-4x = -1/2(10x + 18)

-4x = -1/2(10x + 18) is a mathematical equation that offers an excellent opportunity to explore algebraic principles, techniques for solving equations, and understanding how to manipulate expressions to isolate variables. Equations like this are foundational in algebra, providing skills that are applicable in various fields such as engineering, physics, economics, and everyday problem-solving. In this comprehensive article, we will delve into methods for solving such equations, the importance of understanding algebraic operations, and practical tips for mastering similar problems.

Understanding the Equation: Structure and Components

Before jumping into solutions, it’s crucial to understand the components of the equation:

```
-4x = -1/2(10x + 18)
```

This equation features a linear variable term on both sides, combined with a fractional coefficient on the right side. Breaking down the elements helps in identifying the appropriate steps to solve for x.

Left Side: -4x

- Represents a straightforward linear term, with a coefficient of -4 multiplied by x.
- Indicates that whatever the value of x, it is being scaled by -4.

Right Side: -1/2(10x + 18)

- Is a product of a fraction (-1/2) and a binomial (10x + 18).
- The negative sign in front of the fraction indicates the entire expression is negated.

Step-by-Step Approach to Solving the Equation

Solving equations like this involves systematic steps designed to isolate the variable x on one side of the equation. The general approach involves distributing, combining like terms, and then dividing to solve for x.

Step 1: Distribute the Fraction

The first step is to eliminate the parentheses by distributing the -1/2 across the terms inside:

```
-1/2  10x = -5x
```

```
-1/2  18 = -9
```

So, the right side simplifies to:

```
-5x - 9
```

The new form of the equation is:

```
-4x = -5x - 9
```

Step 2: Move Variable Terms to One Side

To gather all x terms on one side, add 5x to both sides:

```
-4x + 5x = -5x - 9 + 5x
```

Simplifying:

```
x = -9
```

Step 3: Confirm the Solution

Substitute x = -9 back into the original equation to verify:

Original equation:

```
-4x = -1/2(10x + 18)
```

Substitute x = -9:

```
-4  (-9) = -1/2(10  -9 + 18)
```

Calculate each side:

Left:

```
36
```

Right:

```
-1/2  (-90 + 18) = -1/2  (-72) = 36
```

Since both sides equal 36, the solution x = -9 is verified.

Key Algebraic Concepts Illustrated

This problem demonstrates several essential algebraic concepts:

Distributive Property

- Used to remove parentheses by multiplying each term inside by the factor outside.

Combining Like Terms

- Critical in simplifying complex expressions to make solving more straightforward.

Isolating the Variable

- The ultimate goal is to get x alone on one side of the equation.

Verification of Solution

- Substituting the solution back into the original equation to ensure correctness.

Common Mistakes and How to Avoid Them

When solving equations like this, students often encounter pitfalls:

- **Forgetting to distribute:** Failing to distribute -1/2 across the binomial leads to incorrect simplification.
- **Sign errors:** Misplacing negative signs can cause incorrect solutions.
- **Incorrect algebra operations:** When moving terms across the equation, it's essential to perform inverse operations carefully.
- **Skipping verification:** Always substitute back to confirm the solution's accuracy.

To avoid these mistakes, proceed cautiously, double-check each step, and ensure understanding of each operation performed.

Practice Problems for Mastery

Applying the skills learned to similar problems enhances understanding. Here are some practice equations:

1. Solve for x: 3x + 5 = -1/3(9x - 6)
2. Solve for y: -2(y - 4) = 3(2y + 1)
3. Find x: 1/2(4x - 8) = x + 3

Attempt solving these on your own, following the step-by-step method outlined above, and verify your solutions.

Real-World Applications of Algebraic Equations

Understanding how to solve equations like -4x = -1/2(10x + 18) is not merely an academic exercise. These skills are vital in various real-world contexts:

Physics

- Calculating forces, velocities, and other quantities often involves solving linear equations.

Economics

- Determining break-even points or optimizing profit involves solving similar algebraic expressions.

