F(x)=-2x2+8x-17 F(-34)

F(x)=-2x2+8x-17 F(-34): An In-Depth Analysis of Quadratic Functions and Their Applications

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Understanding the Function F(x) = $-2x^2 + 8x - 17$

The function F(x) = -2x2 + 8x - 17 is a quadratic function, a fundamental concept in algebra and mathematics at large. Quadratic functions are characterized by their degree, which is 2, and their parabolic graph shape. This particular function provides an excellent example to understand key quadratic properties, such as vertex, axis of symmetry, roots, and their real-world applications.

Breaking Down the Quadratic Function

Standard Form and Coefficients

The quadratic function is expressed in the standard form:

$$F(x) = ax^2 + bx + c$$

where:

- a = -2 (coefficient of x2)
- b = 8 (coefficient of x)
- c = -17 (constant term)

Understanding these coefficients helps determine the parabola's orientation, width, and position.

Graphing the Function

Given that a = -2 (negative), the parabola opens downward. The absolute value of a influences the "width" of the parabola: the larger the absolute value, the narrower the parabola.

Evaluating F(-34): Calculating the Function at x = -34

The expression F(-34) refers to evaluating the quadratic function when x is replaced by -34.

Step-by-Step Calculation

Let's compute F(-34):

$$
\begin{aligned}
F(-34) &= -2(-34)^2 + 8(-34) - 17 \\
&= -2(1156) + (-272) - 17 \\
&= -2312 - 272 - 17 \\
&= -2312 - 289 \\
&= -2601
\end{aligned}
$$

Thus, F(-34) = -2601.

This value indicates the function's output at x = -34, which is significantly negative, reflecting the parabola's position relative to the x-axis.

Vertex of the Parabola

The vertex of a parabola given in standard form can be found using the formulas:

$$
x_{v} = -\frac{b}{2a}
$$

$$
F(x_{v}) = \text{the maximum or minimum value of the function}
$$

Calculating the Vertex Coordinates

Given a = -2, b = 8:

$$
x_{v} = -\frac{8}{2 \times -2} = -\frac{8}{-4} = 2
$$

Now, compute $F(2)$:

$$
F(2) = -2(2)^2 + 8(2) - 17 = -2(4) + 16 - 17 = -8 + 16 - 17 = -9
$$

Vertex Coordinates: (2, -9)

The vertex at (2, -9) is the highest point on the parabola because the parabola opens downward ($a < 0$). Here, -9 is the maximum value of the function.

Axis of Symmetry and Parabola Properties

The axis of symmetry is a vertical line passing through the vertex, given by:

$$
x = x_{v} = 2
$$

This line divides the parabola into two mirror images and is essential for graphing and analyzing the function.

Key properties:

- Maximum point: At (2, -9)
- Direction: Opens downward because $a < 0$
- Width: Narrower than a standard parabola with smaller $|a|$ values
- Roots or x-intercepts: Points where $F(x) = 0$

Finding the Roots of the Function

The roots or zeros of the quadratic function are solutions to:

$$
-2x^2 + 8x - 17 = 0
$$

Using the Quadratic Formula

The quadratic formula:

$$
x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}
$$

Plugging in the coefficients:

$$
x = \frac{-8 \pm \sqrt{8^2 - 4 \times (-2) \times (-17)}}{2 \times -2}
$$

Calculate discriminant:

$$
\Delta = 8^2 - 4 \times -2 \times -17 = 64 - 4 \times -2 \times -17
$$

Note that:

$$
4 \times -2 \times -17 = 4 \times 34 = 136
$$

Therefore,

$$
\Delta = 64 - 136 = -72
$$

Since the discriminant is negative, the quadratic has no real roots but two complex roots.

Complex Roots Calculation

$$
x = \frac{-8 \pm \sqrt{-72}}{ -4 }
$$

$$
\sqrt{-72} = \sqrt{72} \times i = 6 \sqrt{2} \times i
$$

Substituting back:

$$
x = \frac{-8 \pm 6 \sqrt{2} i}{ -4 } = \frac{-8}{ -4 } \pm \frac{6 \sqrt{2} i}{ -4 } = 2 \mp \frac{3 \sqrt{2} i}{ 2 }
$$

Complex roots:

$$
x = 2 \pm \frac{3 \sqrt{2} i}{ 2 }
$$

This indicates the parabola does not intersect the x-axis.

Real-World Applications of Quadratic Functions

Quadratic functions are prevalent across various fields, demonstrating their importance beyond pure mathematics.

Common Applications Include:

- **Projectile Motion:** The path of objects thrown or launched follows a quadratic curve, where the height depends on time, gravity, initial velocity, and angle.
- **Economics and Business:** Maximizing profit or minimizing cost, often modeled with quadratic functions to determine optimal points.
- **Engineering:** Design of parabolic reflectors and satellite dishes relies on parabola properties.
- **Biology and Ecology:** Population models sometimes employ quadratic equations to project growth or decline phases.

