Factorise (x-2y)^2-6(x-2y)+5

Factorise (x-2y)^2 - 6(x-2y) + 5

Understanding how to factorize algebraic expressions is a fundamental skill in mathematics, especially in algebra. Today, we explore the process of factorising the quadratic expression (x - 2y)^2 - 6(x - 2y) + 5. This expression is a perfect example of how substitution can simplify complex algebraic forms into familiar quadratic equations, making the process of factorisation more straightforward. By the end of this guide, you'll be equipped with the techniques to approach similar problems confidently, whether you're a student preparing for exams or someone looking to strengthen your algebraic skills.

Understanding the Expression

Before diving into the factorisation process, it’s essential to understand what the expression involves. The expression is:

$$
(x - 2y)^2 - 6(x - 2y) + 5
$$

At first glance, it appears complex due to the binomial $(x - 2y)$ being repeated in different terms. To simplify the process, we can use substitution, treating $(x - 2y)$ as a single variable, say t:

$$
t = x - 2y
$$

This substitution transforms the original expression into a quadratic in t:

$$
t^2 - 6t + 5
$$

This approach is common in algebra because it reduces the complexity of the expression, allowing us to focus on familiar quadratic factorisation techniques.

Step-by-Step Factorisation Process

Now, let's go through the process of factorising the quadratic expression carefully.

Step 1: Substitution

- Recognise the substitution $t = x - 2y$ to simplify the expression.
- Rewrite the original expression as:

$$
t^2 - 6t + 5
$$

This form is easier to factorise using standard quadratic techniques.

Step 2: Factorising the quadratic in t

The quadratic $t^2 - 6t + 5$ can be factorised by looking for two numbers that:

- Multiply to the constant term, 5
- Add up to the coefficient of the middle term, -6

Let's find these numbers:

- Factors of 5: 1 and 5
- Sum of 1 and 5: 6
- Since the middle term is -6, we need factors that sum to -6, which are -1 and -5.

Check:

$$
-1 \times -5 = 5
$$
$$
-1 + -5 = -6
$$

Perfect! So, the factorisation of the quadratic is:

$$
(t - 1)(t - 5)
$$

Step 3: Replacing t back with $(x - 2y)$

Now, substitute back $t = x - 2y$:

$$
(x - 2y - 1)(x - 2y - 5)
$$

This is the factorised form of the original expression.

Final Factorised Form

$$
\boxed{
(x - 2y - 1)(x - 2y - 5)
}
$$

This factorisation indicates that the quadratic expression in the variables x and y decomposes neatly into two binomials.

Applications and Importance of Factorisation

Factorisation isn't just an academic exercise; it has a wide range of applications in mathematics and related fields. Here are some areas where understanding factorisation is crucial:

1. Solving Quadratic Equations

Factorising the quadratic expression allows you to find roots easily:

- Set each factor equal to zero:

$$
x - 2y - 1 = 0 \quad \text{or} \quad x - 2y - 5 = 0
$$

- Solve for x:

$$
x = 2y + 1 \quad \text{or} \quad x = 2y + 5
$$

This solution set is vital in solving systems of equations and in various algebraic applications.

2. Simplifying Algebraic Expressions

Factorisation helps in simplifying complex expressions, making calculations more manageable, especially when integrating or differentiating in calculus.

3. Polynomial Division and Factor Theorem

Understanding how to factor polynomials underpins the Polynomial Remainder Theorem and Polynomial Division techniques, which are essential for higher-level algebra.

4. Solving Real-world Problems

Many problems in physics, engineering, and economics involve quadratic relationships that can be simplified through factorisation, facilitating easier analysis and solutions.

Additional Techniques for Factorisation

While the current problem is straightforward, it’s helpful to review other common methods of factorising algebraic expressions:

1. Factoring Quadratic Trinomials

- Method: Find two numbers that multiply to the constant term and add to the middle coefficient.
- Example: $x^2 + 5x + 6 = (x + 2)(x + 3)$

2. Difference of Squares

- Form: $a^2 - b^2 = (a - b)(a + b)$
- Example: $x^2 - 9 = (x - 3)(x + 3)$

3. Perfect Square Trinomials

- Form: $a^2 \pm 2ab + b^2 = (a \pm b)^2$
- Example: $x^2 + 6x + 9 = (x + 3)^2$

4. Factoring by Grouping

- Suitable for four-term expressions where common factors can be extracted in pairs.