Engineering

- Analyzing circuit equations or mechanical systems requires proficiency in solving such equations.

Finance

- Computing interest rates and repayment schedules often involves algebraic manipulation.

Conclusion: Mastering the Art of Solving Algebraic Equations

The equation **-4x = -1/2(10x + 18)** encapsulates fundamental algebraic techniques that serve as building blocks for more complex mathematical problems. By understanding the structure of the equation, applying distribution, combining like terms, and isolating the variable, students can confidently solve for unknowns in various contexts. Remember to verify your solutions to ensure accuracy and develop good habits that will serve you well in advanced mathematics and practical applications. With practice and attention to detail, mastering equations like these becomes an achievable and rewarding skill that forms the foundation for higher-level problem-solving.

Frequently Asked Questions

How do I solve the equation -4x = -1/2(10x + 18)?

First, distribute the -1/2 across the parentheses: -1/2 10x = -5x and -1/2 18 = -9. So, the equation becomes -4x = -5x - 9. Then, add 5x to both sides: -4x + 5x = -9, resulting in x = -9.

What is the value of x in the equation -4x = -1/2(10x + 18)?

The value of x is -9, as shown by solving the equation step-by-step.

Can I simplify the equation -4x = -1/2(10x + 18) by eliminating the fraction?

Yes, multiply both sides of the equation by 2 to clear the fraction: 2 -4x = 2 -1/2(10x + 18), which simplifies to -8x = -(10x + 18). Then, proceed to solve for x.

What are common mistakes to avoid when solving -4x = -1/2(10x + 18)?

Common mistakes include forgetting to distribute the -1/2 correctly, neglecting to combine like terms properly, or mismanaging the signs during the process. Always carefully distribute and track signs throughout your steps.

Is the solution x = -9 valid for the equation -4x = -1/2(10x + 18)?

Yes, substituting x = -9 back into the original equation confirms that both sides are equal, so x = -9 is the valid solution.

Additional Resources

Understanding and Solving the Equation -4x = -1/2(10x + 18)

Mathematics is often viewed as a universal language, providing clear, logical pathways to solve problems that might initially seem complex. Equations like -4x = -1/2(10x + 18) serve as excellent examples of algebraic principles in action. This detailed exploration aims to dissect this equation comprehensively, offering insights into the underlying concepts, step-by-step solving strategies, common pitfalls, and broader applications.

Introduction to the Equation

The given equation:

$$
-4x = -\frac{1}{2}(10x + 18)
$$

is a linear algebraic expression involving a variable x, constants, and fractional coefficients. At first glance, it appears straightforward but warrants careful attention to detail in its manipulation.

Understanding this equation involves recognizing several key components:

- The presence of a fractional coefficient $-\frac{1}{2}$
- The linear combination inside the parentheses
- The sign conventions and potential for errors in handling negatives

This equation can serve as an excellent teaching tool for algebraic principles such as distributing, combining like terms, and solving for the unknown.

Fundamental Concepts Needed

Before solving, it’s essential to understand some foundational algebra concepts:

Distributive Property

- Allows us to multiply a single term across terms inside parentheses:

$$
a(b + c) = ab + ac
$$

Handling Fractions

- Fractions can complicate equations but can be managed by multiplying through by the least common denominator (LCD) to clear fractions.

Solving for a Variable

- The goal is to isolate x on one side of the equation, typically by performing inverse operations.

Sign Management

- Paying close attention to signs is crucial, especially with negative coefficients.

Step-by-Step Solution Strategy

Let's delve into the systematic approach to solving the equation.

Step 1: Recognize and Write Down the Equation

$$
-4x = -\frac{1}{2}(10x + 18)
$$

Step 2: Eliminate the Fraction

Multiplying both sides of the equation by 2 (the denominator of the fraction) to clear the fraction simplifies the process:

$$
2 \times (-4x) = 2 \times \left(-\frac{1}{2}(10x + 18)\right)
$$

which becomes:

$$
-8x = - (10x + 18)
$$

This step is crucial because working with whole numbers simplifies algebraic manipulations and reduces errors.

