Find F (x). F(x)=2e X +5xlnx F (x)=

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Introduction to the Function F(x)

In calculus, understanding how to differentiate complex functions is essential. The function F(x) = 2e^x + 5x ln x combines exponential and logarithmic components, making it a rich subject for derivative analysis. To find F′(x), the derivative of F(x), we need to apply various differentiation rules systematically, including the sum rule, product rule, and derivatives of exponential and logarithmic functions. This article will guide you through the process step-by-step, providing detailed explanations and insights to ensure a comprehensive understanding.

Breaking Down the Function F(x)

Components of F(x)

The function F(x) is composed of two terms:

- 2e^x — an exponential function scaled by 2
- 5x ln x — a product of x and ln x, scaled by 5

To differentiate F(x), we will differentiate each term separately and then sum the results, based on the sum rule of derivatives.

Differentiating F(x): Step-by-Step Approach

1. Derivative of 2e^x

The derivative of an exponential function e^x with respect to x is straightforward:

- d/dx [e^x] = e^x

Since the term is scaled by 2, the derivative becomes:

- d/dx [2e^x] = 2e^x

2. Derivative of 5x ln x

The second term is a product of two functions:

- $u(x) = x$
- $v(x) = \ln x$

Applying the product rule:

- $d/dx\,[u(x)\,v(x)] = u'(x)\,v(x) + u(x)\,v'(x)$

Calculating derivatives of u and v:

- $u'(x) = 1$
- $v'(x) = 1/x$

Therefore, the derivative of 5x ln x is:

$d/dx\,[5x \ln x] = 5\,[u'(x)\,v(x) + u(x)\,v'(x)] = 5\,[1 \ln x + x\,(1/x)] = 5\,[\ln x + 1]$

Combining the Derivatives to Find F'(x)

Adding the derivatives of both terms:

F'(x) = derivative of 2e^x + derivative of 5x ln x

$F'(x) = 2e^x + 5\,(\ln x + 1)$

Simplified further:

$F'(x) = 2e^x + 5 \ln x + 5$

Final Expression for the Derivative F'(x)

The derivative of the given function F(x) is:

$F'(x) = 2e^x + 5 \ln x + 5$

This expression provides the rate of change of F(x) at any point x in its domain ($x > 0$, since ln x is defined for $x > 0$).

Additional Considerations

Domain of F(x) and F'(x)

- The original function F(x) includes ln x, which is only defined for $x > 0$.
- Therefore, the domain of F(x) and F'(x) is $x > 0$.

Behavior of $f'(x)$

- As x approaches $0+$ (from the right), $\ln x$ approaches $-\infty$, making $f'(x)$ tend toward $-\infty$.
- As x increases, $2e^x$ dominates the growth of $f'(x)$, leading to exponential growth.
- The constant term $+5$ influences the overall trend, adding a steady positive shift.

Applications and Implications

Understanding $f'(x)$ allows us to analyze the behavior of $f(x)$:

- **Critical points:** Set $f'(x) = 0$ to find potential maxima, minima, or inflection points.
- **Increasing/Decreasing intervals:** $f(x)$ is increasing where $f'(x) > 0$ and decreasing where $f'(x) < 0$.
- **Concavity and inflection points:** Further second derivative analysis can reveal concavity and points of inflection.

Summary and Key Takeaways

- The derivative of $f(x) = 2e^x + 5x \ln x$ is $f'(x) = 2e^x + 5 \ln x + 5$.
- Applying sum rule, product rule, and derivatives of exponential and logarithmic functions is essential.
- Domain considerations are crucial due to the $\ln x$ component.
- The derivative provides insights into the function's increasing/decreasing behavior and critical points.

Conclusion

Differentiating complex functions like $f(x) = 2e^x + 5x \ln x$ requires a methodical approach, combining multiple differentiation rules. Mastery of these techniques enables deeper analysis of function behavior, critical for solving real-world problems involving exponential and logarithmic relationships. Whether for academic purposes or practical applications, understanding how to find derivatives like $f'(x)$ is fundamental in calculus and mathematical analysis.

Frequently Asked Questions

How do I find the derivative of the function $f(x) = 2e^x + 5x \ln x$?

To find $f'(x)$, differentiate each term separately: The derivative of $2e^x$ is $2e^x$; for $5x \ln x$, use the product rule: $(5)(d/dx[x \ln x]) = 5 [\ln x + 1]$. So, $f'(x) = 2e^x + 5 (\ln x + 1)$.

What is the derivative of $f(x) = 2e^x + 5x \ln x$?

The derivative is $f'(x) = 2e^x + 5 (\ln x + 1)$.

How can I simplify the derivative of $F(x) = 2e^x + 5x \ln x$?

The simplified derivative is $F'(x) = 2e^x + 5 \ln x + 5$.

Are there any special rules I should use to differentiate $F(x) = 2e^x + 5x \ln x$?

Yes, you should use the sum rule for differentiation and apply the product rule to the term $5x \ln x$.

Does the derivative of $F(x) = 2e^x + 5x \ln x$ exist for all x?

The derivative exists for all $x > 0$, because $\ln x$ is only defined for positive x.

How do I interpret the derivative $F'(x) = 2e^x + 5 \ln x + 5$ in terms of the function's growth?

The derivative indicates the rate of change of F(x). The term $2e^x$ grows exponentially, while $5 \ln x$ grows slowly, and the +5 is constant. Overall, F(x) increases rapidly for large x.

