Change Y=9/4x^2+3x-1 To Vertex Form

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Understanding how to convert a quadratic function from standard form to vertex form is an essential skill in algebra, especially when analyzing the graph's properties. The quadratic equation $Y = \frac{9}{4}x^2 + 3x - 1$ is initially presented in standard form, which is generally written as $y = ax^2 + bx + c$. While standard form is useful for identifying the y-intercept, converting it to vertex form allows us to easily find the vertex of the parabola, its axis of symmetry, and opens up avenues for graphing and solving quadratic equations efficiently.

In this article, we will explore step-by-step how to change the quadratic function $Y = \frac{9}{4}x^2 + 3x - 1$ into its vertex form. We will cover the concepts of completing the square, the significance of vertex form, and practical tips for mastering this transformation, making this guide valuable for students, educators, and anyone interested in quadratic functions.

Understanding the Forms of Quadratic Functions

Quadratic functions are typically presented in two main forms:

Standard Form

- Expressed as $y = ax^2 + bx + c$
- Useful for identifying the y-intercept directly (which is c)
- Not immediately obvious where the vertex of the parabola is located

Vertex Form

- Expressed as $y = a(x - h)^2 + k$
- Clearly displays the vertex at point (h, k)
- Facilitates graphing and understanding the parabola's properties

Converting from standard to vertex form involves rewriting the equation so that it explicitly shows the vertex, which is particularly useful for graphing and analyzing the parabola's symmetry.

Step-by-Step Guide to Convert $Y = \frac{9}{4}x^2 + 3x - 1$ to Vertex Form

Let's now proceed with the detailed steps:

Step 1: Factor out the leading coefficient from the quadratic terms

Since the coefficient of x^2 is $\frac{9}{4}$, factor it out from the quadratic and linear terms:

$$
Y = \frac{9}{4}x^2 + 3x - 1
$$

Rewrite the linear term to factor out $\frac{9}{4}$:

$$
Y = \frac{9}{4} \left(x^2 + \frac{3}{1} \times \frac{4}{9} x \right) - 1
$$

But to make it clearer, express $3x$ as a fraction with denominator 1:

$$
Y = \frac{9}{4}x^2 + \frac{3}{1}x - 1
$$

Now, factor $\frac{9}{4}$:

$$
Y = \frac{9}{4} \left(x^2 + \frac{3}{1}x \times \frac{4}{9} \right) - 1
$$

Simplify the coefficient inside the parentheses:

$$
\frac{3}{1} \times \frac{4}{9} = \frac{3 \times 4}{1 \times 9} = \frac{12}{9} = \frac{4}{3}
$$

Therefore:

$$
Y = \frac{9}{4} \left(x^2 + \frac{4}{3}x \right) - 1
$$

Step 2: Complete the square inside the parentheses

To complete the square, take half of the coefficient of x, square it, and add/subtract it inside the parentheses.

- Coefficient of x: $\frac{4}{3}$
- Half of this: $\frac{1}{2} \times \frac{4}{3} = \frac{2}{3}$

- Square of this half: $\left(\frac{2}{3} \right)^2 = \frac{4}{9}$

Add and subtract $\frac{4}{9}$ inside the parentheses to complete the square:

$$
Y = \frac{9}{4} \left(x^2 + \frac{4}{3}x + \frac{4}{9} - \frac{4}{9} \right) - 1
$$

Group the perfect square trinomial:

$$
Y = \frac{9}{4} \left(\left(x + \frac{2}{3} \right)^2 - \frac{4}{9} \right) - 1
$$

Step 3: Simplify the expression

Distribute $\frac{9}{4}$:

$$
Y = \frac{9}{4} \left(x + \frac{2}{3} \right)^2 - \frac{9}{4} \times \frac{4}{9} - 1
$$

Calculate:

$$
\frac{9}{4} \times \frac{4}{9} = 1
$$

So:

$$
Y = \frac{9}{4} \left(x + \frac{2}{3} \right)^2 - 1 - 1
$$

$$
Y = \frac{9}{4} \left(x + \frac{2}{3} \right)^2 - 2
$$

Final Vertex Form

The quadratic function in vertex form is:

$$

Y = \frac{9}{4} \left(x + \frac{2}{3} \right)^2 - 2
\]

Vertex Coordinates:

- The vertex \((h, k) \) is at:

\[
h = -\frac{2}{3}, \quad k = -2
\]

Interpreting the Vertex Form and Its Graphical Significance

Converting to vertex form provides immediate insight into the graph of the parabola:

- Vertex: \(\left(-\frac{2}{3}, -2 \right) \)
- Axis of symmetry: \(x = -\frac{2}{3} \)
- Direction of opening: Since the coefficient \(\frac{9}{4} \) is positive, the parabola opens upward.
- Width of the parabola: The coefficient \(\frac{9}{4} \) affects how "wide" or "narrow" the parabola appears; larger coefficients produce narrower graphs.

