

Find The Value Of $X-6x+13$ When $X=3+2i$

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Understanding how to evaluate expressions involving complex numbers is an essential skill in algebra and higher mathematics. When you're asked to find the value of an expression like $X - 6X + 13$ given that $X = 3 + 2i$, it involves substituting the complex number into the expression and simplifying accordingly. This process not only tests your algebraic skills but also deepens your understanding of complex arithmetic, including addition, multiplication, and the properties of imaginary numbers. In this comprehensive guide, we will explore step-by-step how to evaluate this expression, the mathematical concepts involved, and practical tips for handling similar problems.

Understanding Complex Numbers and Their Properties

Before diving into the calculation, it's crucial to review the fundamentals of complex numbers, as they form the basis of the problem.

What Are Complex Numbers?

Complex numbers are numbers that have a real part and an imaginary part, expressed in the form:

$$[a + bi]$$

where:

- a is the real part
- b is the imaginary part
- i is the imaginary unit, defined as $i^2 = -1$

In our case, $X = 3 + 2i$ has a real part of 3 and an imaginary part of 2.

Properties of Complex Numbers

Key properties include:

- Addition and subtraction: performed component-wise.
- Multiplication: involves distributive property and using $i^2 = -1$.
- Conjugate: $\overline{a + bi} = a - bi$.
- Magnitude (modulus): $|a + bi| = \sqrt{a^2 + b^2}$.

These properties are essential when simplifying expressions involving complex numbers.

Step-by-Step Solution of the Expression

The expression to evaluate is:

$$\sqrt{X^2 - 6X + 13}$$

with

$$\sqrt{X = 3 + 2i}$$

Let's proceed step-by-step.

Step 1: Substitute the value of X

Replace X with $3 + 2i$:

$$\sqrt{(3 + 2i)^2 - 6(3 + 2i) + 13}$$

Step 2: Distribute the multiplication

Multiply 6 by $(3 + 2i)$:

$$\sqrt{6 \times (3 + 2i) = 6 \times 3 + 6 \times 2i = 18 + 12i}$$

Now, rewrite the original expression:

$$\sqrt{(3 + 2i)^2 - (18 + 12i) + 13}$$

Note that subtracting $(18 + 12i)$ is equivalent to:

$$\sqrt{3 + 2i - 18 - 12i + 13}$$

Step 3: Combine like terms

Group real parts and imaginary parts:

- Real parts: $(3 - 18 + 13)$
- Imaginary parts: $(2i - 12i)$

Compute each:

Real parts:

$$\begin{aligned} &[\\ 3 - 18 + 13 &= (3 - 18) + 13 = -15 + 13 = -2 \\ &] \end{aligned}$$

Imaginary parts:

$$\begin{aligned} &[\\ 2i - 12i &= -10i \\ &] \end{aligned}$$

Step 4: Write the simplified expression

Combine the real and imaginary parts:

$$\begin{aligned} &[\\ -2 - 10i & \\ &] \end{aligned}$$

This is the final simplified form of the expression when $(X = 3 + 2i)$.

Interpreting the Result: Complex Number Output

The value of the expression $(X - 6X + 13)$ given $(X = 3 + 2i)$ is:

$$[-2 - 10i]$$

This is a complex number with:

- Real part: -2
- Imaginary part: -10

Implications:

- The result is a complex number, indicating the original expression, when

evaluated at this complex value, yields a complex output.

- The real part reflects the scalar component, while the imaginary part indicates the influence of the imaginary component of X .

Additional Considerations and Variations

Understanding how to handle similar problems can be expanded by exploring related concepts.

1. Evaluating Expressions with Different Complex Numbers

For any complex number $(a + bi)$, the process remains the same:

- Substitute the value into the expression.
- Distribute and simplify.
- Combine like terms.

2. Handling Expressions Involving Powers of X

In cases where the expression involves X^2 , X^3 , etc.,

- Use the binomial expansion or algebraic multiplication.
- Remember to simplify using $i^2 = -1$.

3. Using Conjugates and Magnitudes

These tools are useful when:

- Dividing complex numbers.
- Simplifying complex fractions.
- Finding the magnitude or phase of the resulting complex number.

Real-World Applications of Complex Number Evaluations

Complex numbers are not just theoretical; they have practical applications across various fields:

- Electrical Engineering: Alternating current (AC) circuit analysis uses complex impedance.
- Control Systems: Stability analyses often involve complex eigenvalues.
- Signal Processing: Fourier transforms utilize complex numbers.
- Quantum Physics: Wave functions are expressed with complex amplitudes.

Evaluating complex expressions accurately is vital for these applications.

Tips for Mastering Complex Number Calculations

To become proficient in handling complex algebra:

- Practice substitution: Always carefully substitute the complex value into expressions.
- Remember key identities: $i^2 = -1$, and how to simplify powers of i .
- Organize calculations: Separate real and imaginary parts for clarity.
- Use conjugates wisely: For division or simplifying complex fractions.
- Check your work: Verify each step, especially signs and distribution.

Summary and Final Thoughts

In conclusion, evaluating the expression $X^2 - 6X + 13$ at $X = 3 + 2i$ involves straightforward substitution, distribution, and combination of like terms, all within the framework of complex number arithmetic. The final value, $-2 - 10i$, exemplifies how complex numbers can produce complex results that have both real and imaginary components. Mastering these techniques enables you to tackle a wide range of problems in mathematics, physics, engineering, and beyond.

By understanding the underlying principles and practicing regularly, you'll develop confidence and skill in handling complex expressions, which are foundational in advanced science and engineering disciplines. Whether you're solving algebraic problems for homework or applying these concepts in real-world scenarios, the ability to evaluate and interpret complex numbers is an invaluable mathematical tool.

