Factorise 15x– 10x^3

Factorise 15x - 10x^3 is a fundamental algebraic skill that helps simplify expressions, solve equations, and understand the underlying structure of algebraic formulas. Whether you're a student learning algebra for the first time or someone brushing up on factorisation techniques, mastering how to factorise expressions like 15x - 10x^3 is essential. In this article, we will explore the step-by-step process of factorising this expression, understand the concepts behind factorisation, and look at various methods and examples to deepen your understanding.

Understanding the Expression 15x - 10x^3

Before diving into the process of factorisation, it’s important to analyze the given algebraic expression:

- Expression: 15x - 10x^3
- Type: Polynomial, specifically a binomial (two terms).

This expression consists of two terms with variables and coefficients, which suggests that common factorisation techniques can be applied. Recognising the greatest common factor (GCF) among both terms is often the first step.

Step-by-Step Guide to Factorising 15x - 10x^3

Step 1: Find the Greatest Common Factor (GCF)

The initial step in factorising any polynomial is to identify the GCF of all terms. Here are the coefficients:

- 15
- 10

The GCF of 15 and 10 is 5.

Next, look at the variable parts:

- x
- x^3

The GCF of x and x^3 is x, since x is the lowest power among the variables.

Therefore, the GCF of the entire expression is: 5x

Step 2: Factor out the GCF

Now, divide each term by the GCF:

- $15x \div 5x = 3$
- $10x^3 \div 5x = 2x^2$

Express the original polynomial as:

$15x - 10x^3 = 5x(3 - 2x^2)$

This is the factorised form, where 5x is factored out, leaving a binomial inside the parentheses.

Understanding the Factorised Form

The factored form $5x(3 - 2x^2)$ simplifies the original expression, making it easier to work with in equations or further algebraic manipulations. It reveals the structure of the polynomial, especially the roots or solutions when set equal to zero.

Further Techniques for Factorisation

While the above method is straightforward for this expression, sometimes more complex polynomials require advanced techniques:

1. Recognising Difference of Squares

The expression inside the parentheses, $3 - 2x^2$, can be examined to see if it's a difference of squares or can be rewritten as such.

- 3 can be written as $(\sqrt{3})^2$
- $2x^2$ can be written as $(\sqrt{2}\,x)^2$

But since these are not perfect squares, this approach doesn't directly apply here.

2. Factoring Quadratic Forms

If after factoring out GCF, the remaining polynomial is quadratic, techniques like factoring by grouping or quadratic formula may be used.

In our case, $3 - 2x^2$ is a quadratic in form:

- Rewrite as: $-2x^2 + 3$

To factor further, you might consider factoring out -1:

- $-1 (2x^2 - 3)$

But since $2x^2 - 3$ does not factor nicely over rationals, it remains as is unless working over complex numbers.

Applications of Factoring $15x - 10x^3$

Factoring expressions like $15x - 10x^3$ is more than an academic exercise; it has practical applications:

- **Simplifying Algebraic Expressions:** Simplifies complex formulas for easier calculation.
- **Solving Equations:** Setting the factored form equal to zero helps find the roots or solutions quickly.
- **Graphing:** Identifying zeros of the function by solving the factored form aids in graphing.
- **Calculus:** Factoring is a preliminary step in differentiation or integration of polynomial functions.

Solving Equations Using the Factorised Form

Suppose you want to solve the equation:

$15x - 10x^3 = 0$

Using the factorised form:

$5x (3 - 2x^2) = 0$

Set each factor equal to zero:

1. $5x = 0 \Rightarrow x = 0$
2. $3 - 2x^2 = 0 \Rightarrow 2x^2 = 3 \Rightarrow x^2 = 3/2 \Rightarrow x = \pm\sqrt{3/2}$

Solutions: $x = 0$, $x = \sqrt{3/2}$, $x = -\sqrt{3/2}$

This demonstrates how factorisation simplifies solving polynomial equations.

