

Identifying the Pattern of the Sequence

Initial Terms and Observations

The sequence provided is:
- 1st term (a_1) = 2
- 2nd term (a_2) = 5
- 3rd term (a_3) = 25/2

Our goal is to determine the general rule or formula that generates this sequence, allowing us to find the 9th term.

Analyzing the Relationship Between Terms

Let's examine the relationship between consecutive terms:
- From a_1 to a_2:
$a_2 = 5$
- From a_2 to a_3:
$a_3 = 25/2$

Check if the sequence follows a multiplicative pattern:
- $a_2 / a_1 = 5 / 2 = 2.5$
- $a_3 / a_2 = (25/2) / 5 = (25/2) (1/5) = (25/2) (1/5) = (25/2) (1/5) = (25 1) / (2 5) = 25 / 10 = 5 / 2 = 2.5$

This indicates that each term is obtained by multiplying the previous term by 2.5.

Confirming the Pattern

Since the ratio between consecutive terms is constant:
- Common ratio (r) = 2.5

This suggests that the sequence is geometric, with:
- First term (a_1) = 2

- Common ratio (r) = 2.5

Formulating the General Term

Geometric Sequence Formula

For a geometric sequence, the nth term is given by:
$$ a_n = a_1 \times r^{n-1} $$

Applying our identified values:
$$ a_n = 2 \times (2.5)^{n-1} $$

Calculating the 9th Term

To find a_9:
$$ a_9 = 2 \times (2.5)^{8} $$

Now, we compute $(2.5)^8$.

Calculating the 9th Term

Step-by-Step Computation

Calculate $(2.5)^8$:
- First, note that $2.5 = \frac{5}{2}$. Therefore:
$$ (2.5)^8 = \left(\frac{5}{2}\right)^8 = \frac{5^8}{2^8} $$

Calculate numerator and denominator separately:
- 5^8
- 2^8

Compute 5^8:
- $5^1 = 5$
- $5^2 = 25$
- $5^3 = 125$
- $5^4 = 625$
- $5^5 = 3125$
- $5^6 = 15625$
- $5^7 = 78125$
- $5^8 = 390625$

Compute 2^8:
- $2^1 = 2$
- $2^2 = 4$
- $2^3 = 8$
- $2^4 = 16$

- $2^5 = 32$
- $2^6 = 64$
- $2^7 = 128$
- $2^8 = 256$

Thus:
$$ (2.5)^8 = \frac{390625}{256} $$

Now, multiply by 2:
$$ a_9 = 2 \times \frac{390625}{256} = \frac{2 \times 390625}{256} = \frac{781250}{256} $$

Convert to decimal:
$$ a_9 = \frac{781250}{256} \approx 3050.9765625 $$

Therefore, the 9th term is approximately 3050.98.

Alternative Approach: Simplified Exact Value

If an exact fractional form is preferred, the 9th term is:
$$ a_9 = \frac{781250}{256} $$

This fraction can be simplified further by dividing numerator and denominator by their greatest common divisor (GCD). Since 256 = 2^8 and 781250 is divisible by 2:
- Divide numerator and denominator by 2:
$$ \frac{781250 / 2}{256 / 2} = \frac{390625}{128} $$

Check divisibility:
- 390625 is odd, so no further division by 2 is possible.
- 128 is 2^7, and 390625 has prime factors of 5, so the fraction is in lowest terms.

Final exact value:
$$ a_9 = \frac{390625}{128} $$

Approximate decimal value: 3050.98.

Summary

- The sequence is geometric with initial term $a_1 = 2$ and common ratio $r = 2.5$.
- The nth term is given by $a_n = 2 \times (2.5)^{n-1}$.
- The 9th term is $a_9 = 2 \times (2.5)^8 = \frac{781250}{256} \approx 3050.98$.

Conclusion

By recognizing the pattern as a geometric sequence, we efficiently derived the general formula and computed the 9th term. Such methods are vital in solving problems involving sequences, enabling quick calculations once the underlying pattern is understood. This approach can be extended to find any term in the sequence, illustrating the power of algebraic formulas in sequence analysis.

