Find The Solution Set X^2+2x-15=0

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Introduction to Quadratic Equations and Their Solutions

Quadratic equations are a fundamental part of algebra and appear frequently in various mathematical contexts. They are equations of the form $ax^2 + bx + c = 0$, where a, b, and c are constants, and $a \neq 0$. Solving these equations involves finding the values of x (called roots or solutions) that satisfy the equation. In this article, we will focus on solving the specific quadratic equation:

$x^2 + 2x - 15 = 0$

and explore different methods to find its solution set. Whether you're a student brushing up on algebra skills or someone seeking to deepen your understanding of quadratic equations, this comprehensive guide will provide clear explanations and practical steps.

Understanding the Equation $x^2 + 2x - 15 = 0$

Before diving into solution methods, it’s essential to understand the structure of the quadratic equation at hand.

Components of the Equation

- Quadratic term: x^2
- Linear term: 2x
- Constant term: -15

The equation is set equal to zero, which is typical in quadratic equations because solutions are the x-values where the parabola intersects the x-axis.

What Does the Solution Set Mean?

The solution set of the quadratic equation consists of all x-values that satisfy the equation. These solutions can be:

- Real and distinct: Two different real solutions
- Real and repeated: One real solution (also called a double root)
- Complex: No real solutions, but two complex solutions

Our goal is to determine which case applies and find the exact solutions.

Methods to Find the Solution Set of $x^2 + 2x - 15 = 0$

There are several methods to solve quadratic equations. The most common are:

- Factoring
- Completing the square
- Quadratic formula

We will explore each method in detail, starting with factoring, which is the simplest when applicable.

Factoring Method

Factoring involves rewriting the quadratic expression as a product of two binomials. For the equation $x^2 + 2x - 15 = 0$, we look for two numbers that multiply to -15 and add to 2.

Step-by-Step Process

1. Identify coefficients: $a = 1$, $b = 2$, $c = -15$
2. Find two numbers: Their product should be $a c = 1 (-15) = -15$, and their sum should be $b = 2$
3. Determine factors of -15: The pairs are (1, -15), (-1, 15), (3, -5), (-3, 5)
4. Find the pair that sums to 2: 3 + (-5) = -2 (not 2), and -3 + 5 = 2 (yes!)

Factoring the quadratic

Since -3 + 5 = 2, the factors are $(x - 3)$ and $(x + 5)$:

$x^2 + 2x - 15 = (x - 3)(x + 5)$

Solving for x

Set each factor equal to zero:

- $x - 3 = 0 \rightarrow x = 3$
- $x + 5 = 0 \rightarrow x = -5$

Solution set:

$x = 3, -5$

Result: The solution set is $\{ -5, 3 \}$

Completing the Square Method

Completing the square provides an alternative approach that can be useful, especially when factoring is difficult.

Step-by-Step Process

1. Start with the original equation:

$x^2 + 2x - 15 = 0$

2. Move the constant to the other side:

$x^2 + 2x = 15$

3. Complete the square:

- Take half of the coefficient of x (which is 2), divide by 2: 2 / 2 = 1
- Square this value: $1^2 = 1$
- Add and subtract this square on the left side:

$x^2 + 2x + 1 - 1 = 15$

which simplifies to:

$(x + 1)^2 - 1 = 15$

4. Isolate the perfect square:

$(x + 1)^2 = 15 + 1 = 16$

5. Take the square root of both sides:

$x + 1 = \pm\sqrt{16} = \pm 4$

6. Solve for x:

- $x + 1 = 4 \rightarrow x = 3$
- $x + 1 = -4 \rightarrow x = -5$

Solution set:

$x = 3, -5$

Result: Same as the factoring method, confirming the solutions.

Quadratic Formula Method

The quadratic formula is a universal method applicable to all quadratic equations, especially when factoring is not straightforward. It states:

$x = [-b \pm \sqrt{(b^2 - 4ac)}] / (2a)$

Applying the Quadratic Formula

Given our equation: $x^2 + 2x - 15 = 0$

- $a = 1$
- $b = 2$
- $c = -15$

Calculate the discriminant:

$D = b^2 - 4ac = (2)^2 - 4\ 1\ (-15) = 4 + 60 = 64$

Since $D > 0$, there are two real solutions.

Calculate the roots:

$x = [-2 \pm \sqrt{64}] / (2\ 1) = [-2 \pm 8] / 2$

Now, find each solution:

- $x = (-2 + 8) / 2 = 6 / 2 = 3$
- $x = (-2 - 8) / 2 = -10 / 2 = -5$

Solution set:

$x = 3, -5$

Summary of the Solution Set

All methods lead to the same solutions:

Solution set: { -5, 3 }

This confirms the solutions are real, distinct roots, and the quadratic can be factored nicely.

Visualizing the Solutions on a Graph

Understanding the graph of $y = x^2 + 2x - 15$ can provide valuable insight into the solutions.