Additional Tips for Converting Quadratic Equations to Vertex Form

- Always factor out the leading coefficient if it is not 1 before completing the square.
- Complete the square carefully: half of the linear coefficient, squared, to find the perfect square trinomial.
- Keep track of constants: remember to add and subtract the same value inside the parentheses to maintain equality.
- Simplify after distributing to find the vertex form clearly.
- Check your work by expanding the vertex form back into standard form to verify the transformation.

Practical Applications of Vertex Form

Converting quadratic equations into vertex form is not purely an algebraic exercise; it has real-world applications:

- Graphing parabolas accurately by knowing the vertex.
- Optimizing functions in economics, physics, and engineering by understanding maximum or minimum points.
- Solving quadratic inequalities by analyzing the parabola's vertex and intercepts.
- Modeling real-world phenomena such as projectile motion, where the vertex indicates the maximum height or range.

Conclusion

Transforming the quadratic function $Y = \frac{9}{4}x^2 + 3x - 1$ into vertex form involves a systematic process of factoring, completing the square, and simplifying. The final form:

$$
Y = \frac{9}{4} \left(x + \frac{2}{3} \right)^2 - 2
$$

reveals the vertex at $\left(-\frac{2}{3}, -2 \right)$, providing valuable information for graphing and analysis. Mastering this technique enhances your understanding of quadratic functions and their properties, making it an essential skill in algebra and calculus.

By practicing these steps and understanding their rationale, you'll be well-equipped to handle similar conversions and deepen your grasp of quadratic equations and their applications.

Frequently Asked Questions

How do I convert the quadratic function $Y = 9/4x^2 + 3x - 1$ to vertex form?

To convert $Y = 9/4x^2 + 3x - 1$ to vertex form, you need to complete the square on the quadratic part and factor out common coefficients if necessary, then rewrite it as $y = a(x - h)^2 + k$.

What is the first step in rewriting $Y = 9/4x^2 + 3x - 1$ in vertex form?

The first step is to factor out the coefficient of x^2 (which is $9/4$) from the quadratic and linear terms to make completing the square easier.

How do I complete the square for the quadratic part in Y =

$9/4x^2 + 3x - 1$?

After factoring out 9/4, rewrite the quadratic and linear terms, then add and subtract the square of half the coefficient of x inside the parentheses to complete the square.

What is the vertex form formula for a quadratic function?

The vertex form is $y = a(x - h)^2 + k$, where (h, k) is the vertex of the parabola.

How do I find the vertex (h, k) after converting to vertex form?

Once the quadratic is in vertex form, the vertex (h, k) can be read directly from the equation: h is the value inside (x - h), and k is the constant term outside the squared term.

Can you provide a step-by-step example of converting $Y = 9/4x^2 + 3x - 1$ to vertex form?

Yes. First, factor out 9/4: $y = 9/4(x^2 + (4/3)x) - 1$. Then, complete the square inside the parentheses: add and subtract $(2/3)^2 = 4/9$. Rewrite as $y = 9/4[(x + 2/3)^2 - 4/9] - 1$. Distribute 9/4: $y = (9/4)(x + 2/3)^2 - 1 - (9/4)(4/9)$. Simplify: $y = (9/4)(x + 2/3)^2 - 1 - 1$. Final form: $y = (9/4)(x + 2/3)^2 - 2$.

Why is converting to vertex form useful in analyzing quadratic functions?

Converting to vertex form makes it easier to identify the vertex, axis of symmetry, and the parabola's direction, which are essential for graphing and understanding the function's behavior.

Additional Resources

Change $Y=9/4x^2 + 3x - 1$ To Vertex Form: An In-Depth Investigation

Understanding how to convert a quadratic function from its standard form to its vertex form is a fundamental skill in algebra, offering critical insights into the graph's shape and key features such as its vertex and axis of symmetry. This article explores the methodical process of transforming the quadratic function \(Y = \frac{9}{4}x^2 + 3x - 1 \) into its vertex form, illuminating the step-by-step procedure, underlying principles, and practical applications.

Introduction: The Significance of Quadratic Forms

Quadratic functions are ubiquitous in mathematics, physics, engineering, and many applied sciences. Their standard form, \(Y = ax^2 + bx + c \), provides an algebraic representation, but often lacks immediate visual or geometric intuition. The vertex form, \(Y = a(x-h)^2 + k \), explicitly reveals the parabola's vertex (h, k), making it invaluable for graphing, optimization problems, and understanding the function's properties.

Transforming from standard to vertex form involves completing the square—a technique that re-expresses the quadratic in a form that clearly identifies the vertex. This process not only aids in graphing but also deepens comprehension of quadratic behavior.

Understanding the Given Function

The function under investigation is:

$$ Y = \frac{9}{4}x^2 + 3x - 1 $$

Key features include:

- Leading coefficient $a = \frac{9}{4}$, which indicates the parabola opens upward and is relatively "narrow" due to the positive coefficient.
- The linear coefficient $b = 3$.
- The constant term $c = -1$.