Frequently Asked Questions

What is the common pattern in the sequence 2, 5, 25/2, ...?

The sequence alternates between multiplying by 2 and dividing by 2; specifically, $2 \times 2 = 4$ (but given as 5, indicating a possible typo), or more accurately, the pattern appears to involve multiplying by 2 and then dividing by 1, but more context is needed. Assuming the sequence is 2, 5, 25/2, ... it suggests a pattern involving multiplying by 2. Clarification is needed for the exact rule.

How do I find the 9th term of the sequence 2, 5, 25/2, ...?

Identify the pattern or formula governing the sequence. If the sequence is geometric or arithmetic, use the corresponding formula to find the 9th term. For this sequence, more information about the pattern is needed to derive an explicit formula.

Is the sequence 2, 5, 25/2, ... an arithmetic or geometric sequence?

The sequence appears neither purely arithmetic nor geometric. It involves multiplication and division in a pattern. Clarification of the rule is necessary to classify it accurately.

What is the common ratio or difference in the sequence 2, 5, 25/2, ...?

Calculating the ratios: $5/2 \approx 2.5$ and $(25/2)/5 = 2.5$, indicating the sequence might follow a pattern with a ratio of 2.5 after the first term. Confirming the pattern is essential for an accurate formula.

Can I use a recursive formula to find the 9th term of this sequence?

Yes, if the pattern between terms is known, a recursive formula can be established to find subsequent terms up to the 9th. However, the specific recursive rule depends on the pattern identified.

What is the explicit formula for the nth term of the sequence 2, 5, 25/2, ...?

Without a clear pattern, it's difficult to determine the explicit formula. More terms or rules are needed to derive a general expression for the nth term.

How do I verify the pattern in this sequence to find the 9th term?

Examine the ratios or differences between terms to identify a pattern. Once the pattern is confirmed, apply the formula or recursive rule to find the 9th term.

Are there common sequences similar to 2, 5, 25/2, ... that can help me find the 9th term?

Sequences like geometric or arithmetic progressions are common. Comparing this sequence to known patterns can help, but first, the exact pattern needs to be clarified.

What steps should I follow to find the 9th term of any sequence?

First, identify the pattern or rule governing the sequence. Then, derive the explicit or recursive formula. Finally, substitute n=9 into the formula to find the 9th term.

Additional Resources

Find The 9th Term in the Sequence 2, 5, 25/2, ...: An Investigative Analysis

Understanding the behavior and pattern of sequences is a foundational aspect of mathematical exploration. The problem at hand—"Find the 9th term in the sequence 2, 5, 25/2, ..."—may seem straightforward at first glance, yet it invites a nuanced examination into the nature of sequences, their classifications, and the methods of deducing general terms. This article delves into an investigative analysis of this sequence, exploring possible patterns, deriving the explicit formula, and contextualizing the significance of such problems within mathematical education and research.

Deciphering the Sequence: Initial Observations

The sequence provided is:

- Term 1: 2
- Term 2: 5
- Term 3: 25/2

Our goal is to find the 9th term: T_9.

Before attempting to derive the general formula, it is essential to analyze the pattern of the initial terms.

Listing the Known Terms

Term Number	Term Value	Decimal Approximation
1	2	2.0
2	5	5.0

3	25/2	12.5

Observation reveals the sequence is increasing rapidly; the jump from the first to the second term is 3, and from the second to the third is 7.5. The pattern is not immediately linear or geometric; thus, alternative methods are needed.

Initial Hypotheses

Possible patterns could include:

- Arithmetic progression: unlikely, as the differences are inconsistent (3 and 7.5).
- Geometric progression: check ratio between terms.
- Recursive relations: perhaps each term depends on previous terms.
- Combination of operations: such as multiplication or division combined with addition/subtraction.

Examining Ratios and Differences

Ratios between consecutive terms

Calculate:

- $\frac{T_2}{T_1} = \frac{5}{2} = 2.5$
- $\frac{T_3}{T_2} = \frac{25/2}{5} = \frac{12.5}{5} = 2.5$

This is intriguing: the ratio between the second and the first term is 2.5, and between the third and the second term is also 2.5.