Graphical Interpretation of $F(x) = -2x^2 + 8x - 17$

Visualizing the function provides insight into its behavior and key features.

Key Steps to Graph

1. Plot the vertex at (2, -9)

2. Draw the axis of symmetry at x = 2

3. Identify the y-intercept by evaluating F(0):

   $$
   F(0) = -2(0)^2 + 8(0) - 17 = -17
   $$

   Plot point (0, -17)

4. Since the roots are complex, the parabola does not cross the x-axis.

5. Use symmetry to plot points on either side of the vertex to complete the parabola.

Summary and Key Takeaways

- The quadratic function F(x) = -2x2 + 8x - 17 opens downward, with a vertex at (2, -9).
- Evaluating F(-34) yields -2601, highlighting the function's value far from the vertex.
- The discriminant is negative, indicating no real roots, and the parabola does not intersect the x-axis.
- The function's maximum value is -9 at x = 2.
- Quadratic functions have wide-ranging applications, from physics to economics, demonstrating their importance in modeling real-world phenomena.
- Graphically, understanding the vertex, axis of symmetry, and intercepts aids in visualizing the function's behavior.

Conclusion

The exploration of F(x) = -2x2 + 8x - 17 and its value at x = -34 offers a comprehensive understanding of quadratic functions' properties, their graphing techniques, and their practical applications. Whether analyzing motion, optimizing business strategies, or designing engineering structures, quadratic functions serve as essential tools in both academic and professional contexts. By mastering the concepts of vertex, roots, and symmetry, students and professionals alike can better interpret and utilize quadratic models in their respective fields.

Frequently Asked Questions

What is the quadratic function given in the problem?

The quadratic function is $F(x) = -2x^2 + 8x - 17$.

How do you evaluate F(-34) for the given function?

To evaluate F(-34), substitute $x = -34$ into the function and simplify.

What is the value of F(-34) for the function $F(x) = -2x^2 + 8x - 17$?

$F(-34) = -2(-34)^2 + 8(-34) - 17 = -2(1156) + (-272) - 17 = -2312 - 272 - 17 = -2601$.

What is the general process to evaluate a quadratic function at a specific x-value?

Substitute the x-value into the function and perform the arithmetic operations to find the result.

Is the parabola opens upward or downward in the given function?

Since the coefficient of x^2 is -2 (negative), the parabola opens downward.

What is the vertex form of the quadratic function $F(x) = -2x^2 + 8x - 17$?

First, complete the square: $F(x) = -2(x^2 - 4x) - 17$. Then, rewrite as $F(x) = -2[(x - 2)^2 - 4] - 17 = -2(x - 2)^2 + 8 - 17 = -2(x - 2)^2 - 9$.

What is the significance of the vertex in this quadratic function?

The vertex represents the maximum point of the parabola since it opens downward; in this case, at (2, -9).

How does the value of x affect the output of the function F(x)?

Since the function is quadratic with a negative leading coefficient, as x moves away from the vertex, F(x) decreases.

Can you determine whether F(-34) is positive or

negative?

$F(-34) = -2601$, which is negative.

Additional Resources

Understanding and Calculating $F(x) = -2x^2 + 8x - 17$ at $F(-34)$: A Comprehensive Guide

When exploring quadratic functions such as $F(x) = -2x^2 + 8x - 17$, evaluating the function at specific points like $F(-34)$ helps us understand its behavior, roots, and graph. In this detailed guide, we will walk through the process of calculating $F(-34)$, analyze the function's characteristics, and provide insights into how quadratic functions operate. Whether you're a student studying algebra or a professional brushing up on mathematical concepts, this article aims to clarify each step involved in tackling such problems.

Understanding the Function $F(x) = -2x^2 + 8x - 17$

Before delving into the calculation of $F(-34)$, it’s important to understand the structure and properties of the given quadratic function.

The Basic Structure of a Quadratic Function

A quadratic function is generally expressed as:

$F(x) = ax^2 + bx + c$

where:
- $a \neq 0$ (if $a = 0$, the function becomes linear),
- b is the coefficient of x,
- c is the constant term.

In our case:
- $a = -2$
- $b = 8$
- $c = -17$

The graph of this quadratic function is a parabola opening downward because a is negative.

Key Features of the Quadratic Function

- Vertex: The highest or lowest point on the parabola.
- Axis of Symmetry: The vertical line passing through the vertex.
- Roots (Zeros): The x-values where the function equals zero.
- Y-intercept: The point where the graph crosses the y-axis.

Step-by-Step Calculation of F(-34)

Now, let’s proceed with calculating F(-34), which involves substituting $x = -34$ into the function.