Key features of the parabola:

- Vertex: The vertex occurs at $x = -b / (2a) = -2 / (2 1) = -1$
- Calculate y-coordinate at $x = -1$:

$$y = (-1)^2 + 2(-1) - 15 = 1 - 2 - 15 = -16$$

- Vertex point: $(-1, -16)$

- X-intercepts: The solutions $x = -5$ and $x = 3$ are points where the parabola crosses the x-axis.

Plotting the parabola:

- The parabola opens upward (since $a = 1 > 0$).
- The roots at $x = -5$ and $x = 3$ are where $y = 0$.

Practical Applications of Solving Quadratic Equations

Quadratic equations like $x^2 + 2x - 15 = 0$ appear in various real-world scenarios:

- Physics: Calculating projectile motion and trajectories.
- Engineering: Analyzing structural stresses and forces.
- Finance: Determining profit maximization points.
- Biology: Modeling population growth with quadratic functions.

Understanding how to find the solution set of quadratic equations enables professionals and students to solve complex problems across multiple disciplines.

Tips for Solving Quadratic Equations Effectively

- Check for factoring opportunities first. If the quadratic factors easily, it’s the quickest method.
- Use the quadratic formula when factoring is difficult or impossible. It works for all quadratics.
- Complete the square for educational insight or when the quadratic formula seems cumbersome.
- Always verify solutions. Substitute solutions back into the original equation to confirm correctness.

Conclusion

The quadratic equation $x^2 + 2x - 15 = 0$ has a solution set of $\{ -5, 3 \}$. Whether you use factoring, completing the square, or the quadratic formula, you arrive at the same roots, demonstrating the consistency and reliability of these methods. Mastering these techniques allows you to efficiently solve a wide range of quadratic equations and enhances your algebraic problem-solving skills.

Remember, understanding the structure of quadratic equations and practicing different solution methods will make you more confident when tackling similar problems in academic and real-world contexts.

Frequently Asked Questions

What are the solutions to the quadratic equation $x^2 + 2x - 15 = 0$?

The solutions are $x = 3$ and $x = -5$.

How do I solve the quadratic equation $x^2 + 2x - 15 = 0$?

You can factor it as $(x + 5)(x - 3) = 0$, leading to solutions $x = -5$ and $x = 3$.

What method is best to solve $x^2 + 2x - 15 = 0$?

Factoring is the most straightforward method here, but you can also use the quadratic formula.

Can I complete the square to solve $x^2 + 2x - 15 = 0$?

Yes. Completing the square gives $(x + 1)^2 = 16$, leading to $x = -1 \pm 4$, so solutions are $x = 3$ and $x = -5$.

Is the quadratic equation $x^2 + 2x - 15 =$ solvable over real numbers?

Yes, because the discriminant is positive, indicating two real solutions.

What is the discriminant of the quadratic $x^2 + 2x - 15 = 0$?

The discriminant is $\Delta = 2^2 - 4(1)(-15) = 4 + 60 = 64$.

How do the solutions $x = 3$ and $x = -5$ relate to the graph of $y = x^2 + 2x - 15$?

They are the x-intercepts (roots) where the parabola crosses the x-axis.

What is the sum and product of the solutions to $x^2 + 2x - 15 = 0$?

Sum of solutions: $-b/a = -2/1 = -2$; product: $c/a = -15/1 = -15$.

Additional Resources

Find The Solution Set $X^2 + 2x - 15 = 0$ is a fundamental problem in algebra that serves as an excellent example for understanding quadratic equations. Quadratic equations are polynomial equations of degree two, and their solutions provide insight into various mathematical concepts, including factoring, the quadratic formula, and the nature of roots. Exploring the solutions to this particular quadratic equation not only enhances problem-solving skills but also deepens understanding of the properties and behaviors of quadratic functions.

Understanding the Quadratic Equation: X^2 + 2x - 15 = 0

The equation X^2 + 2x - 15 = 0 is a standard quadratic form, where the coefficients are clearly identified as:

- a = 1 (coefficient of X^2)
- b = 2 (coefficient of x)
- c = -15 (constant term)

Understanding these coefficients is crucial because they determine the method used to find solutions and the nature of the roots.

Key features:

- It is a quadratic equation due to the squared term.
- The coefficients are integers, which makes factoring feasible in many cases.
- The constant term is negative, indicating that solutions may have different signs.

Methods to Find the Solution Set

Several methods can be employed to find solutions to quadratic equations. The most common ones are factoring, completing the square, and using the quadratic formula. Each method has its pros and cons, and their applicability depends on the specific form of the quadratic.