Practice Problems to Master Factorisation

To reinforce your understanding, here are some practice problems:

1. Factorise $20x - 15x^2$
2. Factorise $24x^3 - 36x^2$
3. Factorise $18x^2 - 8x$
4. Factorise $50x^3 + 75x^2$
5. Factorise $9x^2 - 16$

Solutions:

1. GCF = $5x$; $20x - 15x^2 = 5x(4 - 3x)$
2. GCF = $12x^2$; $24x^3 - 36x^2 = 12x^2(2x - 3)$
3. GCF = $2x$; $18x^2 - 8x = 2x(9x - 4)$
4. GCF = $25x^2$; $50x^3 + 75x^2 = 25x^2(2x + 3)$
5. Recognise as difference of squares: $9x^2 - 16 = (3x)^2 - 4^2 = (3x - 4)(3x + 4)$

Conclusion

Factorising algebraic expressions like $15x - 10x^3$ is an essential skill that simplifies calculations, aids in solving equations, and helps understand the structure of polynomials. The key steps involve identifying the greatest common factor and applying relevant algebraic identities or techniques as needed. With consistent practice, you can develop confidence in factorising a wide range of algebraic expressions, which is foundational for advanced mathematics topics like calculus, algebraic functions, and polynomial theory.

Remember, mastering factorisation not only makes solving equations easier but also deepens your understanding of how algebraic expressions behave and are constructed. Keep practicing with different types of polynomials to enhance your skills!

Frequently Asked Questions

How do I factorise the expression 15x - 10x^3?

First, identify the common factors in both terms, which are $5x$. Factoring out $5x$ gives: $5x(3 - 2x^2)$.

What is the factored form of $15x - 10x^3$?

The factored form is $5x(3 - 2x^2)$.

Can $3 - 2x^2$ be factored further?

No, $3 - 2x^2$ is a binomial that does not factor further over the real numbers.

What common factor can be extracted from $15x - 10x^3$?

The greatest common factor is $5x$.

Is it possible to factorise $15x - 10x^3$ using difference of squares?

No, because $15x - 10x^3$ is not a difference of squares; it factors by extracting the common factor first.

Additional Resources

Factorise $15x - 10x^3$: A Comprehensive Guide to Simplifying Algebraic Expressions

Understanding how to factorise $15x - 10x^3$ is a fundamental skill in algebra that opens the door to simplifying complex expressions and solving equations efficiently. Whether you're a student working through coursework or a professional reviewing algebraic concepts, mastering the process of factorisation is crucial. In this guide, we will explore the step-by-step method to break down the expression $15x - 10x^3$ into its simplest factors, discuss the underlying principles, and provide practical tips to enhance your algebraic toolkit.

Introduction to Factorisation

Factorisation, also known as factoring, involves expressing a mathematical expression as a product of its factors. These factors are elements that, when multiplied together, produce the original expression. The ability to factorise expressions like $15x - 10x^3$ is essential for simplifying expressions, solving equations, and analyzing algebraic functions.

Understanding the Expression: $15x - 10x^3$

Before diving into the factorisation process, let's analyze the given expression:

- Terms involved: $15x$ and $-10x^3$

- Common factors: Both terms contain the variable x and numerical coefficients.

This expression is a binomial, consisting of two terms that are similar in structure but differ in coefficients and powers of x.

Step-by-Step Factorisation Process

Let's explore the systematic process to factorise $15x - 10x^3$.

Step 1: Identify the Greatest Common Factor (GCF)

The first step in factorisation is to find the Greatest Common Factor (GCF) of the terms involved.

- Numerical coefficients: 15 and 10
- Factors of 15: 1, 3, 5, 15
- Factors of 10: 1, 2, 5, 10
- GCF of 15 and 10: 5

- Variables: x and x^3
- Common variable factors: x (since x divides both x and x^3)

- GCF of variables: x (the lowest power among the terms)

Therefore, the overall GCF of the expression is:

- Numerical GCF: 5
- Variable GCF: x

Combined GCF: $5x$

Step 2: Factor out the GCF

Next, rewrite the original expression by factoring out $5x$:

$15x - 10x^3 = 5x (?)$

Divide each term by $5x$:

- $15x \div 5x = 3$
- $-10x^3 \div 5x = -2x^2$

This yields:

Factorised form:
$15x - 10x^3 = 5x (3 - 2x^2)$

Step 3: Examine the Remaining Expression

Now, focus on the expression inside the brackets: $3 - 2x^2$. Is it factorisable further?

- The expression $3 - 2x^2$ is a binomial similar to a quadratic form but with different coefficients.
- Since $3 - 2x^2$ resembles a difference of squares or quadratic expression, check for possible further factorisation.

