$5x + 2y + 19 = 0$ $3x + 4y + 17 = 0$

$5x + 2y + 19 = 0$ $3x + 4y + 17 = 0$ are two fundamental linear equations in two variables, and understanding their properties is essential for students and professionals working with algebra, coordinate geometry, and systems of equations. These equations serve as a foundation for exploring how lines interact in a plane, whether they intersect, are parallel, or coincide. In this comprehensive guide, we will delve into various aspects of these equations, including their solutions, graphical representations, methods of solving, and applications.

Understanding the Equations

What Are Linear Equations?

Linear equations in two variables, such as x and y, are algebraic expressions that can be written in the form $ax + by + c = 0$, where a, b, and c are constants. These equations graph as straight lines in the Cartesian coordinate system. The equations **$5x + 2y + 19 = 0$** and **$3x + 4y + 17 = 0$** are examples of such equations.

Interpreting the Given Equations

Let's analyze the specific equations:

1. Equation 1: $5x + 2y + 19 = 0$
2. Equation 2: $3x + 4y + 17 = 0$

These equations can be rewritten in the slope-intercept form to better understand their graphical behavior.

Rearranged forms:

- $5x + 2y + 19 = 0$
→ $2y = -5x - 19$
→ $y = (-5/2)x - 19/2$

- $3x + 4y + 17 = 0$
→ $4y = -3x - 17$
→ $y = (-3/4)x - 17/4$

From these forms, we see that both lines have negative slopes, indicating they decline from left to right.

Graphical Representation of the Equations

Plotting the Lines

To visualize the equations, plot their respective lines on the Cartesian plane:

- For $5x + 2y + 19 = 0$, find two points:
- When $x = 0$: $2y = -19 \rightarrow y = -19/2 = -9.5$
- When $y = 0$: $5x = -19 \rightarrow x = -19/5 = -3.8$

- For $3x + 4y + 17 = 0$, find two points:
- When $x = 0$: $4y = -17 \rightarrow y = -17/4 = -4.25$
- When $y = 0$: $3x = -17 \rightarrow x = -17/3 \approx -5.6667$

Plot these points:

Equation	Point 1 (x=0)	Point 2 (y=0)
$5x + 2y + 19=0$	(0, -9.5)	(-3.8, 0)
$3x + 4y + 17=0$	(0, -4.25)	(-5.6667, 0)

Drawing these lines on the coordinate plane will reveal their positions relative to each other.

Understanding Relative Positioning

By analyzing the slopes:

- Line 1: slope $= -5/2 = -2.5$
- Line 2: slope $= -3/4 = -0.75$

Since the slopes are different, the lines are not parallel and will intersect at a single point.

Solving the System of Equations

Methods of Solving

There are several methods to find the point of intersection, which is the solution to the system:

- Substitution Method
- Elimination Method
- Graphical Method
- Matrix Method (using determinants or matrices)

For these equations, substitution or elimination are straightforward.

Solving by Substitution

Using the slope-intercept forms:

- $y = (-5/2)x - 19/2$
- $y = (-3/4)x - 17/4$

Set equal:

$$(-5/2)x - 19/2 = (-3/4)x - 17/4$$

Multiply through by 4 to clear denominators:

$4\ (-5/2)x - 4\ (19/2) = 4\ (-3/4)x - 4\ (17/4)$

Simplify:

$-10x - 38 = -3x - 17$

Bring all variables to one side:

$-10x + 3x = -17 + 38$

$-7x = 21$

Solve for x:

$x = 21 / -7 = -3$

Now, substitute $x = -3$ into one of the equations:

$y = (-5/2)(-3) - 19/2$

Calculate:

$y = (15/2) - 19/2 = (15 - 19)/2 = -4/2 = -2$

Solution:

- $x = -3$
- $y = -2$

This point, (-3, -2), is the intersection point of the two lines.

Significance of the Solution

Interpretation in Geometry

The solution (-3, -2) indicates the unique point where the two lines intersect. In coordinate geometry, this point is crucial for understanding the relationship between the two lines.

Applications of Such Systems

Systems of linear equations like these appear in various fields:

- Physics: To determine points of intersection or equilibrium.
- Economics: Cost and revenue analysis.
- Engineering: Designing structures with multiple constraints.
- Computer Graphics: Line intersection calculations.

Additional Concepts Related to the Equations

Parallel and Coincident Lines

While these specific lines intersect, understanding the concepts of parallel and coincident lines is essential:

- Parallel lines: Same slope, different intercepts, no intersection.
- Coincident lines: Same line, infinitely many solutions.

Determining Parallelism

Compare the slopes:

- Slope of line 1: $-5/2$
- Slope of line 2: $-3/4$

Since $-5/2 \neq -3/4$, the lines are not parallel.