Keywords: complex numbers, algebra, evaluate complex expressions, imaginary unit, substitution, simplification, complex arithmetic, real and imaginary parts, advanced mathematics, engineering applications

Frequently Asked Questions

What is the value of the expression $-6x + 13$ when $x = 3 + 2i$?

When $x = 3 + 2i$, substitute into the expression: $-6(3 + 2i) + 13 = -18 - 12i + 13 = (-18 + 13) - 12i = -5 - 12i$.

How do you evaluate the expression $-6x + 13$ for a complex number x ?

Substitute the complex value of x into the expression, then perform algebraic operations (multiplication and addition) treating i as the imaginary unit, simplifying the real and imaginary parts separately.

What is the simplified result of $-6(3 + 2i) + 13$?

The simplified result is $-5 - 12i$.

Can you explain step-by-step how to find the value of $-6x + 13$ when $x = 3 + 2i$?

Yes. First, multiply -6 by x : $-6(3 + 2i) = -18 - 12i$. Then, add 13 : $(-18 - 12i) + 13 = (-18 + 13) - 12i = -5 - 12i$.

Is the value of $-6x + 13$ when $x = 3 + 2i$ a complex number? If so, what is it?

Yes, it is a complex number: $-5 - 12i$.

Additional Resources

Find The Value Of $X-6x+13$ When $X=3+2i$

In the world of mathematics, complex numbers often seem daunting at first glance, especially for students and enthusiasts trying to grasp their operations and applications. Today, we delve into a specific problem that combines algebraic expressions with complex numbers: Find the value of $X - 6X + 13$ when $X = 3 + 2i$. This problem provides an excellent opportunity to explore the fundamental principles of complex number arithmetic, substitution, and simplification, all within a context that emphasizes clarity and precision.

Understanding Complex Numbers: A Primer

Before we jump into solving the expression, it's crucial to understand what complex numbers are and how they operate mathematically.

What Are Complex Numbers?

A complex number is a number that can be expressed in the form $a + bi$, where:

- a is the real part,
- b is the imaginary part,
- i is the imaginary unit, satisfying $i^2 = -1$.

For instance, in the given value $X = 3 + 2i$, 3 is the real part, and $2i$ is the imaginary part. Complex numbers are fundamental in various fields, including engineering, physics, and applied mathematics, owing to their ability to model oscillations, waves, and other phenomena.

Basic Operations with Complex Numbers

To accurately evaluate the given expression, one must be comfortable with:

- Addition and subtraction: Combine the real parts and the imaginary parts separately.
- Multiplication: Use distributive property, considering that $i^2 = -1$.
- Substitution: Replacing the variable with the given complex number.

Step-by-Step Solution Approach

Let's analyze the problem systematically:

Given:

- $X = 3 + 2i$
- Expression to evaluate: $X - 6X + 13$

The process involves:

1. Substituting the value of X into the expression.
2. Simplifying the algebraic terms involving X .
3. Combining like terms.
4. Arriving at the final result.

Substituting the Value of X

Start with the original expression:

$[X - 6X + 13]$

Substitute $X = 3 + 2i$:

$$\sqrt{(3 + 2i) - 6(3 + 2i) + 13}$$

Simplifying the Expression

Step 1: Distribute the -6 across the complex number

$$\sqrt{(3 + 2i) - (6 \times 3 + 6 \times 2i) + 13}$$

Calculating:

$$\sqrt{(3 + 2i) - (18 + 12i) + 13}$$

Step 2: Combine like terms

Now, combine the real parts and imaginary parts separately:

- Real parts: $3 - 18 + 13$
- Imaginary parts: $2i - 12i$

Calculations:

- Real: $\sqrt{(3 - 18 + 13 = (3 - 18) + 13 = -15 + 13 = -2)}$
- Imaginary: $\sqrt{(2i - 12i = -10i)}$

Final Simplified Result:

$$\sqrt{-2 - 10i}$$

Interpreting the Result

The expression evaluates to $-2 - 10i$. This is a complex number with a real part of -2 and an imaginary part of -10i.

Significance of the Result

This result illustrates that operations involving complex numbers follow similar rules to real numbers, with the added consideration of the imaginary unit i . The final answer being a complex number emphasizes that the original algebraic expression, when evaluated at a complex input, yields a complex output.

Broader Implications and Applications

Understanding how to manipulate and evaluate expressions like $X^2 - 6X + 13$ with complex numbers has real-world significance:

- Electrical engineering: Phasor calculations involve complex numbers.
- Quantum physics: Wave functions and probability amplitudes often involve complex expressions.
- Control systems: Stability and response analysis frequently utilize complex plane representations.

Furthermore, this exercise underscores vital algebraic skills that are foundational in advanced mathematics and engineering disciplines.

Additional Tips for Handling Complex Expressions

To enhance proficiency, consider the following:

- Practice substitution: Always carefully replace variables with their values, especially complex ones.
- Remember $i^2 = -1$: This identity simplifies multiplication involving imaginary units.
- Combine like terms systematically: Separate real and imaginary components to avoid errors.
- Use complex conjugates when necessary to simplify expressions involving division or modulus calculations.

Conclusion

Evaluating the expression $X^2 - 6X + 13$ at $X = 3 + 2i$ demonstrates the beauty and consistency of algebraic operations extended into the complex plane. The process involves straightforward substitution, distribution, and combination of like terms, ultimately revealing that the expression simplifies to $-2 - 10i$. Mastery of such computations is essential for students and professionals working in fields that rely on complex analysis, signal processing, and quantum mechanics. As complex numbers continue to underpin many technological advances, understanding their manipulation remains a vital skill in the modern scientific toolkit.

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