Factorising the Inner Expression: $3 - 2x^2$

The expression $3 - 2x^2$ can be viewed as:

$3 - 2x^2 = A - B$, where

- $A = 3$ (a constant)
- $B = 2x^2$

Is this a difference of squares? Let's analyze:

- Difference of squares has the form $a^2 - b^2$.
- 3 is not a perfect square, so cannot be written as a^2 directly.
- $2x^2$ can be written as $(\sqrt{2}\,x)^2$.

But since 3 is not a perfect square, the expression $3 - 2x^2$ cannot be factored into real quadratic factors directly via difference of squares.

Alternative approach: Recognize quadratic form

Rewrite $3 - 2x^2$ as:

- $-2x^2 + 3$

Or,

- $-(2x^2 - 3)$

This form indicates that the expression $2x^2 - 3$ resembles a quadratic in x, and can be factored further if it factors over real numbers.

Step 4: Consider quadratic factorisation

Since $2x^2 - 3$ is quadratic in x, see if it factors:

- The quadratic $2x^2 - 3$ has no middle term, indicating it might not factor nicely over the reals.

Check the discriminant:

- Discriminant, $\Delta = b^2 - 4ac$
- Here, $a=2$, $b=0$, $c=-3$
- $\Delta = 0^2 - 42(-3) = 0 + 24 = 24$

Since the discriminant is positive, the quadratic has real roots and can be factored over real numbers:

$$2x^2 - 3 = 0$$

Solve for roots:

$$x^2 = 3/2$$

$$x = \pm\sqrt{(3/2)}$$

Expressed as factors:

$$2x^2 - 3 = 2(x - \sqrt{(3/2)})(x + \sqrt{(3/2)})$$

But note that $\sqrt{(3/2)}$ can be simplified:

$$\sqrt{(3/2)} = \sqrt{3} / \sqrt{2} = (\sqrt{6}) / 2$$

Therefore,

$$2x^2 - 3 = 2 (x - \sqrt{6}/2)(x + \sqrt{6}/2)$$

Multiplying out:

$$2 (x - \sqrt{6}/2)(x + \sqrt{6}/2) = 2 [x^2 + x(\sqrt{6}/2) - x(\sqrt{6}/2) - (\sqrt{6}/2)^2] = 2 [x^2 - (6/4)] = 2 [x^2 - 3/2] = 2x^2 - 3$$

So, the factorisation over reals is:

$$2x^2 - 3 = 2 (x - \sqrt{6}/2)(x + \sqrt{6}/2)$$

Step 5: Rewrite the inner expression

Recall that:

$$3 - 2x^2 = - (2x^2 - 3) = - 2 (x - \sqrt{6}/2)(x + \sqrt{6}/2)$$

Thus, the entire original expression becomes:

$$15x - 10x^3 = 5x \times [- 2 (x - \sqrt{6}/2)(x + \sqrt{6}/2)]$$

Simplify constants:

$$= -10x (x - \sqrt{6}/2)(x + \sqrt{6}/2)$$

Final Factored Form

The fully factorised form of $15x - 10x^3$ over the real numbers is:

$-10x\ (x - \sqrt{6}/2)(x + \sqrt{6}/2)$

Alternatively, you can write it as:

$-10x\ (x - \sqrt{6}/2)(x + \sqrt{6}/2)$

which explicitly shows the roots of the quadratic component.

Summary of the Factorisation Process

1. Identify the GCF: The GCF of $15x$ and $-10x^3$ is $5x$.
2. Factor out the GCF:
$15x - 10x^3 = 5x\ (3 - 2x^2)$
3. Analyze the inner expression:
Recognize $3 - 2x^2$ as related to the quadratic $2x^2 - 3$.
4. Factor the quadratic:
Over real numbers, $2x^2 - 3$ factors as $2(x - \sqrt{6}/2)(x + \sqrt{6}/2)$.
5. Express the complete factorisation:
Incorporate the negative sign and constants for the final expression.

Practical Tips for Factorising Similar Expressions

- Always start with the GCF, as it simplifies the expression and reduces complexity.
- Recognize common patterns such as difference of squares, quadratic trinomials, or sum/difference of cubes.
- When dealing with quadratic expressions, determine whether they can be factored over real numbers or require complex roots.
- Use the quadratic formula for roots that are not perfect squares to facilitate further factorisation.
- Maintain awareness of the signs and constants throughout the process to avoid errors.

Conclusion

Factorising algebraic expressions like $15x - 10x^3$ involves systematic steps:

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finding the GCF,
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