

Factorize $10ax - 15bx - 4ay + 6by$

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Understanding how to factorize algebraic expressions is a fundamental skill in mathematics, especially in algebra. The expression $10ax - 15bx - 4ay + 6by$ is a linear algebraic expression involving four terms with multiple variables. Proper factorization of such expressions not only simplifies calculations but also enhances problem-solving efficiency in various mathematical contexts, including solving equations, simplifying expressions, and analyzing algebraic structures. In this article, we will explore step-by-step methods to factorize the expression $10ax - 15bx - 4ay + 6by$, highlighting important concepts, common techniques, and practical tips to master such algebraic manipulations.

Understanding the Expression

Before diving into the factorization process, it's crucial to understand the structure of the expression:

$$10ax - 15bx - 4ay + 6by$$

- It consists of four terms.
- The variables involved are a , b , x , and y .
- Each term involves a combination of these variables multiplied by coefficients.

The goal is to factorize this expression completely, ideally expressing it as a product of simpler factors, such as binomials or monomials.

Step-by-Step Approach to Factorization

1. Group the Terms Strategically

The initial step involves grouping terms to identify common factors:

- Group 1: $10ax - 15bx$
- Group 2: $-4ay + 6by$

This grouping aims to reveal common factors within each group, making it easier to factor out common elements.

2. Factor Out the Common Factors from Each Group

Analyze each group separately:

$$\text{Group 1: } 10ax - 15bx$$

- Common factor: $5b$ (since 5 divides both 10 and 15, and both terms include a and x, b is common in the second term)
- Alternatively, look at the coefficients:
- $10a x$ and $-15b x$: the common variable part is x, and coefficients 10 and 15 have a common factor of 5.

Let's factor carefully:

- For $10ax$, factors are $5 \cdot 2 \cdot a \cdot x$
- For $-15bx$, factors are $5 \cdot -3 \cdot b \cdot x$

Common numerical factor: $5x$

So, factor $5x$ out:

$$10ax - 15bx = 5x(2a - 3b)$$

Similarly, for the second group:

$$-4ay + 6by$$

- Common factor: $-2y$

Because:

- $-4ay$: factors are $-2 \cdot 2 \cdot a \cdot y$
- $6by$: factors are $2 \cdot 3 \cdot b \cdot y$

Extract:

$$-2y:$$

- From $-4ay$: $-2 \cdot 2a \cdot y$
- From $6by$: $-2 \cdot -3b \cdot y$

So,

$$-4ay + 6by = -2y(2a - 3b)$$

3. Rewrite the Expression with Factored Groups

Putting the factorizations together:

$$\begin{aligned} & \backslash [\\ & 10ax - 15bx - 4ay + 6by = 5x(2a - 3b) - 2y(2a - 3b) \\ & \backslash] \end{aligned}$$

Notice that both terms contain the common binomial $(2a - 3b)$.

4. Factor Out the Common Binomial

Since $(2a - 3b)$ is common, we can factor it out:

```
\[
(2a - 3b)(5x - 2y)
\]
```

This is the fully factorized form of the original expression.

Final Factorized Form

```
\[
\boxed{
10ax - 15bx - 4ay + 6by = (2a - 3b)(5x - 2y)
}
\]
```

This factorization simplifies the original expression into a product of two binomials, making further operations or solutions more straightforward.

Key Concepts and Techniques Used

1. Grouping Terms

- Group terms to find common factors within subsets of the expression.
- Useful when the expression involves multiple terms with similar variables or coefficients.

2. Extracting the Greatest Common Factor (GCF)

- Identify the largest common factor (number and/or variables) in each group.
- Factor out the GCF to simplify the expression.

3. Recognizing Common Binomials

- After factoring groups, look for common binomial factors to combine the expression into a product.

4. Factoring by Grouping

- A method where the expression is manipulated into groups with common factors, leading to binomial factors.

Common Mistakes to Avoid

- Not fully identifying the GCF in each group, leading to incomplete

factorization.

- Overlooking the negative signs, which can cause incorrect factoring.
- Ignoring common variables or coefficients that can be factored out.
- Failing to check if the factors can be simplified further.

