$-3(x^2 - 4x + 5)$ Pls Show Work

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Understanding how to work with algebraic expressions is fundamental in mathematics, especially when dealing with polynomials and their transformations. The expression $-3(x^2 - 4x + 5)$ is a quadratic expression multiplied by a scalar, and simplifying or expanding such expressions is a common task in algebra. In this comprehensive guide, we will delve into the step-by-step process of analyzing, expanding, and understanding the expression $-3(x^2 - 4x + 5)$. Whether you're a student preparing for exams or someone looking to sharpen your algebra skills, this detailed explanation will provide clarity and confidence in handling similar expressions.

Breaking Down the Expression $-3(x^2 - 4x + 5)$

Before diving into the computations, it's essential to understand the components of this expression:

Components of the Expression

- **Coefficient:** -3, which is a scalar multiplying the entire quadratic expression.
- **Quadratic Expression Inside the Parentheses:** $x^2 - 4x + 5$, which is a quadratic polynomial.

The goal is to apply the distributive property to eliminate the parentheses and simplify the expression into a standard quadratic form.

Step-by-Step Process to Show Work for $-3(x^2 - 4x + 5)$

Let's explore each step involved in expanding this expression thoroughly.

Step 1: Understand the Distributive Property

The distributive property states that for any numbers or expressions a, b, and c:

$$a(b + c) = ab + ac$$

In this context, the scalar -3 is distributed across each term inside the parentheses:

$$ -3 \times x^2, \quad -3 \times (-4x), \quad -3 \times 5 $$

Step 2: Multiply Each Term by -3

Applying the distributive property:

- Multiply the first term:
$$ -3 \times x^2 = -3x^2 $$
- Multiply the second term:
$$ -3 \times (-4x) = +12x $$
- Multiply the third term:
$$ -3 \times 5 = -15 $$

Step 3: Write the Expanded Expression

Combine all the products to write the simplified form:

$$ -3x^2 + 12x - 15 $$

This is the fully expanded form of the original expression.

Understanding the Resulting Expression

The expression $-3x^2 + 12x - 15$ is a quadratic polynomial in standard form. Let's analyze its components:

Coefficients and Constants

- **Quadratic coefficient:** -3, indicating the parabola opens downward.
- **Linear coefficient:** 12, which affects the slope and direction of the parabola's tilt.
- **Constant term:** -15, which is the y-intercept when $x=0$.

Key Features of the Quadratic Expression

1. **Vertex:** The highest or lowest point of the parabola, found using the vertex formula.
2. **Axis of symmetry:** The vertical line passing through the vertex.
3. **Roots or zeros:** Values of x where the quadratic equals zero.

We'll explore these features further to deepen your understanding.

Finding the Vertex of the Quadratic $-3x^2 + 12x - 15$

The vertex form of a quadratic is helpful in graphing and understanding the parabola's shape.

Step 1: Use the Vertex Formula

The x-coordinate of the vertex is given by:

$$x_{v} = -\frac{b}{2a}$$

where $a = -3$ and $b = 12$.

Calculating:

$$x_{v} = -\frac{12}{2 \times (-3)} = -\frac{12}{-6} = 2$$

Step 2: Find the y-coordinate of the vertex

Substitute $x=2$ into the quadratic:

$$y = -3(2)^2 + 12(2) - 15$$
$$y = -3(4) + 24 - 15$$
$$y = -12 + 24 - 15 = -3$$

Result:

The vertex is at $(2, -3)$.

Determining the Roots of the Quadratic

Roots are the solutions to the quadratic equation:

$$ -3x^2 + 12x - 15 = 0 $$

Applying the quadratic formula:

$$ x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} $$

where $a = -3$, $b = 12$, and $c = -15$.

Step 1: Calculate the discriminant

$$ D = b^2 - 4ac = (12)^2 - 4 \times (-3) \times (-15) $$
$$ D = 144 - 4 \times (-3) \times (-15) $$
$$ D = 144 - 180 = -36 $$

Step 2: Interpret the discriminant

Since $D = -36 < 0$, there are no real roots. The quadratic has complex roots.

