2/5x - 1/2 = 1/3 + 5/6x

2/5x - 1/2 = 1/3 + 5/6x

Solving algebraic equations involving fractions can seem daunting at first, especially when multiple fractions with different denominators are involved. Understanding how to approach and simplify these types of equations is essential for mastering algebra and progressing to more complex mathematics. In this article, we will explore how to solve the equation 2/5x - 1/2 = 1/3 + 5/6x step-by-step, providing detailed explanations, strategies, and tips to help you become confident in handling similar fractional equations.

Understanding the Equation

Before diving into solving the equation, let's analyze its structure:

Equation: 2/5x - 1/2 = 1/3 + 5/6x

This equation contains:

- Variables with fractional coefficients (2/5x and 5/6x)
- Constant fractions (-1/2 and +1/3)
- Multiple fractions with different denominators

The goal is to find the value of x that satisfies this equation.

Step-by-Step Approach to Solving the Equation

Solving fractional equations generally involves similar steps:

1. Eliminate the fractions by finding a common denominator
2. Combine like terms
3. Isolate the variable
4. Solve for the variable
5. Verify the solution (optional but recommended)

Let's apply these steps to our specific equation.

Step 1: Find the Least Common Denominator (LCD)

The denominators in the equation are 5, 2, 3, and 6. To eliminate fractions, find the LCD of these denominators.

Denominators: 5, 2, 3, 6

Prime factorization:

- 5 (prime)
- 2 (prime)
- 3 (prime)
- 6 = 2 × 3

LCD calculation:

- Take the highest powers of each prime from all denominators:
- 2 (from 2 and 6)
- 3 (from 3 and 6)
- 5 (from 5)

LCD = 2 × 3 × 5 = 30

Step 2: Multiply the entire equation by the LCD to clear fractions

Multiply both sides of the equation by 30:

$$
30 \times \left(\frac{2}{5}x - \frac{1}{2} \right) = 30 \times \left(\frac{1}{3} + \frac{5}{6}x \right)
$$

Distribute 30:

$$
30 \times \frac{2}{5}x - 30 \times \frac{1}{2} = 30 \times \frac{1}{3} + 30 \times \frac{5}{6}x
$$

Simplify each term:

- $30 \times \frac{2}{5}x = (30/5) \times 2x = 6 \times 2x = 12x$
- $30 \times \frac{1}{2} = 15$
- $30 \times \frac{1}{3} = 10$
- $30 \times \frac{5}{6}x = (30/6) \times 5x = 5 \times 5x = 25x$

So, the equation becomes:

\[
12x - 15 = 10 + 25x
\]

Step 3: Rearrange the equation to gather like terms

Bring all terms involving x to one side and constants to the other side:

\[
12x - 25x = 10 + 15
\]

\[
-13x = 25
\]

Step 4: Solve for x

Divide both sides by -13:

\[
x = \frac{25}{-13} = -\frac{25}{13}
\]

Solution:

\[
x = -\frac{25}{13}
\]

Step 5: Verify the solution (Optional but Recommended)

Substitute x = -25/13 back into the original equation to verify correctness.

Original equation:

\[

\frac{2}{5}x - \frac{1}{2} = \frac{1}{3} + \frac{5}{6}x
\]

Calculations:

- \(\frac{2}{5} \times -\frac{25}{13} = \frac{2 \times -25}{5 \times 13} = \frac{-50}{65} = -\frac{10}{13}\)
- \(\frac{5}{6} \times -\frac{25}{13} = \frac{5 \times -25}{6 \times 13} = \frac{-125}{78}\)

Now, evaluate each side:

Left side:

\[
-\frac{10}{13} - \frac{1}{2}
\]

Express \(-\frac{1}{2}\) as \(-\frac{13}{26}\), and \(-\frac{10}{13}\) as \(-\frac{20}{26}\):

\[
-\frac{20}{26} - \frac{13}{26} = -\frac{33}{26}
\]

Right side:

\[
\frac{1}{3} + \frac{-125}{78}
\]

Express \(\frac{1}{3}\) as \(\frac{26}{78}\):

\[
\frac{26}{78} - \frac{125}{78} = -\frac{99}{78}
\]

Simplify \(-\frac{99}{78}\):

\[
-\frac{99}{78} = -\frac{33}{26}
\]

Since both sides equal \(-\frac{33}{26}\), the solution is verified.

Additional Tips for Solving Fractional Equations

- Always find the LCD first: Eliminating fractions makes the algebra much easier.
- Be careful with signs: When multiplying or dividing both sides, keep track of positive and negative

signs.
- Simplify fractions at each step: Simplifying helps avoid errors and makes calculations clearer.
- Check your solution: Substituting back into the original equation confirms accuracy.

Common Mistakes to Avoid

- Forgetting to multiply all terms by the LCD: Missing this step can lead to incorrect solutions.
- Incorrectly simplifying fractions: Always reduce fractions to their simplest form.
- Mixing up signs: Negative signs can easily cause errors if not handled carefully.
- Neglecting to verify the solution: Always substitute your answer back into the original equation.

