Factor Completely.2x 18x +36 =

Factor Completely.2x 18x +36 =: A Comprehensive Guide to Factoring Quadratic Expressions

When tackling algebraic expressions, one of the fundamental skills students and mathematicians alike need to master is factoring. Understanding how to factor expressions completely not only simplifies solving equations but also deepens comprehension of algebraic structures. In this article, we focus on a specific quadratic expression: **2x^2 + 18x + 36**. Our goal is to guide you step by step through the process of factoring this quadratic expression completely, ensuring you grasp the underlying principles and techniques involved.

Understanding the Expression: 2x^2 + 18x + 36

Breaking Down the Components

The quadratic expression in question is:

2x^2 + 18x + 36

At first glance, this appears to be a standard quadratic trinomial. To factor it effectively, it's essential to analyze its components:

- The leading coefficient (a) is 2.
- The middle coefficient (b) is 18.
- The constant term (c) is 36.

Recognizing these components allows us to determine the most appropriate factoring method, such as factoring out the greatest common factor (GCF) or using the quadratic formula if necessary.

Step-by-Step Process to Factor Completely

Step 1: Find the Greatest Common Factor (GCF)

Start by checking if there's a common factor among all terms. For 2x^2 + 18x

+ 36, observe the coefficients:

- 2 (from $2x^2$)
- 18 (from 18x)
- 36 (constant term)

All these numbers are divisible by 6, so the GCF is 6. Factoring out 6 simplifies the expression and makes further factoring easier:

$$6\left(\frac{2x^2}{6} + \frac{18x}{6} + \frac{36}{6} \right) = 6\left(\frac{1}{3}x^2 + 3x + 6 \right)$$

However, to keep the coefficients integral, it's better to factor out the GCF as 6:

$$6\left(2x^2/6 + 18x/6 + 36/6 \right) = 6\left((1/3)x^2 + 3x + 6 \right)$$

But notice that dividing $2x^2$ by 6 results in a fractional coefficient, which complicates factoring. To avoid fractions, it's preferable to factor out 2 (the leading coefficient) rather than 6.

Step 2: Factor out the GCF (preferably the leading coefficient)

Since the leading coefficient is 2, and all coefficients are divisible by 2, factor out 2:

$$2(x^2 + 9x + 18)$$

Now, our task is to factor the quadratic inside the parentheses: $x^2 + 9x + 18$.

Step 3: Factor the quadratic $x^2 + 9x + 18$

This quadratic has coefficients:

- Coefficient a = 1
- Coefficient b = 9
- Constant c = 18

Since the leading coefficient is 1, we can apply factoring methods such as trial, decomposition, or the ac method directly.

Step 4: Find two numbers that multiply to c (18) and add to b (9)

To factor x^2 + 9x + 18, look for two numbers:

- Product = 18
- Sum = 9

Possible pairs of factors of 18:

1. 1 and 18 (sum = 19)
2. 2 and 9 (sum = 11)
3. 3 and 6 (sum = 9)

Here, 3 and 6 satisfy the conditions:

- 3 × 6 = 18
- 3 + 6 = 9

Step 5: Write the factored form of the quadratic

Using these numbers, we can express the quadratic as:

$$(x + 3)(x + 6)$$

Step 6: Write the completely factored expression

Recall that we factored out a 2 earlier, so the complete factorization of the original quadratic is:

$$2(x + 3)(x + 6)$$

Final Answer and Explanation

The quadratic expression **$2x^2 + 18x + 36$** factors completely as:

$$2(x + 3)(x + 6)$$

This is the fully factored form, and it cannot be simplified further. Recognizing the common factor and applying the middle-term factoring method allowed us to factor this quadratic efficiently.

Additional Tips for Factoring Quadratics

Recognize When to Use Different Methods

Depending on the quadratic, different methods may be more efficient:

- **Factoring out GCF:** Always check for common factors first.
- **Simple trinomial ($a=1$):** Use trial, decomposition, or the ac method.
- **Quadratic with $a \neq 1$:** Use factoring, completing the square, or quadratic formula as needed.

Quadratic Formula as a Backup

If factoring proves difficult, remember the quadratic formula:

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Apply this when the quadratic doesn’t factor nicely, or to verify factors.

Common Mistakes to Avoid

- Forgetting to factor out the GCF first.
- Misidentifying the pair of numbers that multiply to c and add to b.
- Not checking for the greatest common factor after factoring the quadratic.

Practice Problems for Mastery

1. Factor $3x^2 + 12x + 12$ completely.
2. Factor $x^2 + 5x + 6$.
3. Factor $4x^2 + 25x + 24$.
4. Factor $6x^2 + 7x + 1$.