Step 1: Write the function with x replaced by -34

$F(-34) = -2(-34)^2 + 8(-34) - 17$

Step 2: Calculate each term carefully

- $(-34)^2$: Since squaring a negative number results in a positive,

$(-34)^2 = 1156$

- Multiply by -2:

$-2 \times 1156 = -2312$

- Calculate 8×-34:

$8 \times -34 = -272$

- Constant term:

-17 (unchanged)

Step 3: Combine all the parts

$F(-34) = -2312 + (-272) - 17$

Step 4: Simplify stepwise

- First, combine the first two:

$-2312 - 272 = -2584$

- Then, subtract 17:

$-2584 - 17 = -2601$

Final Result:

$F(-34) = -2601$

Analyzing the Result and the Function's Behavior

Having calculated F(-34), let's explore what this value tells us about the quadratic function and how it fits into the broader context of the function's properties.

Significance of the Value $F(-34) = -2601$

- The value is highly negative, indicating that at $x = -34$, the parabola dips significantly below the x-axis.
- This suggests that $x = -34$ is well outside the range of the parabola's vertex, which is typically where the maximum or minimum occurs depending on the parabola's orientation.

Exploring the Characteristics of the Quadratic Function

1. Finding the Vertex

The vertex form of a quadratic function is useful for understanding its maximum or minimum point.

Vertex x-coordinate:

$x_v = -b / (2a)$

Plugging in the values:

$x_v = -8 / (2 \times -2) = -8 / -4 = 2$

Vertex y-coordinate:

$F(2) = -2(2)^2 + 8(2) - 17$

$= -2(4) + 16 - 17$

$= -8 + 16 - 17$

$= (8) - 17 = -9$

Vertex point: $(2, -9)$

Interpretation: The parabola reaches its maximum value of -9 at $x = 2$.

2. Axis of Symmetry

The axis of symmetry is the vertical line passing through the vertex:

$x = 2$

3. Roots (Zeros) of the Function

To find where $F(x) = 0$, solve:

$-2x^2 + 8x - 17 = 0$

Use the quadratic formula:

$x = [-b \pm \sqrt{(b^2 - 4ac)}] / (2a)$

Plugging in the coefficients:

$x = [-8 \pm \sqrt{(8^2 - 4\times(-2)\times(-17))}] / (2\times-2)$

$x = [-8 \pm \sqrt{(64 - 4\times-2\times-17)}] / -4$

Calculate discriminant:

$4\times-2\times-17 = 4\times34 = 136$

But note that:

$-4\times a\times c = -4\times-2\times-17 = -4\times-2\times-17 = -4\times34 = -136$

Wait, correction: The discriminant is:

$\Delta = b^2 - 4ac = 64 - 4\times(-2)\times(-17)$

Calculate $4\times(-2)\times(-17)$:

$4\times(-2) = -8$

$-8\times(-17) = 136$

Now, the discriminant:

$\Delta = 64 - 136 = -72$

Since the discriminant is negative, the quadratic has no real roots; the parabola does not cross the x-axis.

Implication: The parabola is entirely below or above the x-axis depending on its orientation, but since the vertex is at $y = -9$ (negative), and it opens downward, the entire parabola is below the x-axis, consistent with no real roots.

Visualizing the Function

Understanding the graph is crucial:

- The parabola opens downward (since $a = -2$).
- The vertex is at $(2, -9)$, the maximum point.
- The parabola extends downward, crossing the y-axis at $y = -17$.
- At $x = -34$, the function is at $y = -2601$, far below the vertex, illustrating the parabola's steep decline on that side.
- No real roots indicate the parabola doesn't intersect the x-axis.

Practical Applications and Contexts

Quadratic functions like $F(x) = -2x^2 + 8x - 17$ are prevalent across various fields:

- Physics: modeling projectile motion where the downward parabola represents the trajectory.
- Economics: analyzing profit functions with maximum points.
- Engineering: designing parabolic structures.
- Mathematics Education: understanding quadratic behavior and solving for specific values.

Knowing how to evaluate functions at specific points, such as F(-34), allows for precise predictions and analyses within these applications.

Summary and Key Takeaways

- To evaluate F(-34) in the quadratic function $F(x) = -2x^2 + 8x - 17$, substitute x = -34 and carefully perform the calculations.
- The computed value, F(-34) = -2601, reflects the function's value at that point, illustrating the parabola's steep decline on that side.
- The vertex of the parabola is at (2, -9), representing the maximum point of the function.
- The discriminant indicates no real roots, so the parabola remains below the x-axis.
- Understanding these properties enhances comprehension of quadratic functions and their behaviors.

Final Thoughts

Mastering the process of evaluating quadratic functions at specific points, such as F(-34), is fundamental in algebra and beyond. It provides insights into the shape and features of the graph, helps solve real-world problems, and deepens understanding of quadratic behavior. Whether you're analyzing projectile paths, optimizing profit functions, or simply exploring mathematical concepts, these skills are invaluable tools in your analytical toolkit.

Remember: Always double-check your calculations, understand the structure of the quadratic, and visualize the graph for a comprehensive grasp of the function's behavior.

F X 2x2 8x 17 F 34

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