Additional Resources

Find F(x) for the function $F(x) = 2e^x + 5x \ln x$

Introduction

In the realm of calculus and mathematical analysis, understanding the behavior and properties of functions is fundamental. The task of finding the derivative of a function, denoted as F'(x), is central to analyzing the function's rate of change, identifying critical points, and understanding its overall behavior. The function under consideration, $F(x) = 2e^x + 5x \ln x$, presents an interesting blend of exponential and logarithmic components. This combination requires a careful application of differentiation rules, including the product rule, chain rule, and basic derivatives of exponential and logarithmic functions.

This article offers a comprehensive exploration of how to find F'(x), providing detailed explanations, step-by-step derivations, and insightful analysis of the function's properties. The goal is to serve as a thorough resource for students, educators, and enthusiasts seeking to deepen their understanding of differentiation techniques applied to complex functions.

Understanding the Function F(x)

Breaking Down F(x)

The function is given as:

$$F(x) = 2e^x + 5x \ln x$$

This composite function combines two distinct parts:

1. Exponential component: $2e^x$
2. Product of a polynomial and logarithm: $5x \ln x$

Understanding the individual derivatives of these parts is essential before combining them to find the derivative of the entire function.

Differentiation Techniques Required

To differentiate F(x), we must recall and apply several core rules:

1. Derivative of e^x

- The derivative of e^x with respect to x is e^x.

2. Constant multiple rule

- The derivative of $c \cdot f(x)$ is $c \cdot f'(x)$, where c is a constant.

3. Product rule

- For the product $u(x) \cdot v(x)$, the derivative is:

$$\frac{d}{dx}[u(x) v(x)] = u'(x) v(x) + u(x) v'(x)$$

4. Derivative of $\ln x$

- The derivative of $\ln x$ is $\frac{1}{x}$.

5. Chain rule

- When differentiating composite functions, such as $e^{g(x)}$, the derivative is $e^{g(x)} g'(x)$.

Step-by-Step Derivation of F'(x)

Differentiating the First Term: $2e^x$

Applying the constant multiple rule:

$$
\frac{d}{dx} [2e^x] = 2 \frac{d}{dx} [e^x] = 2e^x
$$

This part is straightforward, as the exponential function's derivative is itself, scaled by 2.

Differentiating the Second Term: $5x \ln x$

Since this is a product of two functions, $u(x) = x$ and $v(x) = \ln x$, the product rule applies:

$$
\frac{d}{dx} [x \ln x] = u'(x) v(x) + u(x) v'(x)
$$

Calculating each derivative:

- $u'(x) = 1$
- $v'(x) = \frac{1}{x}$

Substituting into the product rule:

$$
\frac{d}{dx} [x \ln x] = 1 \cdot \ln x + x \cdot \frac{1}{x} = \ln x + 1
$$

Now, applying the constant multiple rule for the entire term $5x \ln x$:

$$
\frac{d}{dx} [5x \ln x] = 5 (\ln x + 1) = 5 \ln x + 5
$$

Final Expression for $f'(x)$

Combining the derivatives of both parts:

$$
f'(x) = \frac{d}{dx} [2e^x] + \frac{d}{dx} [5x \ln x] = 2e^x + 5 \ln x + 5
$$

Thus, the derivative of the function $f(x)$ is:

$$
\boxed{
f'(x) = 2e^x + 5 \ln x + 5
}
$$

Analytical Insights and Interpretations

Critical Points and Behavior

Understanding $f'(x)$ allows us to analyze the increasing or decreasing nature of $f(x)$:

- Where $f'(x) > 0$: $f(x)$ is increasing.
- Where $f'(x) < 0$: $f(x)$ is decreasing.
- Critical points: Values of x where $f'(x) = 0$.

The critical points are solutions to:

$$
2e^x + 5 \ln x + 5 = 0
$$

Due to the combination of exponential and logarithmic terms, solving this equation analytically is complex. Numerical methods or graphing tools are typically employed to approximate solutions.

Domain Considerations

Since $\ln x$ appears in the derivative, the domain of $f(x)$ is $x > 0$. At $x \to 0^+$, $\ln x \to -\infty$, which influences the function's behavior near zero.

Asymptotic Behavior

- As $x \to \infty$:

e^x dominates, and $f'(x) \to \infty$, indicating that $f(x)$ increases rapidly.

- As $x \to 0^+$:

$\ln x \to -\infty$, and $F'(x) \to -\infty$, suggesting F(x) decreases sharply near zero.

Graphical Representation

Graphing F(x) and F'(x) provides intuitive insights into the function's shape, critical points, and inflection points. Such visualizations are invaluable for understanding the function's dynamics over its domain.

Practical Applications and Significance

Understanding derivatives such as F'(x) is crucial in various fields:

- Physics: Rate of change in exponential decay or growth processes.
- Economics: Marginal analysis involving logarithmic and exponential models.
- Engineering: Signal processing where exponential and logarithmic functions model system behaviors.
- Biology: Population models with growth rates.

The derivation exemplifies the importance of mastering differentiation rules to handle complex functions blending exponential and logarithmic components.

Conclusion

The process of finding the derivative of $F(x) = 2e^x + 5x \ln x$ illuminates the elegance and power of calculus. By methodically applying differentiation rules—constant multiple rule, product rule, chain rule, and basic derivatives—we arrive at a concise expression for F'(x):

$$
F'(x) = 2e^x + 5 \ln x + 5
$$

This derivative encapsulates the instantaneous rate of change of the function at any point $x > 0$, offering insights into its increasing and decreasing intervals, critical points, and overall behavior. Such analytical skills are essential in mathematical analysis, enabling professionals and students to interpret complex functions and apply them across diverse scientific and engineering disciplines.

Understanding and computing derivatives like this form the backbone of calculus, unlocking deeper insights into the functions that describe our world.

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