1. Factoring Method

Factoring involves expressing the quadratic as a product of two binomials. For the equation X^2 + 2x - 15 = 0, the goal is to find two numbers that:

- Multiply to give c = -15
- Add to give b = 2

Step-by-step:

- List factor pairs of -15: (1, -15), (-1, 15), (3, -5), (-3, 5)
- Find the pair that sums to 2: (5, -3), because 5 + (-3) = 2

Rewrite the quadratic:

$X^2 + 5x - 3x - 15 = 0$

Group terms:

$(X^2 + 5x) + (-3x - 15) = 0$

Factor out common factors:

$X(X + 5) - 3(X + 5) = 0$

Factor out common binomial:

$(X - 3)(X + 5) = 0$

Set each factor equal to zero:

$X - 3 = 0 \rightarrow X = 3$

$X + 5 = 0 \rightarrow X = -5$

Solution set:

$X = 3, -5$

Pros:

- Quick and straightforward if the quadratic factors easily.
- No need for complex calculations.

Cons:

- Not always applicable, especially if the quadratic does not factor nicely.
- Can be time-consuming if factors are not obvious.

2. Completing the Square

Completing the square involves rewriting the quadratic in the form $(X + d)^2 = e$, then solving for X.

Step-by-step:

Start with:

$X^2 + 2x - 15 = 0$

Move the constant to the other side:

$X^2 + 2x = 15$

Complete the square:

- Take half of the coefficient of x: $2/2 = 1$
- Square it: $1^2 = 1$

Add this square to both sides:

$X^2 + 2x + 1 = 15 + 1$

Express as a perfect square:

$(X + 1)^2 = 16$

Solve for X:

$X + 1 = \pm\sqrt{16}$

$X + 1 = \pm 4$

$X = -1 \pm 4$

Solutions:

$X = -1 + 4 = 3$

$X = -1 - 4 = -5$

Pros:

- Useful for equations with coefficients that do not factor easily.
- Provides insight into the structure of the quadratic.

Cons:

- Slightly more algebraically intensive.
- Not always straightforward if coefficients are complex.

3. Quadratic Formula

The quadratic formula provides a universal method:

$X = [-b \pm \sqrt{(b^2 - 4ac)}] / 2a$

For our equation:

$a = 1, b = 2, c = -15$

Calculate discriminant:

$\Delta = b^2 - 4ac = (2)^2 - 4(1)(-15) = 4 + 60 = 64$

Since $\Delta > 0$, there are two real roots.

Calculate roots:

$X = [-2 \pm \sqrt{64}] / 2$

$X = [-2 \pm 8] / 2$

Calculate each:

- $X = (-2 + 8) / 2 = 6 / 2 = 3$
- $X = (-2 - 8) / 2 = -10 / 2 = -5$

Solution set:

$X = 3, -5$

Pros:

- Works for all quadratic equations.
- Precise and systematic.

Cons:

- Slightly more computational effort.
- Requires understanding of square roots.

Analysis of the Solution Set

The solutions $X = 3$ and $X = -5$ are real roots, which aligns with the positive discriminant (64). These solutions are also rational numbers, making the problem relatively straightforward. The symmetry of the solutions around the axis of symmetry $x = -b/2a = -1$ reflects the properties of quadratic functions.

Visual interpretation:

Plotting the quadratic function $y = X^2 + 2x - 15$ reveals a parabola opening upwards, intersecting the x-axis at the solutions. The roots at $X = -5$ and $X = 3$ are the points where the parabola touches or crosses the x-axis.

Features and Applications of the Solution Set

Understanding the solutions to this quadratic equation has broad applications:

- Mathematical modeling: Quadratic equations model projectile motion, area problems, and economic functions.
- Graphing: Roots determine intercepts and shape of the parabola.
- Problem-solving: Recognizing that solutions are integers simplifies many real-world calculations.

Features:

- The solutions are rational and easy to interpret.
- The quadratic's symmetry simplifies graphical analysis.
- The solutions are distinct, indicating two points of intersection with the x-axis.

Pros and Cons of Different Solution Methods

Method	Pros	Cons
Factoring	Fast for simple quadratics; intuitive	Not always possible; requires factorization skills
Completing the Square	Deep understanding; versatile for all quadratics	Slightly more complex algebra; longer process
Quadratic Formula	Universal; systematic	More calculations; requires understanding of square roots

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Conclusion

The quadratic equation $X^2 + 2x - 15 = 0$ exemplifies fundamental algebraic principles and showcases multiple methods to find solutions. Whether through factoring, completing the square, or applying the quadratic formula, the solutions consistently emerge as $X = 3$ and $X = -5$. These solutions not only solve the equation but also serve as foundational knowledge for more advanced algebra, calculus, and applied mathematics.

Understanding how to derive these roots enhances problem-solving skills and deepens comprehension of the behavior of quadratic functions. The simplicity of the solutions, combined with the various methods to obtain them, makes this an ideal case study for students and educators alike. Mastery of these techniques ensures a solid mathematical foundation and prepares learners for tackling more complex polynomial equations and applications across sciences and engineering.

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