Checking for Coincidence

If the equations are multiples of each other, lines are coincident. Here, they are not multiples, so lines are distinct.

Real-World Applications and Examples

Problem 1: Intersection Point in Urban Planning

Suppose two streets are represented by the lines:

- Street A: $5x + 2y + 19 = 0$
- Street B: $3x + 4y + 17 = 0$

The intersection point $(-3, -2)$ helps urban planners identify where the streets cross. This information is vital for designing traffic flow, placing traffic signals, or planning utilities.

Problem 2: Engineering Design

In mechanical engineering, these equations could represent the constraints of two components. Finding the intersection point ensures proper fit and alignment.

Advanced Topics and Further Study

Matrix Representation and Determinants

Expressing the system in matrix form:

\[
\begin{bmatrix}

5 & 2 \\
3 & 4
\end{bmatrix}
\begin{bmatrix}
x \\
y
\end{bmatrix}
=
\begin{bmatrix}
-19 \\
-17
\end{bmatrix}
\]

The determinant:

\[
D = (5)(4) - (2)(3) = 20 - 6 = 14 \neq 0
\]

Since D ≠ 0, the system has a unique solution, which aligns with our previous calculation.

Using Cramer's Rule

Calculate determinants for x and y:

\[
D_x = \begin{bmatrix}
-19 & 2 \\
-17 & 4
\end{bmatrix} = (-19)(4) - (2)(-17) = -76 + 34 = -42
\]

\[
D_y = \begin{bmatrix}
5 & -19 \\
3 & -17
\end{bmatrix} = (5)(-17) - (-19)(3) = -85 + 57 = -28
\]

Solutions:

\[
x = \frac{D_x}{D} = \frac{-42}{14} = -3
\]

\[
y = \frac{D_y}{D} = \frac{-28}{14} = -2
\]

Matching our earlier solution.

Conclusion

The equations **5x + 2y + 19 = 0** and **3x + 4y + 17 = 0** exemplify fundamental concepts in algebra and geometry. Their intersection point, (-3,

Frequently Asked Questions

How can I find the intersection point of the lines 5x + 2y + 19 = 0 and 3x + 4y + 17 = 0?

To find the intersection, solve the system of equations simultaneously. Using substitution or elimination methods will give the point where both lines intersect.

Are the lines 5x + 2y + 19 = 0 and 3x + 4y + 17 = 0 parallel or intersecting?

Since their slopes are different, the lines are intersecting. You can confirm this by solving the system; a unique solution exists.

What is the slope of the line 5x + 2y + 19 = 0?

Rearranged in slope-intercept form: $y = -(5/2)x - 19/2$, so the slope is $-5/2$.

What is the slope of the line 3x + 4y + 17 = 0?

Rearranged in slope-intercept form: $y = -(3/4)x - 17/4$, so the slope is $-3/4$.

Can these two lines be perpendicular?

No, because their slopes are $-5/2$ and $-3/4$. Since the product of the slopes is not -1, they are not perpendicular.

How do I convert the equations to slope-intercept form?

For each equation, solve for y: for example, $5x + 2y + 19 = 0$ becomes $y = -(5/2)x - 19/2$.

What is the solution to the system of equations?

Solve the system using substitution or elimination to find the values of x and y where both equations intersect.

Are the lines 5x + 2y + 19 = 0 and 3x + 4y + 17 = 0 consistent?

Yes, they are consistent and intersect at a single point because the system has a unique solution.

How do I verify the solution for the intersection point?

Plug the x and y values obtained into both equations to see if they satisfy both equations simultaneously.

What is the significance of these equations in coordinate geometry?

They represent straight lines in the coordinate plane, and solving them helps find their intersection point and analyze their relationship.

Additional Resources

Understanding the System of Equations: $5x + 2y + 19 = 0$ and $3x + 4y + 17 = 0$

When exploring algebraic concepts, particularly systems of equations, the pair of equations $5x + 2y + 19 = 0$ and $3x + 4y + 17 = 0$ serve as classic examples illustrating how to analyze, solve, and interpret such systems. These equations, which are linear in two variables, form the basis for understanding concepts like intersection points, solution sets, and methods of solving such systems—be it graphically, algebraically, or through substitution and elimination. This detailed guide aims to break down these equations step-by-step, providing clarity and insight into their properties and solutions.

Understanding the Equations: An Overview

Before diving into solving techniques, it’s essential to understand the structure and significance of the given system:

- Equation 1: $5x + 2y + 19 = 0$
- Equation 2: $3x + 4y + 17 = 0$

These are linear equations, each representing a straight line in the xy-plane. The solutions to the system are the points where these lines intersect, which can be a single point, no points (if they are parallel and distinct), or infinitely many points (if they are coincident).

