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Quadratic equations are fundamental in algebra, appearing frequently in various fields such as physics, engineering, economics, and everyday problem-solving. Among these, equations like $2x^2 + x - 6 = 0$ are classic examples that serve as excellent practice for understanding the quadratic formula, factoring methods, and completing the square. If you've encountered the quadratic equation $2x^2 + x - 6 = 0$ and wondered about its solutions, this comprehensive guide will walk you through the process of solving it step-by-step, exploring various methods, and understanding the significance of its solutions.

Understanding the Quadratic Equation $2x^2 + x - 6 = 0$

A quadratic equation is any second-degree polynomial equation of the form $ax^2 + bx + c = 0$, where a , b , and c are constants, and $a \neq 0$. The equation in question, $2x^2 + x - 6 = 0$, is a quadratic because the highest power of x is 2.

Key components:

- Quadratic coefficient (a): 2
- Linear coefficient (b): 1
- Constant term (c): -6

Understanding these components is essential because they determine which solving method to use and how the solutions, or roots, are calculated.

Methods to Solve $2x^2 + x - 6 = 0$

Different methods exist for solving quadratic equations. The most common are factoring, using the quadratic formula, and completing the square. Each method has its advantages depending on the specific form of the quadratic equation.

1. Factoring Method

Factoring involves expressing the quadratic polynomial as a product of two binomials. This method is quick and straightforward if the quadratic factors nicely.

Steps to factor $2x^2 + x - 6$:

- Multiply the coefficient of x^2 (2) and the constant term (-6): $2 \cdot -6 = -12$
- Find two numbers that multiply to -12 and add to the middle coefficient (1)

- Possible pairs: (4, -3), (-4, 3), (6, -2), (-6, 2)
- The pair that sums to 1 is 4 and -3 because $4 + (-3) = 1$

- Rewrite the middle term using these numbers:

$$2x^2 + 4x - 3x - 6 = 0$$

- Group terms:

$$(2x^2 + 4x) + (-3x - 6) = 0$$

- Factor each group:

$$2x(x + 2) - 3(x + 2) = 0$$

- Factor out the common binomial:

$$(x + 2)(2x - 3) = 0$$

Solutions from factoring:

Set each factor equal to zero:

$$- x + 2 = 0 \rightarrow x = -2$$

$$- 2x - 3 = 0 \rightarrow x = 3/2$$

Result: The solutions are $x = -2$ and $x = 1.5$ (or $3/2$).

2. Quadratic Formula Method

The quadratic formula is a universal method applicable to all quadratic equations:

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Applying the quadratic formula to $2x^2 + x - 6 = 0$:

- Identify coefficients: $a = 2$, $b = 1$, $c = -6$

- Calculate the discriminant (D):

$$D = b^2 - 4ac = 1^2 - 4 \times 2 \times (-6) = 1 + 48 = 49$$

- Since $D > 0$, there are two real solutions.

- Find the solutions:

$$x = \frac{-1 \pm \sqrt{49}}{2 \times 2} = \frac{-1 \pm 7}{4}$$

- Calculate each solution:

- Using the plus sign:

$$x = \frac{-1 + 7}{4} = \frac{6}{4} = \frac{3}{2}$$

- Using the minus sign:

$$x = \frac{-1 - 7}{4} = \frac{-8}{4} = -2$$

Solutions: $x = 3/2$ and $x = -2$, confirming the factoring method results.

3. Completing the Square

Completing the square rewrites the quadratic into a perfect square trinomial. Although more algebraically intensive, it provides insight into the structure of quadratic equations.

Steps for $2x^2 + x - 6 = 0$:

- Divide the entire equation by 2 to simplify:

$$\left[x^2 + \frac{1}{2}x - 3 = 0 \right]$$

- Move the constant to the other side:

$$\left[x^2 + \frac{1}{2}x = 3 \right]$$

- Take half of the coefficient of x: $\left(\frac{1/2}{2} = \frac{1}{4}\right)$

- Square it: $\left(\left(\frac{1}{4}\right)^2 = \frac{1}{16}\right)$

- Add $\left(\frac{1}{16}\right)$ to both sides:

$$\left[x^2 + \frac{1}{2}x + \frac{1}{16} = 3 + \frac{1}{16} \right]$$

- Rewrite the left side as a perfect square:

$$\left[\left(x + \frac{1}{4}\right)^2 = \frac{48}{16} + \frac{1}{16} = \frac{49}{16} \right]$$

- Take square root of both sides:

$$\left[x + \frac{1}{4} = \pm \frac{7}{4} \right]$$

- Solve for x:

$$\left(x = -\frac{1}{4} + \frac{7}{4} = \frac{6}{4} = \frac{3}{2} \right)$$

$$\left(x = -\frac{1}{4} - \frac{7}{4} = -\frac{8}{4} = -2 \right)$$

Again, solutions are $x = -2$ and $x = 3/2$.

Summary of Solutions for $2x^2 + x - 6 = 0$

- Solutions: $x = -2$ and $x = 1.5$ (or $3/2$)

- Methods used: Factoring, quadratic formula, completing the square

- Discriminant (D): 49, indicating two real solutions

Significance of Quadratic Solutions

Understanding the solutions to quadratic equations like $2x^2 + x - 6 = 0$ is vital because:

- They help determine the roots or x-intercepts of quadratic functions, which are essential in graphing parabolas.

- They provide insights into the nature of the solutions (real vs. complex) based on the discriminant.

- They are used in various applications such as projectile motion, optimization problems,

and financial calculations.

Applications of Solving Quadratic Equations

Quadratic equations appear in numerous real-world scenarios, including:

- Physics: Calculating the trajectory of projectiles
- Engineering: Designing systems with quadratic relationships
- Economics: Modeling profit maximization or cost minimization
- Biology: Population modeling with quadratic growth factors
- Computer Science: Algorithms involving quadratic functions

Key Points to Remember When Solving $2x^2 + x - 6 = 0$

- Always identify coefficients correctly: a, b, c
- Calculate the discriminant to determine the number and type of solutions
- Choose the appropriate solving method based on the quadratic form
- Cross-verify solutions by substituting back into the original equation
- Simplify solutions as fractions or decimals for clarity

Conclusion

Solving quadratic equations like $2x^2 + x - 6 = 0$ is a fundamental skill in algebra that opens the door to understanding various mathematical concepts and real-world applications. Whether through factoring, the quadratic formula, or completing the square, mastering these methods ensures you can confidently find solutions to any quadratic equation. Remember, the key steps involve identifying the coefficients, calculating the discriminant, and choosing the method best suited to the specific problem. With practice, solving quadratics will become an intuitive part of your mathematical toolkit, helping you analyze and solve complex problems efficiently.

FAQs about $2x^2 + x - 6 = 0$

1. What are the solutions to $2x^2 + x - 6 = 0$?

Answer: The solutions are $x = -2$ and $x = 1.5$ (or $3/2$).