Summary

Solving the fractional equation 2/5x - 1/2 = 1/3 + 5/6x involves a systematic approach:

- Find the LCD (30) to clear fractions.
- Multiply the entire equation by the LCD.
- Simplify and combine like terms.
- Isolate the variable x.
- Solve for x.
- Verify the solution for accuracy.

Understanding these steps enhances your algebra skills and prepares you for tackling more complex equations involving fractions, variables, and constants.

Conclusion

Mastering the process of solving fractional equations like 2/5x - 1/2 = 1/3 + 5/6x is a fundamental skill in algebra. With practice, you'll become more efficient at identifying common denominators, simplifying equations, and verifying your solutions. Remember, patience and careful calculation are key. Keep practicing with different equations to build confidence and deepen your understanding of algebraic principles.

Further Resources

- Algebra textbooks and online courses
- Interactive algebra problem solvers
- Practice worksheets on solving equations with fractions
- Video tutorials explaining similar algebraic concepts

By consistently practicing and applying these strategies, you'll improve your problem-solving skills and excel in algebra and beyond.

Frequently Asked Questions

How do I solve the equation 2/5x - 1/2 = 1/3 + 5/6x for x?

First, identify the variable terms and constants. Bring all x terms to one side and constants to the other: 2/5x - 5/6x = 1/3 + 1/2. Find common denominators to combine like terms: (12/30x - 25/30x) = (2/6 + 3/6). Simplify: (-13/30x) = (5/6). Then, solve for x by multiplying both sides by the reciprocal: x = (5/6) (-30/13) = -150/78, which simplifies to -75/39 or approximately -1.923.

What are the steps to isolate x in the equation 2/5x - 1/2 = 1/3 + 5/6x?

The steps are: 1) Move all x terms to one side: 2/5x - 5/6x = 1/3 + 1/2. 2) Find common denominators to combine x terms: (12/30x - 25/30x). 3) Combine constants on the right: 1/3 + 1/2 = 2/6 + 3/6 = 5/6. 4) Simplify the x terms: (-13/30x) = 5/6. 5) Solve for x by multiplying both sides by the reciprocal of -13/30: x = (5/6) (-30/13).

Is the equation 2/5x - 1/2 = 1/3 + 5/6x linear in x?

Yes, the equation is linear in x because it can be written in the form ax + b = cx + d, with all x terms on one side and constants on the other. The x appears to the first power only, making it a linear equation.

Can I use a calculator to solve 2/5x - 1/2 = 1/3 + 5/6x?

Yes, using a calculator can help with calculating common denominators, combining fractions, and performing multiplication or division steps accurately. It simplifies solving the equation efficiently, especially when dealing with fractions.

What is the final solution for x in the equation 2/5x - 1/2 = 1/3 + 5/6x?

The solution is x = -150/78, which simplifies to approximately -1.923. You can also express it as a simplified fraction: x = -75/39, or approximately -1.923.

Additional Resources

2/5x - 1/2 = 1/3 + 5/6x

In the realm of algebra, equations serve as the foundational language through which we understand relationships between variables. Among the myriad of equations students and professionals encounter, rational equations—those involving fractions—stand out due to their unique complexity and the necessity for careful manipulation. One such equation, 2/5x - 1/2 = 1/3 + 5/6x, exemplifies the challenges and techniques involved in solving for an unknown variable, typically denoted as x. This article explores this equation in depth, breaking down the steps involved in solving it, elucidating the underlying principles, and presenting practical insights for learners aiming to master rational equations.

Understanding the Equation: Components and Challenges

At first glance, the equation:

2/5x - 1/2 = 1/3 + 5/6x

may seem straightforward, but beneath its surface lie several critical features:

- Fractional Coefficients: The presence of multiple fractions with different denominators.
- Variable Terms in Fractions: The variable x appears within fractions, complicating straightforward addition or subtraction.
- Mixed Operations: Addition, subtraction, and the mixture of constants and variable terms.

These elements necessitate a systematic approach, emphasizing the importance of clarity and precision in algebraic manipulation.

Step 1: Clarify the Equation and Identify Goals

The primary goal is to solve for x, i.e., find its value that satisfies the equation. To do this effectively, the first step is to clear the fractions to simplify the algebraic manipulation.

Why clear fractions?
Fractions can complicate the process because denominators are different. Eliminating them transforms the equation into a form that's easier to work with—namely, a linear equation with only integers or simplified fractions.

Step 2: Find the Least Common Denominator (LCD)

To eliminate the fractions, determine the least common denominator (LCD) of all the denominators involved:

- Denominators are 2, 3, 5, and 6.

Calculating the LCD:
- Prime factors:
- 2 = 2
- 3 = 3
- 5 = 5
- 6 = 2 × 3

The LCD must include each prime factor at its highest power:
- 2 (from 2 and 6),
- 3 (from 3 and 6),
- 5 (from 5).

Thus, LCD = 2 × 3 × 5 = 30.

