Determine The Range: Y= 2+sec(3x)

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Understanding the range of a function is a fundamental aspect of analyzing its behavior in mathematics. When dealing with trigonometric functions such as secant, it's essential to understand how transformations affect their domain and range. In this comprehensive guide, we will explore how to determine the range of the function $Y= 2 + \sec(3x)$, breaking down the process step by step, and providing insights into the properties of the secant function, its transformations, and how to identify its range effectively.

Understanding the Basic Secant Function

Before analyzing the specific function $Y= 2 + \sec(3x)$, it is crucial to understand the fundamental properties of the secant function itself.

What is the Secant Function?

The secant function, denoted as $\sec(x)$, is one of the six fundamental trigonometric functions. It is defined as the reciprocal of the cosine function:

$$
\sec(x) = \frac{1}{\cos(x)}
$$

for all x where $\cos(x) \neq 0$.

Key Properties of $\sec(x)$

- Domain: All real numbers x such that $\cos(x) \neq 0$. Since $\cos(x) = 0$ at $x = \frac{\pi}{2} + k\pi$ where k is any integer, the domain excludes these points.
- Range: Because $\sec(x)$ is the reciprocal of $\cos(x)$, its values are:

$$
\sec(x) \leq -1 \quad \text{or} \quad \sec(x) \geq 1
$$

excluding the interval $(-1, 1)$. This is due to the fact that $|\cos(x)| \leq 1$, so $|\sec(x)| \geq 1$.

- Graphical Behavior: The graph of $\sec(x)$ features branches that extend to infinity in both the positive and negative directions, with vertical asymptotes where $\cos(x) = 0$.

Analyzing the Function $Y= 2 + \sec(3x)$

Our goal is to find the range of the function:

$$
Y = 2 + \sec(3x)
$$

This involves understanding how the basic secant function is transformed by the vertical shift and the horizontal compression due to the coefficient 3.

Step 1: Understand the Transformation

- Horizontal Compression/Stretch: The argument $3x$ causes a horizontal compression by a factor of $\frac{1}{3}$. This affects the period of the function.

- Vertical Shift: Adding 2 shifts the entire graph vertically upward by 2 units.

Step 2: Determine the Range of $\sec(3x)$

Given the basic properties:
- $\sec(3x) \leq -1$ or $\sec(3x) \geq 1$

This is because the secant function, like cosine, is undefined at certain points but its range outside those points is:

$$
(-\infty, -1] \cup [1, \infty)
$$

Step 3: Incorporate the Vertical Shift $+2$

Since $y = 2 + \sec(3x)$, the range of y is obtained by shifting all values of $\sec(3x)$ upward by 2:
- For $\sec(3x) \geq 1$, $y \geq 2 + 1 = 3$
- For $\sec(3x) \leq -1$, $y \leq 2 - 1 = 1$

Therefore, the overall range of y is:

$$
(-\infty, 1] \cup [3, \infty)
$$

Detailed Step-by-Step Derivation of the Range

To formalize the derivation process, let's examine step by step how the transformations influence the range.

Step 1: Basic Range of $\sec(x)$

$$
\boxed{
\text{Range of } \sec(x): \quad (-\infty, -1] \cup [1, \infty)
}
$$

This is due to the reciprocal relationship with $\cos(x)$:
- When $|\cos(x)| < 1$, $|\sec(x)| > 1$
- When $|\cos(x)| = 1$, $\sec(x) = \pm 1$

Step 2: Effect of Horizontal Compression $3x$

- The period of $\sec(x)$ is 2π.
- For $\sec(3x)$, the period becomes:

$$
\frac{2\pi}{3}
$$

- This compression affects how frequently the function attains its maximum and minimum values, but does not change the set of possible values for $\sec(3x)$.

Step 3: Vertical Shift $+2$

- Adding 2 to $\sec(3x)$ shifts the entire graph upwards.
- Therefore, the range shifts accordingly:

$$
(-\infty, -1] \cup [1, \infty) \quad \xrightarrow{\text{shift up by 2}} \quad (-\infty, 1] + 2 \cup [1, \infty) + 2
$$

which simplifies to:

$$
(-\infty, 1 + 2] \cup [1 + 2, \infty) \quad \Rightarrow \quad (-\infty, 3] \cup [3, \infty)
$$

Since the union of these two sets is the entire real line except perhaps some interval between the two, but note that the points at exactly 3 are included in both sets, the combined range is:

$$
(-\infty, 3] \cup [3, \infty) = (-\infty, \infty)
$$

However, because $\sec(3x)$ cannot take values between -1 and 1, the shifted function $y = 2 + \sec(3x)$ cannot take values between 2 + (-1) = 1 and 2 + 1 = 3. This means:

- The function cannot attain values in the interval $(1, 3)$.