- Example: $ax + ay + bx + by = (a + b)(x + y)$

Practice Problems for Mastery

To reinforce your understanding, try factorising the following expressions:

1. $x^2 - 4x - 5$
2. $9x^2 - 16$
3. $x^2 + 7x + 12$
4. $3x^2 + 6x$
5. $x^3 - 3x^2 + 2x$

Solutions:

1. $(x - 5)(x + 1)$

2. $(3x - 4)(3x + 4)$ (Difference of squares)

3. $(x + 3)(x + 4)$

4. $3x(x + 2)$ (Factor out common term)

5. $x(x - 1)(x - 2)$ (Factor out x, then factor quadratic)

Conclusion

Factorising the expression $(x - 2y)^2 - 6(x - 2y) + 5$ reveals the power of substitution techniques in simplifying and solving algebraic problems. By treating $(x - 2y)$ as a single variable, the process reduces to familiar quadratic factorisation, making the problem manageable and transparent. Mastery of such techniques enhances problem-solving skills, enabling you to approach more complex algebraic expressions with confidence. Remember to explore different methods of factorisation, as they are fundamental tools in advanced mathematics and practical applications across various scientific disciplines.

Meta Description:
Learn how to factorise the algebraic expression $(x - 2y)^2 - 6(x - 2y) + 5$ with step-by-step guidance, tips, and application insights. Perfect for students and math enthusiasts aiming to strengthen their algebra skills.

Frequently Asked Questions

How do I factorise the expression $(x - 2y)^2 - 6(x - 2y) + 5$?

Let $u = x - 2y$. Then the expression becomes $u^2 - 6u + 5$, which can be factored as $(u - 1)(u - 5)$. Substituting back $u = x - 2y$, the factorised form is $(x - 2y - 1)(x - 2y - 5)$.

What is the first step to factorise $(x - 2y)^2 - 6(x - 2y) + 5$?

The first step is to substitute $u = x - 2y$, rewriting the expression as $u^2 - 6u + 5$ for easier factorisation.

Can I factorise $(x - 2y)^2 - 6(x - 2y) + 5$ directly without substitution?

It's difficult to factor directly; substituting $u = x - 2y$ simplifies the expression to a quadratic in u, making it easier to factor.

What are the factors of $u^2 - 6u + 5$?

The factors are $(u - 1)(u - 5)$, since these multiply to $u^2 - 6u + 5$.

How can I verify that $(x - 2y - 1)(x - 2y - 5)$ is the correct factorisation?

You can expand $(x - 2y - 1)(x - 2y - 5)$ and check if it simplifies back to the original expression.

Is the expression $(x - 2y)^2 - 6(x - 2y) + 5$ a quadratic in disguise?

Yes, when viewed as a quadratic in $u = x - 2y$, it becomes $u^2 - 6u + 5$.

What is the importance of substitution in factorising this expression?

Substitution simplifies the expression to a standard quadratic form, making factorisation straightforward.

Are the factors (x - 2y - 1) and (x - 2y - 5) linear factors?

Yes, both are linear factors in terms of x and y.

Can this factorisation be useful in solving equations involving (x - 2y)^2 - 6(x - 2y) + 5 = 0?

Absolutely, setting each factor equal to zero can help find solutions for x and y.

What type of quadratic is formed when substituting u = x - 2y?

It forms a standard quadratic expression, specifically u^2 - 6u + 5.

Additional Resources

Factorise (x-2y)^2 - 6(x-2y) + 5: A Comprehensive Guide to Simplification and Understanding

When it comes to algebraic expressions, factorisation is a fundamental skill that enables mathematicians and students alike to simplify complex expressions, solve equations, and understand the underlying structure of algebraic problems. The expression factorise (x-2y)^2 - 6(x-2y) + 5 offers an excellent example of how substitution and recognition of quadratic forms can streamline the process of factorisation. This article provides an in-depth review of the step-by-step approach to factorising this expression, along with insights into its features, potential pitfalls, and best practices.

Understanding the Expression

Before diving into the factorisation process, it’s essential to understand the structure and components of the expression:

$$ (x-2y)^2 - 6(x-2y) + 5 $$

This expression is quadratic in the variable (x - 2y), which suggests that substitution can be a useful strategy. Recognising the quadratic form allows us to treat (x - 2y) as a single variable, say t, simplifying the problem significantly.