Practical Tips for Efficient Factorization

- Always look for the greatest common factors both numerically and variable-wise.
- Group terms logically based on common variables or coefficients.
- Double-check the factorized form by expanding it back to verify correctness.
- Practice similar problems to recognize patterns quickly.
- Use algebraic identities when applicable, such as difference of squares or perfect square trinomials.

Applications of Factorization

Factorization plays a crucial role in various mathematical applications:

1. **Simplifying Expressions:** Reducing complex algebraic expressions to simpler forms.
2. **Solve Equations:** Setting factors equal to zero to find solutions in algebraic equations.
3. **Polynomial Division:** Simplifying polynomials before division.
4. **Graphing:** Finding roots and intercepts of algebraic functions.
5. **Calculus:** Simplifying functions to evaluate derivatives and integrals.

Additional Practice Problems

To strengthen your understanding, try factoring the following expressions:

- $3x^2 - 6x$
- $6a^2b - 9ab^2 + 12a^2b^2$
- $x^3 - 27$
- $4m^2 + 4mn + n^2$

Attempt these problems using the techniques discussed, focusing on grouping, extracting GCF, and recognizing special algebraic identities.

Conclusion

Factorizing algebraic expressions like $10ax - 15bx - 4ay + 6by$ is an essential skill that combines understanding of common factors, grouping strategies, and pattern recognition. By systematically applying these techniques, you can simplify complex expressions efficiently, paving the way for solving equations, analyzing functions, and exploring deeper algebraic concepts. Remember to practice regularly, verify your factorizations, and explore various types of expressions to build confidence and proficiency in algebraic manipulation.

Frequently Asked Questions

How do I factorize the expression $10ax - 15bx - 4ay + 6by$?

To factorize $10ax - 15bx - 4ay + 6by$, group terms: $(10ax - 15bx) + (-4ay + 6by)$. Factor each group: $5x(2a - 3b) - 2y(2a - 3b)$. Then, factor out the common binomial: $(2a - 3b)(5x - 2y)$.

What common factors can be extracted from the expression $10ax - 15bx - 4ay + 6by$?

The common factors are 5 for the first two terms and 2 for the last two. Grouping helps identify the common binomial factor $(2a - 3b)$ after factoring out common coefficients.

Can the expression $10ax - 15bx - 4ay + 6by$ be written as a product of two binomials?

Yes, after factoring by grouping, it can be written as $(2a - 3b)(5x - 2y)$.

What is the step-by-step process to factorize $10ax - 15bx - 4ay + 6by$?

First, group the terms: $(10ax - 15bx)$ and $(-4ay + 6by)$. Next, factor out common factors from each group: $5x(2a - 3b)$ and $-2y(2a - 3b)$. Finally, factor out the common binomial $(2a - 3b)$: $(2a - 3b)(5x - 2y)$.

Are there alternative methods to factorize the expression $10ax - 15bx - 4ay + 6by$?

The primary method is grouping, but you could also attempt to rearrange terms or look for common factors across the entire expression, though grouping is most straightforward here.

Is the factorization of $10ax - 15bx - 4ay + 6by$ useful in solving equations?

Yes, factorization simplifies the expression and can help solve equations involving it by setting each factor equal to zero and solving for variables.

What is the factored form of $10ax - 15bx - 4ay + 6by$?

The factored form is $(2a - 3b)(5x - 2y)$.

Can the expression $10ax - 15bx - 4ay + 6by$ be factored further?

No, after factoring out the common binomial, it cannot be simplified further as the binomial factors are fully factored.

Additional Resources

Factorize $10ax - 15bx - 4ay + 6by$: A Comprehensive Guide to Simplifying Algebraic Expressions

When tackling algebraic expressions, one of the most fundamental skills is factorization – breaking down complex expressions into simpler, multiplied factors. Today, we'll explore the detailed process of factorizing the expression $10ax - 15bx - 4ay + 6by$, a task that involves recognizing common factors, applying algebraic identities, and ensuring the expression is broken down to its most simplified form. This step-by-step guide aims to clarify the method and help you become more confident in handling similar algebraic problems.