Step 3: Remove the Parentheses

Since the right side is $- (10x + 18)$, distribute the negative sign:

$$
-8x = -10x - 18
$$

Step 4: Collect Like Terms

The goal is to get all terms involving x on one side, constants on the other:

- Add $10x$ to both sides to move all x terms to the left:

$$
-8x + 10x = -18
$$

which simplifies to:

$$
2x = -18
$$

Step 5: Solve for x

Divide both sides by 2:

$$
x = \frac{-18}{2} = -9
$$

Result:

$$
\boxed{
x = -9
}
$$

Verification of the Solution

Verifying the solution ensures that it satisfies the original equation. Let's substitute $x = -9$ back into the original:

$$
-4(-9) = -\frac{1}{2}(10 \times -9 + 18)

\]

Calculate each side:

- Left side:

\[
-4 \times -9 = 36
\]

- Right side:

\[
-\frac{1}{2}(-90 + 18) = -\frac{1}{2}(-72) = -\frac{1}{2} \times -72 = 36
\]

Since both sides equal 36, our solution \(x = -9\) is correct.

Deeper Insights into the Equation

Beyond merely solving, understanding the structure and properties of this equation provides valuable learning:

Linear Equations and Their Graphs

- The equation represents a line when graphed. Because it's linear, the graph will be a straight line crossing the \(x\)-axis at \(x = -9\).

The Role of Fractions and Negatives

- Handling fractions requires careful distribution or multiplication to clear denominators.
- Negative coefficients influence the slope and intercepts, which are vital in understanding the behavior of the equation graphically.

Significance of the Solution

- The solution \(x = -9\) indicates the point where both sides of the equation are equal.
- In applied contexts, such solutions can represent equilibrium points, break-even points, or other critical values depending on the scenario.

Common Pitfalls and How to Avoid Them

When solving equations like this, students often encounter errors. Recognizing these pitfalls enhances problem-solving skills:

- Mismanaging negatives: Always double-check the distribution of negative signs.
- Forgetting to multiply through by the LCD: Failing to clear fractions can complicate calculations.
- Incorrect distribution: Ensure the negative sign is correctly distributed across all terms inside parentheses.
- Arithmetic errors: Verify calculations at each step to prevent compounding mistakes.

Alternative Methods and Approaches

While the step-by-step method above is straightforward, other strategies can also be applied:

Method 1: Cross-Multiplication

- Less practical here due to the structure but useful in equations involving ratios.

Method 2: Graphical Solution

- Plotting $y = -4x$ and $y = -\frac{1}{2}(10x + 18)$ to find the intersection point visually, which should occur at $x = -9$.

Method 3: Using Algebraic Substitution

- Substitute specific x values to test solutions during the process.

Broader Applications of Solving Such Equations

Equations like this are foundational in various fields:

- Physics: Calculating relationships between variables such as force, mass, and acceleration.
- Economics: Finding equilibrium points in supply and demand models.
- Engineering: Analyzing circuit equations or stress-strain relationships.
- Data Science: Solving for parameters within linear models.

Understanding how to manipulate and solve equations like $-4x = -1/2(10x + 18)$ is essential for applied problem-solving across these disciplines.

Conclusion: Mastering the Equation

The process of solving -4x = -1/2(10x + 18) illustrates core algebraic principles—distributive property, handling fractions, and isolating variables. It emphasizes the importance of meticulousness, especially regarding signs and arithmetic operations.

By following a structured approach: clearing fractions, distributing negatives, combining like terms, and dividing, students can confidently tackle similar equations. Moreover, understanding the broader implications and applications of such equations enhances both mathematical literacy and real-world problem-solving skills.

Mastery of these concepts forms a stepping stone toward more advanced algebra, calculus, and applied mathematics, laying a strong foundation for academic and professional pursuits.

4x 1 2 10x 18

4x 1 2 10x 18

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