Converting Equations into Slope-Intercept Form

A common first step is rewriting each equation in the slope-intercept form to visualize their graphs easily:

Equation 1:

Starting with:

$$5x + 2y + 19 = 0$$

Solve for y:

$$2y = -5x - 19$$

$$y = (-5/2) x - 19/2$$

Equation 2:

Starting with:

$3x + 4y + 17 = 0$

Solve for y:

$4y = -3x - 17$

$y = (-3/4) x - 17/4$

Graphical Interpretation

Plotting these equations on a Cartesian plane offers immediate visual insight:

- Line 1: Slope = $-5/2$, y-intercept = $-19/2$ (~ -9.5)
- Line 2: Slope = $-3/4$, y-intercept = $-17/4$ (~ -4.25)

Since the slopes are different ($-5/2 \neq -3/4$), the lines are not parallel, indicating they intersect at exactly one point. This intersection point is the unique solution to the system.

Solving the System Algebraically

Two common methods are used:

1. Substitution Method

This involves solving one of the equations for one variable and substituting into the other.

Step 1: Solve Equation 2 for y:

$y = (-3/4) x - 17/4$

Step 2: Substitute into Equation 1:

$5x + 2((-3/4) x - 17/4) + 19 = 0$

Simplify:

$5x + 2 (-3/4) x + 2 (-17/4) + 19 = 0$

$= 5x - (3/2) x - (17/2) + 19 = 0$

Step 3: Combine like terms:

$(5x - 1.5x) + (-8.5 + 19) = 0$

$= (3.5x) + (10.5) = 0$

Step 4: Solve for x:

$3.5x = -10.5$

$x = -10.5 / 3.5$

$x = -3$

Step 5: Find y using the expression from earlier:

$y = (-3/4)(-3) - 17/4$

Calculate:

$y = (9/4) - 17/4 = (-8/4) = -2$

Solution: The intersection point is (-3, -2).

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2. Elimination Method

This involves aligning the equations to eliminate one variable.

Step 1: Write equations:

Equation 1: $5x + 2y = -19$

Equation 2: $3x + 4y = -17$

Step 2: Multiply to match coefficients of y:

Multiply Equation 1 by 2:

$(2)\ (5x + 2y) = 2\ (-19)$

$10x + 4y = -38$

Now subtract Equation 2 from this:

$(10x + 4y) - (3x + 4y) = -38 - (-17)$

$(10x - 3x) + (4y - 4y) = -38 + 17$

$7x + 0 = -21$

Step 3: Solve for x:

$x = -21 / 7 = -3$

Step 4: Substitute x back into one of the original equations:

$5(-3) + 2y = -19$

$-15 + 2y = -19$

$2y = -19 + 15 = -4$

$y = -2$

Result: The solution again is (-3, -2).

Verifying the Solution

Always good practice to verify the solution by plugging into both original equations:

- Equation 1: $5(-3) + 2(-2) + 19 = -15 - 4 + 19 = 0$ ✅
- Equation 2: $3(-3) + 4(-2) + 17 = -9 - 8 + 17 = 0$ ✅

The solution satisfies both equations, confirming its accuracy.

Analyzing the Nature of the System

Since the lines intersect at exactly one point, the system is consistent and independent. This is typical for two lines with different slopes.

Special Cases to Consider:

- Parallel lines: Same slope, different intercepts, no solution.
- Coincident lines: Same slope and intercept, infinitely many solutions.
- Intersecting lines: Different slopes, exactly one solution.

In this case, the equations depict two intersecting lines, hence a unique solution.

Applications and Real-World Contexts

Systems like these are more than academic exercises; they model real-world situations:

- Economics: Equilibrium points where supply and demand curves intersect.
- Physics: Intersection of trajectories or forces.
- Engineering: Compatibility points between different structural components.

Understanding how to solve such systems allows professionals in various fields to make informed decisions based on intersection points, constraints, and relationships.

Summary and Key Takeaways

- Converting linear equations into slope-intercept form facilitates visualization.
- Algebraic methods like substitution and elimination are effective for solving systems.
- The intersection point found (here, $(-3, -2)$) is the unique solution.
- Graphical interpretation confirms the algebraic solution.
- Recognizing the type of system (consistent, inconsistent, dependent) is crucial for understanding the solution set.

Final Thoughts

Mastering the analysis and solution of systems like $5x + 2y + 19 = 0$ and $3x +$

4y + 17 = 0 builds a foundation for tackling more complex algebraic problems. Whether approached graphically, algebraically, or through a combination, understanding these equations enhances problem-solving skills and deepens comprehension of linear relationships. Keep practicing with various systems to develop intuition and confidence in handling diverse mathematical challenges.

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