Key features:
- Quadratic Structure: The expression resembles the standard quadratic form $t^2 - 6t + 5$.
- Linear in $(x-2y)$: The expression involves a square and a linear term in the same binomial.

Step-by-Step Factorisation Process

1. Substitution

To make the factorisation more straightforward, introduce a substitution:

$$
t = x - 2y
$$

Rewriting the original expression:

$$
t^2 - 6t + 5
$$

This transformation reduces the problem to factorising a quadratic polynomial in one variable.

2. Factorising the Quadratic in t

The quadratic expression:

$$
t^2 - 6t + 5
$$

can be factorised using standard methods:

- Find two numbers that multiply to 5 and add to -6.

Factors of 5: 1 and 5.

- Since the middle term is -6, and the product is 5, the numbers are -1 and -5 because:

$$
-1 \times -5 = 5 \quad \text{and} \quad -1 + -5 = -6
$$

Thus, the quadratic factors as:

$$
t^2 - 6t + 5 = (t - 1)(t - 5)
$$

3. Re-substitute to original variables

Recall that $t = x - 2y$. Therefore, the factorised form is:

$$
(x - 2y - 1)(x - 2y - 5)
$$

Final Factorised Form and Interpretation

The fully factorised form of the given expression is:

$$
\boxed{(x - 2y - 1)(x - 2y - 5)}
$$

This form reveals the roots of the quadratic in terms of the original variables and provides insight into the structure of the expression.

Features and Insights:

- Linear Factors: Each factor is linear in x and y, making it easier to analyze roots and graph the expression.
- Zeroes of the Expression: The roots occur when either:

$$
x - 2y - 1 = 0 \quad \text{or} \quad x - 2y - 5 = 0
$$

- Graphical Interpretation: These roots represent lines in the xy-plane, indicating where the original expression evaluates to zero.

Advantages of the Factorisation

Understanding the factorised form offers several benefits:

- Simplification: The expression becomes easier to work with, especially when solving equations or integrating.
- Solution Finding: Roots are immediately apparent, facilitating the solving process.
- Graphical Analysis: The roots correspond to lines, providing geometric interpretation.
- Problem Solving: Factorised expressions can be used to evaluate limits, derivatives, or to perform partial fraction decomposition.

Potential Pitfalls and Challenges

While the process appears straightforward, there are common mistakes and challenges that students and practitioners should be aware of:

- Incorrect Substitution: Failing to correctly substitute or revert back to original variables can lead to errors.
- Misidentification of Factors: Not checking factors thoroughly can result in incomplete or incorrect factorisation.
- Overlooking Quadratic Forms: Recognising quadratic expressions within more complex polynomials is crucial; missing this can complicate the problem unnecessarily.
- Sign Errors: Careful attention to signs when factoring quadratics is essential to avoid mistakes.

Broader Context and Applications

Factorising expressions like $(x-2y)^2 - 6(x-2y) + 5$ is not merely an academic exercise; it has practical relevance in various fields:

- Algebraic Simplification: Simplifies complex algebraic expressions in calculus, algebra, and beyond.
- Solving Equations: Facilitates finding solutions to polynomial equations, especially quadratic ones embedded within larger expressions.
- Geometry and Graphing: Roots and factors help in plotting functions and understanding their behavior.
- Engineering and Physics: Used in modeling situations where relationships are quadratic in nature.

Summary and Final Thoughts

The factorisation of $(x-2y)^2 - 6(x-2y) + 5$ exemplifies the power of substitution in algebra. Recognising the quadratic form in $t = x - 2y$ simplifies the process significantly, transforming a seemingly complex expression into a manageable product of linear factors. The resulting factors:

$$(x - 2y - 1)(x - 2y - 5)$$

not only facilitate problem-solving but also enhance understanding of the expression's structure and roots.

Features:

- Efficient and straightforward when identified correctly.
- Provides immediate insights into solutions and graphing.

Pros:

- Simplifies complex expressions.
- Aids in solving equations efficiently.
- Offers geometric interpretation through roots.

Cons:

- Requires recognition of quadratic form.
- May be prone to signs and substitution errors if not careful.

In summary, mastering the factorisation of such expressions enriches one's algebraic toolkit, paving the way for tackling more advanced problems with confidence. Whether in pure mathematics or applied sciences, understanding how to factorise expressions like (x-2y)^2 - 6(x-2y) + 5 is a valuable skill that enhances analytical capabilities and deepens mathematical insight.

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