Practicing these problems will reinforce your understanding of factoring quadratics and improve your skill in recognizing different factoring techniques.

Conclusion

Mastering the process of factoring quadratic expressions like **$2x^2 + 18x + 36$** is a vital skill in algebra. By systematically identifying the greatest common factor, applying the middle-term factoring method, and verifying your factors, you can efficiently factor any quadratic expression completely. Remember to always check for common factors first and practice a variety of problems to strengthen your skills. With these techniques, you'll be well-equipped to handle more complex algebraic expressions and solve equations with confidence.

Frequently Asked Questions

What is the first step in factoring the quadratic expression $2x^2 + 18x + 36$?

The first step is to factor out the greatest common factor (GCF), which is 2, resulting in $2(x^2 + 9x + 18)$.

How do you factor the quadratic expression $x^2 + 9x + 18$?

You look for two numbers that multiply to 18 and add up to 9; these numbers are 3 and 6, so the factored form is $(x + 3)(x + 6)$.

What is the completely factored form of $2x^2 + 18x + 36$?

The completely factored form is $2(x + 3)(x + 6)$.

Can the quadratic 2x^2 + 18x + 36 be factored using the quadratic formula?

Yes, but factoring by finding two numbers that multiply to ac and add to b is more straightforward here. The quadratic formula can also be used to find roots if needed.

What are the roots of the quadratic equation 2x^2 + 18x + 36 = 0?

The roots are $x = -3$ and $x = -6$, found by setting the factored form equal to zero.

Is factoring completely the preferred method for solving quadratic expressions like 2x^2 + 18x + 36?

Yes, factoring is often the simplest method when the quadratic can be factored easily, as in this case.

What are common mistakes to avoid when factoring quadratic expressions like 2x^2 + 18x + 36?

Common mistakes include missing the greatest common factor, incorrect pairing of factors, or forgetting to factor out the GCF first.

How can you verify that your factored form of 2x^2 + 18x + 36 is correct?

By expanding the factored form $(2(x + 3)(x + 6))$ to ensure it simplifies back to the original expression.

Are there alternative methods to factor 2x^2 + 18x + 36 besides factoring by grouping?

Yes, methods like completing the square or using the quadratic formula can be used if factoring directly is challenging, but in this case, simple factoring is most efficient.

Additional Resources

Factorization

When tackling algebraic expressions, especially polynomial expressions, the ability to factor completely is an essential skill for students, educators, and math enthusiasts alike. Properly factoring an expression like $2x^2 + 18x + 36$ not only simplifies the problem but also reveals underlying properties of the quadratic, such as its roots and graph behavior. In this comprehensive review, we will delve into the step-by-step process of factoring the quadratic expression $2x^2 + 18x + 36$, exploring various methods, tips, and common pitfalls to ensure mastery.

Understanding the Expression: $2x^2 + 18x + 36$

Before diving into the factorization process, it’s vital to understand the structure of the given quadratic:

- Coefficients:
- Leading coefficient (a): 2
- Middle coefficient (b): 18
- Constant term (c): 36

- Degree:
- The quadratic is degree 2, which generally means it can be factored into the product of two binomials.

- Goals:
- Rewrite the quadratic as a product of binomials, such as (mx + n)(px + q), where m, n, p, q are numbers or expressions.

Why factor completely?
Factoring an expression like $2x^2 + 18x + 36$ simplifies solving equations, graphing, and understanding the behavior of the quadratic function. It allows us to find roots, analyze intercepts, and perform further algebraic operations efficiently.

Step-by-Step Approach to Factor Completely

The process involves several steps, starting with identifying the greatest common factor (GCF), then applying methods like factoring by grouping or the quadratic formula if necessary.

Step 1: Factor out the Greatest Common Factor (GCF)

Every term in the quadratic involves a common factor:

- $2x^2$
- 18x
- 36

The GCF of these coefficients is 2.

Factoring out 2:

$$
2x^2 + 18x + 36 = 2(x^2 + 9x + 18)
$$

This simplifies the problem to factoring the quadratic inside the parentheses:

$$
x^2 + 9x + 18
$$

Step 2: Focus on the Quadratic Inside the Parentheses

Now, the goal is to factor:

$$
x^2 + 9x + 18
$$

This is a standard quadratic, which we can attempt to factor by:

- Factoring Trinomial Method
- AC Method
- Quadratic Formula (if necessary)

Method 1: Factoring by Inspection (Trial and Error)

Look for two numbers that:

1. Multiply to the constant term (18)
2. Add up to the middle coefficient (9)

Factors of 18:

- 1 and 18
- 2 and 9
- 3 and 6

Check sums:

- 1 + 18 = 19
- 2 + 9 = 11
- 3 + 6 = 9

The pair 3 and 6 sums to 9, matching the middle coefficient.