Step 3: Find the complex roots

Use the quadratic formula:

$$ x = \frac{-12 \pm \sqrt{-36}}{2 \times -3} $$

Recall that:

$$ \sqrt{-36} = 6i $$

Thus,

$$ x = \frac{-12 \pm 6i}{-6} $$

Simplify numerator:

$$ x = \frac{-12}{-6} \pm \frac{6i}{-6} $$
$$ x = 2 \mp i $$

Final roots:

$$ x = 2 + i \quad \text{and} \quad x = 2 - i $$

These are complex conjugate roots, indicating the parabola does not cross the x-axis.

Graphing the Quadratic $-3x^2 + 12x - 15$

Understanding the graph of the quadratic provides visual insight into its behavior.

Features to note:

- **Direction:** Since $a = -3 < 0$, the parabola opens downward.
- **Vertex:** Located at $(2, -3)$, representing the maximum point.
- **Y-intercept:** When $x=0$, $y = -15$.
- **Complex roots:** No x-intercepts, as roots are imaginary.

Plotting tips:

1. Plot the vertex at $(2, -3)$.
2. Mark the y-intercept at $(0, -15)$.
3. Since the roots are complex, the parabola does not cross the x-axis.
4. Use symmetry about the vertical line $x=2$ to sketch the parabola.

Applications and Importance of Understanding $-3(x^2 - 4x + 5)$

Studying such quadratic expressions and their transformations has numerous practical applications:

Real-world Applications

- **Physics:** Parabolic trajectories, such as projectile motion, are modeled using quadratic equations.
- **Engineering:** Structural design often involves quadratic functions to analyze stresses and loads.
- **Economics:** Quadratic models describe profit maximization or cost functions.
- **Mathematics Education:** Mastering expansion, factoring, and graphing quadratic expressions enhances problem-solving skills.

Importance of Showing Work

- Demonstrates understanding of algebraic properties.
- Ensures clarity and correctness in solutions.
- Prepares students for more advanced topics like calculus.
- Facilitates troubleshooting and learning from mistakes.

Summary and Key Takeaways

To summarize, the process of handling the expression -3(x^2 - 4x + 5) involves:

1. Applying the distributive property to expand the expression.
2. Multiplying each term inside the parentheses by -3.
3. Combining like terms to write the quadratic in standard form.
4. Analyzing the quadratic's features: vertex, roots, axis of symmetry.

Frequently Asked Questions

How do I expand the expression -3(x^2 - 4x + 5)?

To expand, distribute -3 to each term inside the parentheses: $-3 \times x^2 = -3x^2$, $-3 \times (-4x) = +12x$, and $-3 \times 5 = -15$. So, the expanded form is $-3x^2 + 12x - 15$.

What is the simplified form of -3(x^2 - 4x + 5)?

The simplified form after distribution is $-3x^2 + 12x - 15$.

Can you show step-by-step work for expanding -3(x^2 - 4x + 5)?

Yes. Step 1: Write the expression: $-3(x^2 - 4x + 5)$. Step 2: Distribute -3 to each term: $-3 \times x^2 = -3x^2$, $-3 \times (-4x) = +12x$, $-3 \times 5 = -15$. Step 3: Combine the results: $-3x^2 + 12x - 15$.

What is the degree of the polynomial after expansion?

The degree of the polynomial is 2, since the highest power of x is x^2.

How do I factor the expression -3x^2 + 12x - 15?

First, factor out the common factor -3: $-3(x^2 - 4x + 5)$. Since $x^2 - 4x + 5$ does not factor nicely over real numbers, the factored form is -3 times the quadratic expression.

What is the vertex form of this quadratic expression?