Step 3: Multiply Both Sides by the LCD to Clear Fractions

Applying the LCD to each term:

- Multiply entire equation by 30:

$$
30 \times \left(\frac{2}{5}x - \frac{1}{2} \right) = 30 \times \left(\frac{1}{3} + \frac{5}{6}x \right)
$$

Distribute:

$$
30 \times \frac{2}{5}x - 30 \times \frac{1}{2} = 30 \times \frac{1}{3} + 30 \times \frac{5}{6}x
$$

Calculate each term:

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- $30 \times \frac{1}{2} = 15$
- $30 \times \frac{1}{3} = 10$
- $30 \times \frac{5}{6}x = (30/6) \times 5x = 5 \times 5x = 25x$

Now, rewrite the equation:

$$
12x - 15 = 10 + 25x
$$

Step 4: Rearrange the Equation to Isolate Variable Terms

Bring all x terms to one side and constants to the other:

$$
12x - 25x = 10 + 15
$$

Simplify both sides:

$$
-13x = 25
$$

Step 5: Solve for x

Divide both sides by -13:

$$
x = \frac{25}{-13} = -\frac{25}{13}
$$

Solution:

$$
x = -\frac{25}{13}
$$

This fraction is already in its simplest form, providing an exact value for x.

Practical Insights and Common Pitfalls

While the above steps seem straightforward, several common pitfalls can trip up students or practitioners:

- Forgetting to multiply all terms:
When clearing fractions, it's crucial to multiply every term by the LCD, not just some of them.

- Miscalculating the LCD:
Errors in finding the LCD can lead to incorrect solutions. Double-check prime factorization.

- Sign errors:
When moving terms across the equals sign, remember to reverse signs appropriately.

- Not checking the solution:
Substituting the value back into the original equation confirms whether the solution satisfies the equation, especially important when dealing with fractions.

Step 6: Verify the Solution

Verification is an essential step in algebraic problem-solving:

Original equation:

\[
\frac{2}{5}x - \frac{1}{2} = \frac{1}{3} + \frac{5}{6}x
\]

Substitute \(x = -\frac{25}{13}\):

Calculate the left side:

\[
\frac{2}{5} \times -\frac{25}{13} - \frac{1}{2} = \left(\frac{2 \times -25}{5 \times 13} \right) - \frac{1}{2} = \left(\frac{-50}{65} \right) - \frac{1}{2}
\]

Simplify:

\[
\frac{-50}{65} = -\frac{10}{13}
\]

Express \(\frac{1}{2}\) with denominator 13:

\[
\frac{1}{2} = \frac{13}{26} \text{ or better, } \frac{6.5}{13}
\]

But to combine with the other fraction, convert \(\frac{1}{2}\) to denominator 13:

\[
\frac{1}{2} = \frac{13}{26} = \frac{6.5}{13}
\]

Better to convert to denominator 13:

\[
\frac{1}{2} = \frac{13}{26} \Rightarrow \text{but since } \frac{-10}{13} \text{ is given, convert } \frac{1}{2} \text{ as } \frac{13}{26} = \frac{6.5}{13}
\]

Alternatively, write \(\frac{1}{2} = \frac{13}{26}\), then:

\[
-\frac{10}{13} - \frac{13}{26} = -\frac{20}{26} - \frac{13}{26} = -\frac{33}{26}
\]

Calculate the right side:

\[

$$
\frac{1}{3} + \frac{5}{6} \times -\frac{25}{13} = \frac{1}{3} + \left(\frac{5 \times -25}{6 \times 13} \right) = \frac{1}{3} - \frac{125}{78}
$$

Express $\frac{1}{3}$ with denominator 78:

$$
\frac{1}{3} = \frac{26}{78}
$$

Thus:

$$
\frac{26}{78} - \frac{125}{78} = -\frac{99}{78} = -\frac{33}{26}
$$

Both sides evaluate to $-\frac{33}{26}$, confirming the solution's correctness.

Broader Implications and Applications

While this particular equation may seem isolated, the techniques involved—finding the least common denominator, multiplying through to clear fractions, and carefully isolating the variable—are fundamental skills in algebra and higher mathematics.

Real-world applications include:

- Engineering: Calculating component values in circuit design that involve fractional relationships.
- Finance: Solving equations involving ratios and fractions for interest calculations.
- Data Analysis: Manipulating ratios and proportions to derive meaningful insights.

Mastering the solving process enhances problem-solving skills across disciplines, fostering logical reasoning and computational accuracy.

Final Thoughts: The Art of Systematic Algebra

Solving equations like 2/5x - 1/2 = 1/3 + 5/6x demonstrates the importance of a methodical approach—identifying denominators, eliminating fractions, and carefully performing algebraic operations. While fractions introduce an added layer of complexity, they also provide an excellent platform for practicing precision and logical reasoning.

For students and professionals alike, developing fluency in these techniques is invaluable. With practice, what initially appears complicated becomes intuitive, empowering learners to tackle increasingly sophisticated mathematical challenges with confidence.

Whether you're preparing for exams, applying algebra in your career, or simply seeking to sharpen your mathematical toolkit, understanding the step-by-step process behind solving rational equations is a vital achievement—one

2 5x 1 2 1 3 5 6x

2 5x 1 2 1 3 5 6x

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