Therefore:

$$
x^2 + 9x + 18 = (x + 3)(x + 6)
$$

Step 3: Write the Fully Factored Form

Recall that we factored out 2 initially:

$$
2(x + 3)(x + 6)
$$

This is the completely factored form of the original quadratic.

Additional Methods and Considerations

While the above method is straightforward, let’s explore other approaches and their relevance to similar problems.

Method 2: AC Method (or Factoring by Grouping)

For quadratic expressions where the middle term's coefficients are larger, the AC method involves:

- Multiplying the coefficient of x^2 (a) and the constant term (c): $a \times c$
- Finding two numbers that multiply to $a \times c$ and add to b

For our quadratic:

$$
a = 1, \quad c = 18, \quad a \times c = 18
$$

Find two numbers that multiply to 18 and add to 9:

- 3 and 6 (as before)

Rewrite the middle term:

$$
x^2 + 3x + 6x + 18
$$

Group terms:

$$
(x^2 + 3x) + (6x + 18)
$$

Factor each group:

$$
x(x + 3) + 6(x + 3)
$$

Factor out the common binomial:

$$
(x + 3)(x + 6)
$$

Again, multiply by the GCF outside:

$$
2(x + 3)(x + 6)
$$

Method 3: Quadratic Formula (When Factoring Is Not Obvious)

If factoring by inspection or AC method seems complex, the quadratic formula provides a reliable fallback:

$$
x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}
$$

For the quadratic $2x^2 + 18x + 36$:

$$
a=2, \quad b=18, \quad c=36
$$

Calculate the discriminant:

$$
D = b^2 - 4ac = (18)^2 - 4 \times 2 \times 36 = 324 - 288 = 36
$$

Since D is positive and a perfect square, roots are real and rational:

$$
x = \frac{-18 \pm \sqrt{36}}{2 \times 2} = \frac{-18 \pm 6}{4}
$$

Calculate both roots:

- $x = \frac{-18 + 6}{4} = \frac{-12}{4} = -3$
- $x = \frac{-18 - 6}{4} = \frac{-24}{4} = -6$

Expressed as factors:

$$
(x + 3)(x + 6)
$$

And factoring out the GCF again:

$$
2(x + 3)(x + 6)
$$

Final Fully Factored Form and Interpretation

The complete factorization of the original quadratic expression $2x^2 + 18x + 36$ is:

$$
\boxed{2(x + 3)(x + 6)}
$$

Applications and Significance of Fully Factoring

Understanding how to factor quadratics thoroughly has multiple practical applications:

- Solving quadratic equations: Setting the factored form equal to zero quickly yields solutions.
- Graphing parabolas: Roots obtained from factors determine x-intercepts.
- Simplifying algebraic expressions: Factoring allows for cancellation of common factors in rational expressions.
- Identifying properties: The factors reveal the roots, vertex, and axis of symmetry of the parabola.

In this case:

- Roots are at $x = -3$ and $x = -6$.
- The parabola opens upward (since the leading coefficient 2 is positive).
- The vertex lies between these roots, at the midpoint -4.5.

Common Pitfalls and Tips for Effective Factoring

Pitfalls:

- Overlooking the GCF: Always check for a GCF before attempting more complex methods.
- Incorrect factor pairs: Make sure to verify pairs by multiplication and addition.
- Forgetting to check for perfect square discriminant: The quadratic formula is most useful when the discriminant is a perfect square.
- Not simplifying fully: Always include the GCF in the final answer unless specified otherwise.

Tips:

- Always start with the GCF.
- Use mental math to quickly identify factor pairs.
- Practice multiple methods to increase flexibility.

- When in doubt, use the quadratic formula to find roots first, then write factors.

Conclusion

Factoring quadratic expressions like $2x^2 + 18x + 36$ is a fundamental skill that underpins much of algebra. Through systematic steps—factoring out the GCF, applying the AC method, or employing the quadratic formula—you can efficiently arrive at the fully factored form:

$$
\boxed{2(x + 3)(x + 6)}
$$

This process not only simplifies solving equations but also deepens understanding of the quadratic's properties. Mastery of these techniques empowers students and professionals to approach more complex algebraic problems with confidence, making factorization an indispensable tool in their mathematical toolkit.

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