First, rewrite the quadratic as $-3(x^2 - 4x + 5)$. Complete the square inside the parentheses: $x^2 - 4x + 4 - 4 + 5 = (x - 2)^2 + 1$. So, the vertex form is $-3(x - 2)^2 - 3$.

What are the roots of the quadratic expression -3x^2 + 12x - 15?

Set the expression equal to zero: $-3x^2 + 12x - 15 = 0$. Divide through by -3: $x^2 - 4x + 5 = 0$. The discriminant is $(-4)^2 - 4\times1\times5 = 16 - 20 = -4$, which is negative, so there are no real roots.

How can I graph the parabola represented by $-3(x^2 - 4x + 5)$?

First, find the vertex using the vertex form: $-3(x - 2)^2 - 3$, which indicates the vertex is at (2, -3). The parabola opens downward because of the negative leading coefficient. Plot the vertex and additional points to sketch the graph.

Additional Resources

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Mathematics often appears as a complex language, filled with symbols and operations that can seem intimidating at first glance. However, breaking down algebraic expressions into manageable steps reveals the logical structure behind them. One such expression—$-3(x^2 - 4x + 5)$—serves as a perfect example to explore the process of simplifying and understanding algebraic expressions. Whether you're a student brushing up your algebra skills or a curious learner delving into the intricacies of polynomial manipulation, this article aims to clarify the step-by-step approach to working with such an expression.

Let's embark on this mathematical journey, examining the expression in detail, applying the distributive property, and exploring various methods to simplify it. Along the way, we'll discuss key concepts, common pitfalls, and practical tips to master similar algebraic challenges.

Understanding the Expression: Breaking Down $-3(x^2 - 4x + 5)$

At its core, the expression $-3(x^2 - 4x + 5)$ involves a constant multiplier—-3—applied to a quadratic trinomial $(x^2 - 4x + 5)$. The goal is to simplify this expression into a form that's easier to interpret or use in further calculations.

Components of the Expression

Before diving into calculations, let's analyze the individual parts:

- -3: A constant multiplier, which will affect every term inside the parentheses when distributed.

- x^2: The quadratic term, representing an x squared component.
- -4x: The linear term, involving x to the first power.
- +5: The constant term.

The key to simplifying this expression lies in understanding how to correctly distribute the -3 across each term inside the parentheses, applying the distributive property systematically.

Applying the Distributive Property: The First Step in Simplification

What is the Distributive Property?

The distributive property is a fundamental algebraic rule that states:

$a(b + c) = ab + ac$

This property allows us to multiply a single term outside parentheses by each term inside the parentheses individually, then combine like terms if necessary.

Distributing -3 across the polynomial

Applying this to $-3(x^2 - 4x + 5)$:

- Multiply -3 by x^2
- Multiply -3 by -4x
- Multiply -3 by 5

Let's perform each multiplication step-by-step:

1. $-3\ x^2 = -3x^2$

2. $-3\ -4x = (+12x)$

3. $-3\ 5 = -15$

Result of distribution:

Putting it all together, the expression becomes:

$-3x^2 + 12x - 15$

This is the simplified form of the original expression, achieved through straightforward application of the distributive property.

Delving Deeper: Interpreting the Simplified Expression

With the expression now written as $-3x^2 + 12x - 15$, several insights can be gained:

1. Polynomial Form

The expression is a quadratic polynomial, with:

- A leading coefficient -3, indicating the parabola opens downward.
- A linear term 12x, influencing the slope.
- A constant -15, shifting the graph vertically.

2. Factoring the Expression

Factoring quadratic expressions is a common next step, especially when solving equations or analyzing the function's roots.

Let's attempt to factor $-3x^2 + 12x - 15$:

- First, factor out the greatest common factor (GCF). All coefficients are divisible by -3:

$-3(x^2 - 4x + 5)$

- Notice that the expression inside the parentheses is the original quadratic we started with, suggesting that -3 was factored out initially.