Implication

If the ratio between consecutive terms starting from the second term remains constant at 2.5, then the sequence appears to be geometric from the second term onward:

$$
T_2 = T_1 \times 2.5
$$
$$
T_3 = T_2 \times 2.5
$$

Assuming this pattern continues, then:

$$
T_n = T_1 \times (2.5)^{n-1}
$$

for $n \geq 1$.

Let's verify this with the third term:

$$
T_3 = 2 \times (2.5)^{3-1} = 2 \times (2.5)^2 = 2 \times 6.25 = 12.5
$$

which matches the given term $25/2 = 12.5$.

Similarly, check for the second term:

$$
T_2 = 2 \times (2.5)^{2-1} = 2 \times 2.5 = 5
$$

which aligns with the known term.

Thus, the pattern for the sequence is:

$$
T_n = 2 \times (2.5)^{n-1}
$$

This confirms that the sequence is geometric starting from the first term, with the common ratio $r = 2.5$.

Deriving the Explicit Formula

Given the pattern observed, the explicit formula for the n-th term is:

$$
T_n = a \times r^{n-1}
$$

where:

- $a = 2$ (the first term),
- $r = 2.5$.

Therefore:

\[

$$
T_n = 2 \times (2.5)^{n-1}
$$

This formula allows calculation of any term directly without recursive dependency.

Verification of the Formula

We've already verified for $n = 2, 3$. Let's check for $n = 4$:

$$
T_4 = 2 \times (2.5)^{3} = 2 \times 15.625 = 31.25
$$

which would be the next term in the sequence if continued.

Calculating the 9th Term

Applying the explicit formula:

$$
T_9 = 2 \times (2.5)^{8}
$$

Calculating $(2.5)^8$:

- $(2.5)^2 = 6.25$
- $(2.5)^4 = (6.25)^2 = 39.0625$
- $(2.5)^8 = (39.0625)^2$

Compute:

$$
(39.0625)^2 = 39.0625 \times 39.0625
$$

Calculating:

$$
39.0625 \times 39.0625 \approx 1524.90234375
$$

Therefore:

$$
T_9 = 2 \times 1524.90234375 = 3049.8046875
$$

\]

Result:

\[
\boxed{T_9 \approx 3049.80}
\]

or, in fractional form, since \((2.5)^8 = \frac{5^8}{2^8} = \frac{390625}{256} \):

\[
T_9 = 2 \times \frac{390625}{256} = \frac{780,250}{256}
\]

which simplifies to:

\[
T_9 = \frac{780,250}{256} \approx 3049.80
\]

Contextual Significance and Broader Implications

This analysis underscores the importance of pattern recognition in sequence analysis. Recognizing geometric patterns allows for the straightforward derivation of explicit formulas, drastically simplifying the process of finding specific terms. Such skills are fundamental in various domains:

- Mathematics Education: Teaching students to identify patterns enhances problem-solving abilities.
- Computational Mathematics: Efficient algorithms often rely on explicit formulas rather than recursive calculations.
- Applied Sciences: Modeling exponential growth processes, such as population dynamics, radioactive decay, or financial investments, often involves geometric sequences.

Furthermore, this investigation demonstrates the importance of verifying assumptions; initial hypotheses should be tested against known data points, and ratios or differences can reveal underlying structures.

Conclusion

The sequence \(2, 5, 25/2, ... \) is a geometric sequence with the explicit formula:

\[
T_n = 2 \times (2.5)^{n-1}

\]

Using this formula, the 9th term is approximately 3049.80 (exactly \(\frac{780,250}{256} \)). Through meticulous analysis, pattern recognition, and algebraic verification, we've uncovered the sequence's nature, transforming an initial challenge into a comprehensive mathematical understanding.

This exploration exemplifies the analytical rigor necessary for problem-solving in mathematics and highlights the elegance of geometric sequences in modeling exponential growth phenomena.

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