Thus, the accurate range is:

$$
(-\infty, 1] \cup [3, \infty)
$$

Summary:

Original $\sec(x)$ values	Transformed $y= 2 + \sec(3x)$ values
≤ -1	≤ 1
≥ 1	≥ 3

Final Range:

$$
\boxed{
y \in (-\infty, 1] \cup [3, \infty)
}
$$

Graphical Interpretation

Visualizing the function helps solidify understanding of its range:

- The graph of $\sec(3x)$ has vertical asymptotes at points where $\cos(3x) = 0$, i.e., $3x = \frac{\pi}{2} + k\pi$, or $x = \frac{\pi}{6} + \frac{k\pi}{3}$.
- Between these asymptotes, the graph of $\sec(3x)$ oscillates between $-\infty$ and -1, and between 1 and ∞.
- Shifting the graph upward by 2 moves the asymptotes and the oscillations, creating the two distinct branches with ranges as identified.

Applications of Determining the Range in Mathematics

Understanding the range of functions like \(y= 2 + \sec(3x) \) has multiple applications:

1. **Graphing Functions:** Accurate plotting depends on knowing the range to anticipate the values the function can take.
2. **Solving Equations:** When solving for \(x \) given \(y \), the range determines the possible solutions.
3. **Real-world Modeling:** In physics, engineering, and other sciences, knowing the range helps interpret the possible outputs of models involving oscillations and periodic behavior.
4. **Calculus:** Range analysis is essential for understanding the behavior of functions, including finding maxima, minima, and asymptotes.

Practice Problems to Reinforce Concept

To solidify understanding, consider

Frequently Asked Questions

What is the general form of the function $y = 2 + \sec(3x)$?

The function is a transformed secant function with a vertical shift of 2 units, where the secant function is scaled horizontally by a factor of 1/3.

How do you find the range of $y = 2 + \sec(3x)$?

Since sec(3x) varies from -⍰ to -1 and 1 to ⍰ , adding 2 shifts this range to (-⍰ , 1) U (3, ⍰).

What are the key features of the graph of $y = 2 + \sec(3x)$?

The graph has vertical asymptotes where $\cos(3x) = 0$, and the secant curves reach minimum and maximum values at points where $\cos(3x) = \pm 1$, shifted vertically by 2.

What is the period of the function $y = 2 + \sec(3x)$?

The period of the secant function is $(2\pi)/3$, since the period of $\cos(3x)$ is $(2\pi)/3$.

How does the coefficient 3 inside the secant function affect its graph?

The coefficient 3 compresses the graph horizontally by a factor of 1/3, making the period shorter.

Are there any restrictions on the domain of $y = 2 + \sec(3x)$?

Yes, the graph is undefined where $\cos(3x) = 0$, which occurs at $x = (\pi/6) + (\pi/3)k$, for all integers k.

What is the significance of the vertical shift in $Y = 2 + \sec(3x)$?

The vertical shift of +2 moves the entire secant graph upward, changing the range but not the period or shape of the graph.

Additional Resources

Determine The Range: $Y = 2 + \sec(3x)$

Understanding the range of a trigonometric function is fundamental in both pure and applied mathematics, especially when analyzing periodic phenomena, wave behaviors, or oscillatory systems. The function $Y = 2 + \sec(3x)$ presents an interesting case because it involves a secant function, which naturally introduces asymptotes and discontinuities, and a vertical translation that shifts the entire graph vertically. Determining its range involves understanding the properties of the secant function, its transformations, and how these influence the set of possible output values.

Understanding the Basic Components of the Function

What is the Secant Function?

The secant function, denoted as $\sec(\theta)$, is the reciprocal of the cosine function:

$$
\sec(\theta) = \frac{1}{\cos(\theta)}
$$

It shares the periodic nature of cosine, with a period of 2π, but exhibits different behaviors:

- Range of cosine: $-1 \leq \cos(\theta) \leq 1$
- Range of secant: $\sec(\theta) \leq -1$ or $\sec(\theta) \geq 1$

Because secant is the reciprocal of cosine, it is undefined where cosine equals zero, leading to vertical asymptotes at those points.