- To factor further, check if $x^2 - 4x + 5$ can be factored over real numbers:

Discriminant $D = (-4)^2 - 4\ 1\ 5 = 16 - 20 = -4$

Since the discriminant is negative, the quadratic $x^2 - 4x + 5$ does not factor over real numbers and has complex roots. Therefore, the original quadratic expression is prime over the real numbers.

3. Graphical Interpretation

Plotting the quadratic $-3x^2 + 12x - 15$ reveals a downward-opening parabola with its vertex and roots, which can be found via calculus or algebraic methods.

Practical Application: Solving Equations Involving $-3(x^2 - 4x + 5)$

Understanding how to manipulate and simplify such expressions is crucial when solving equations. Suppose we want to solve:

$-3(x^2 - 4x + 5) = 0$

Step 1: Simplify

As shown earlier, this simplifies to:

$-3x^2 + 12x - 15 = 0$

Step 2: Divide through by -3 to simplify coefficients

Dividing both sides by -3:

$x^2 - 4x + 5 = 0$

Step 3: Use quadratic formula

Since the quadratic doesn't factor over the reals, apply the quadratic formula:

$x = [-b \pm \sqrt{(b^2 - 4ac)}] / 2a$

Where $a = 1$, $b = -4$, $c = 5$

Calculate discriminant:

$D = (-4)^2 - 4\ 1\ 5 = 16 - 20 = -4$

Since $D < 0$, solutions are complex:

$x = [4 \pm \sqrt{(-4)}] / 2$

Expressing the square root of negative number:

$x = [4 \pm 2i] / 2$

Simplify:

$x = 2 \pm i$

Thus, the solutions are complex conjugates 2 + i and 2 - i.

This demonstrates how the initial algebraic manipulations lead to understanding the nature of solutions—real or complex—and highlights the importance of clear, step-by-step algebra.

Common Pitfalls and Tips for Mastery

Working with expressions like $-3(x^2 - 4x + 5)$ can involve subtle mistakes if not approached carefully. Here are some common pitfalls and tips:

Pitfalls to Watch For

- Not applying the distributive property correctly: Remember to multiply each term inside the parentheses by the outside coefficient, not just the first term.
- Sign errors: Be cautious with negative signs; distributing a negative coefficient can flip signs unexpectedly.
- Overlooking the need for factoring or quadratic formula: Simplification isn't always just combining like terms; sometimes solving requires factoring or using the quadratic formula.
- Ignoring the nature of roots: A negative discriminant indicates complex solutions; don't assume all quadratics factor nicely over reals.

Tips for Students and Learners

- Write every step clearly: This reduces mistakes and helps track where signs or coefficients might go wrong.
- Check your work: After simplification, plug in a value for x to verify the expression's correctness.
- Practice diverse problems: Exposure to different types of quadratic equations enhances understanding.
- Use graphing tools: Visualizing the quadratic can provide intuition about roots and vertex location.

Extending Beyond: Applications of Such Expressions

Expressions like $-3(x^2 - 4x + 5)$ aren't just abstract algebra exercises—they appear in various fields:

- Physics: Modeling trajectories where quadratic relationships govern motion.
- Engineering: Designing systems with quadratic response curves.
- Economics: Analyzing profit functions that depend quadratically on quantities.
- Computer Graphics: Parabolic curves shape visual elements.

Mastering the simplification and interpretation of such expressions equips learners with vital skills applicable across science and technology disciplines.

Conclusion

Deciphering the expression $-3(x^2 - 4x + 5)$ highlights fundamental algebraic principles—distributive property, factoring, solving quadratic equations—and emphasizes methodical problem-solving. By systematically applying these concepts, learners can confidently manipulate complex expressions, interpret their meaning, and apply them to real-world problems.

Mathematics is a language of patterns and logic. Breaking down complex expressions into understandable steps transforms confusion into clarity, empowering learners to approach even the most daunting problems with confidence. As you continue exploring algebra and beyond, remember that each step taken with patience and precision builds a stronger foundation for mathematical mastery.

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