Understanding the Expression

Before diving into the factorization process, it's essential to understand the structure of the given expression:

$$10ax - 15bx - 4ay + 6by$$

This expression consists of four terms, each involving variables a , b , x , and y with coefficients. The goal is to factorize it completely, which involves:

- Grouping terms that share common factors
- Factoring out the greatest common factors (GCF)
- Recognizing common binomial factors
- Rewriting the expression as a product of simpler expressions

Step 1: Analyze and Group Similar Terms

A good starting point is to look for common factors within pairs of terms. Notice that:

- The first two terms are $10ax$ and $-15bx$
- The last two terms are $-4ay$ and $6by$

Grouping them accordingly:

$$(10ax - 15bx) + (-4ay + 6by)$$

Now, we can focus on factoring each group separately.

Step 2: Factor Out the Greatest Common Factor (GCF) from Each Group

Let's analyze each group to find their GCFs:

Group 1: $10ax - 15bx$

- Coefficients: 10 and 15
- Common factor: 5
- Variables: Both terms contain x and either a or b , but not both

So, extract the GCF:

GCF of coefficients: 5

Variables:

- Both terms have x , so factor out x
- The variables a and b are different; no common variable here

Factoring out:

$$5x(2a - 3b)$$

Group 2: $-4ay + 6by$

- Coefficients: -4 and 6
- GCF of coefficients: 2
- Variables: Both terms contain y ; factors are a in the first and b in the second

Factoring out:

$$2y(-2a + 3b)$$

Note that the negative sign is factored out with -4, so:

$$-4ay = -2 \cdot 2 \cdot a \cdot y$$

$$\text{Similarly, } 6by = 2 \cdot 3 \cdot b \cdot y$$

So, factoring out $2y$:

$$2y(-2a + 3b)$$

Step 3: Rewrite the Expression with Factored Groups

Now, the original expression becomes:

$$5x(2a - 3b) + 2y(-2a + 3b)$$

Notice that the binomial $(2a - 3b)$ appears in the first group, and $(-2a + 3b)$ appears in the second.

Step 4: Recognize and Simplify the Common Binomial Factors

Observe that:

- $(2a - 3b)$ and $(-2a + 3b)$ are negatives of each other. Specifically,

$$(-2a + 3b) = -(2a - 3b)$$

This is crucial because it allows us to factor further.

Express the entire expression:

$$5x(2a - 3b) + 2y(-(2a - 3b))$$

which simplifies to:

$$(2a - 3b)(5x - 2y)$$

Final Factorized Form:

$$(2a - 3b)(5x - 2y)$$

This is the fully factorized form of the original expression.

Summary of the Factorization Process

1. Group similar terms to identify common factors.
2. Factor out the GCF from each group:
 - From the first group: $5x(2a - 3b)$
 - From the second group: $2y(-2a + 3b)$
3. Recognize the relationship between the binomials:
 - $-2a + 3b$ is equivalent to $-(2a - 3b)$
4. Factor out the common binomial:
 - The entire expression becomes $(2a - 3b)(5x - 2y)$

Additional Tips for Factorization

- Always look for common factors in terms and groups before attempting more complex methods.
- Recognize difference of squares, sum and difference of cubes, or other special identities to simplify when applicable.
- When binomials appear with reversed signs, consider factoring out negatives to reveal common factors.
- Practice breaking expressions into groups to find common binomials or

monomials.

Practice Problems for Further Mastery

1. Factorize: $6xy + 9xz - 4y + 6z$
2. Factorize: $8ab - 12ac + 10ad - 15ad$
3. Factorize: $x^2 - 9y^2 + 4 - 16z^2$

By attempting these, you'll reinforce your understanding of factorization techniques and improve your algebraic manipulation skills.

Conclusion

Factorizing the expression $10ax - 15bx - 4ay + 6by$ demonstrates the importance of systematic analysis, grouping, and recognizing algebraic identities. Mastery of such processes enhances your problem-solving capabilities, whether in academic settings or real-world applications involving algebraic expressions. Remember, practice makes perfect – so keep practicing similar problems to sharpen your skills and develop a deeper understanding of algebra.

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