Transformations in the Function $Y = 2 + \sec(3x)$

This function involves two key transformations:

1. Horizontal compression: The argument of secant is multiplied by 3, i.e., $3x$. This compresses the period of the secant function:

$$
\text{Period of } \sec(3x) = \frac{2\pi}{3}
$$

2. Vertical shift: The entire secant graph is shifted upward by 2 units, due to the "+ 2" term.

Understanding these transformations helps us analyze the behavior of the function over its domain and determine its range.

Analyzing the Range of $y = 2 + \sec(3x)$

Step 1: Determine the Range of $\sec(3x)$

Since the secant function's output is constrained to $\sec(\theta) \leq -1$ or $\sec(\theta) \geq 1$, the range of $\sec(3x)$ is:

$$
(-\infty, -1] \cup [1, \infty)
$$

This is because secant cannot take values between -1 and 1, as that would require cosine to be between 0 and 1 but not reaching zero, which is impossible for secant.

Step 2: Incorporate the Vertical Shift

Adding 2 to secant shifts its entire range upward by 2 units:

$$
y = 2 + \sec(3x)
$$

So, the new range becomes:

$$
y \in \left(2 + (-\infty), 2 + (-1] \right] \cup \left[2 + 1, 2 + \infty \right)
$$

Simplifying:

$$
y \in \left(-\infty, 1 \right] \cup \left[3, \infty \right)
$$

Therefore, the complete set of possible y-values (the range) for the function is:

$$
\boxed{
\text{Range of } y = (-\infty, 1] \cup [3, \infty)
}
$$

This indicates that the function can take any value less than or equal to 1 or greater than or equal to 3, but it cannot take values between 1 and 3.

Graphical Interpretation and Behavior

Asymptotes and Discontinuities

Because secant is undefined where cosine is zero, the function will have vertical asymptotes at points where:

$$
\cos(3x) = 0
$$

The zeros of cosine occur at:

$$
3x = \frac{\pi}{2} + k\pi, \quad k \in \mathbb{Z}
$$

Solving for x:

$$
x = \frac{\pi/2 + k\pi}{3} = \frac{\pi}{6} + \frac{k\pi}{3}
$$

At these x-values, sec(3x) approaches infinity or negative infinity, creating vertical asymptotes, and the graph exhibits characteristic "U" or "n" shapes approaching these asymptotes.

Periodic Nature and Key Points

The period of the function is:

$$
\text{Period} = \frac{2\pi}{3}
$$

Within each period, the function reaches its maximum and minimum asymptotes and crosses specific points, such as where secant equals 1 or -1, which corresponds to cosine being ±1.

Features and Characteristics of the Function

Pros:

- Clear periodic behavior: The function repeats every $\frac{2\pi}{3}$, making it predictable over intervals.
- Distinct asymptotes: Vertical asymptotes provide clear markers for the function's undefined points, useful in graphing and analysis.
- Wide range: The function's range includes all values less than or equal to 1 and greater than or equal to 3, offering a broad spectrum of possible outputs.

Cons:

- Discontinuities: The function is not defined at asymptotes, which can complicate analysis or integration.
- Jump discontinuities: The graph jumps between the ranges $(-\infty, 1]$ and $[3, \infty)$, making certain calculus operations more complex.
- Limited domain: The function is periodic, but its domain is implicitly restricted to intervals between asymptotes, which can be limiting in some applications.

Applications of Understanding the Range

Knowing the range of $y = 2 + \sec(3x)$ is crucial in various contexts:

- Signal processing: In analyzing oscillations where amplitude constraints are essential.
- Engineering: For designing systems with predictable voltage or current limits modeled by secant functions.
- Mathematical modeling: When modeling phenomena with periodic discontinuities or asymptotic behaviors.

Summary and Final Thoughts

Determining the range of the function $y = 2 + \sec(3x)$ involves understanding the basic properties of the secant function, its transformations, and their combined effects. The key points are:

- The original secant function has a range of $(-\infty, -1]$ or $[1, \infty)$.
- Vertical shifts translate this range upward by 2 units, resulting in $(- \infty, 1]$ and $[3, \infty)$.
- The function exhibits vertical asymptotes at points where $\cos(3x) = 0$, i.e., at $x = \frac{\pi}{6} + \frac{k\pi}{3}$.

Understanding these aspects provides a comprehensive view of the behavior and characteristics of the function, allowing for accurate graphing, analysis, and application in various fields.

In conclusion, the range of $y = 2 + \sec(3x)$ is:

$$
\boxed{
(-\infty, 1] \cup [3, \infty)
}
$$

This insight is vital for mathematicians, engineers, and scientists working with periodic and oscillatory systems involving secant functions, enabling better modeling and problem-solving strategies.

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