Before converting, it's essential to recognize that the coefficient of x^2 is fractional, which influences the completing-the-square process.

Step-by-Step Conversion to Vertex Form

Step 1: Factor out the Leading Coefficient from the Quadratic Terms

To facilitate completing the square, factor $\frac{9}{4}$ from the quadratic and linear terms:

$$
Y = \frac{9}{4} \left(x^2 + \frac{3}{\frac{9}{4}} x \right) - 1
$$

Calculate $\frac{3}{\frac{9}{4}}$:

$$
\frac{3}{\frac{9}{4}} = 3 \times \frac{4}{9} = \frac{12}{9} = \frac{4}{3}
$$

Thus,

$$
Y = \frac{9}{4} \left(x^2 + \frac{4}{3} x \right) - 1
$$

Step 2: Complete the Square Inside the Parentheses

Completing the square involves transforming:

$$
x^2 + \frac{4}{3} x
$$

into a perfect square trinomial. The general approach:

- Take half of the coefficient of x: $\frac{4/3}{2} = \frac{2/3}$
- Square this value: $\left(\frac{2/3} \right)^2 = \frac{4}{9}$

Add and subtract this square inside the parentheses to maintain equality:

$$
x^2 + \frac{4}{3} x + \frac{4}{9} - \frac{4}{9}
$$

which simplifies to:

$$
\left(x + \frac{2}{3} \right)^2 - \frac{4}{9}
$$

Step 3: Rewrite the Entire Expression

Substitute back into the original expression:

$$
Y = \frac{9}{4} \left[\left(x + \frac{2}{3} \right)^2 - \frac{4}{9} \right] - 1
$$

Distribute $\frac{9}{4}$:

$$
Y = \frac{9}{4} \left(x + \frac{2}{3} \right)^2 - \frac{9}{4} \times \frac{4}{9} - 1

\]

Simplify:

$$
Y = \frac{9}{4} \left(x + \frac{2}{3} \right)^2 - 1 - 1
$$

because $\frac{9}{4} \times \frac{4}{9} = 1$.

Step 4: Final Vertex Form Expression

Combine constants:

$$
Y = \frac{9}{4} \left(x + \frac{2}{3} \right)^2 - 2
$$

This is the quadratic function in vertex form.

Interpreting the Vertex Form

The vertex form:

$$ Y = a(x - h)^2 + k $$

with

- $a = \frac{9}{4}$
- $h = -\frac{2}{3}$
- $k = -2$

indicates the parabola has:

- Vertex: $\left(-\frac{2}{3}, -2 \right)$
- Axis of symmetry: $x = -\frac{2}{3}$
- Direction: Upward opening, since $a > 0$
- Width: Narrower than the standard parabola because $a > 1$

Significance and Applications of the Conversion

Transforming to vertex form is more than a mere algebraic exercise; it has several powerful applications:

- Graphing: The vertex form offers an immediate visual cue for plotting the parabola, as the vertex and axis of symmetry are directly visible.
- Optimization: In real-world problems, the vertex represents the maximum or minimum value; knowing the vertex form simplifies identifying these extremal points.
- Analyzing Transformations: It clarifies how shifts, stretches, and reflections affect the parabola.
- Solving equations: It simplifies the process for solving quadratic equations for specific y-values.

Practical Considerations and Common Pitfalls

While completing the square is straightforward in principle, practitioners should be mindful of:

- Fractional coefficients: Careful handling of fractions is essential to avoid errors.
- Sign errors: Ensure correct signs during the completion of the square and distribution.
- Constant adjustment: Remember to adjust constants outside the parentheses appropriately after completing the square.
- Verification: Confirm the transformation by expanding the vertex form back into standard form.

Conclusion: The Critical Role of Methodical Transformation

The process of converting $Y = \frac{9}{4}x^2 + 3x - 1$ into its vertex form exemplifies the power of algebraic techniques to reveal a quadratic's geometric essence. Through systematic factoring, completing the square, and careful algebraic manipulation, the parabola's key features become transparent: its vertex at $\left(-\frac{2}{3}, -2 \right)$, its axis of symmetry, and its opening direction.

Mastering this conversion enriches mathematical understanding and enhances problem-solving capabilities across disciplines where quadratic relationships are fundamental. Whether for theoretical explorations or practical applications like physics and engineering, the ability to transition seamlessly between forms is an essential skill in the mathematician's toolkit.

In summary:

- The original quadratic: $Y = \frac{9}{4}x^2 + 3x - 1$

- After completing the square and algebraic manipulation:

$$\boxed{
Y = \frac{9}{4} \left(x + \frac{2}{3} \right)^2 - 2
}$$

- The vertex of the parabola is at $\left(-\frac{2}{3}, -2 \right)$.

This transformation exemplifies the elegance and utility of algebraic techniques in uncovering the geometric properties of quadratic functions, solidifying their foundational role in mathematics education and application.

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