If R(t)= Sint, Then Find R(14)(/3)

Understanding the Problem: If R(t) = sin t, Then Find R(14)(π/3)

Introduction to the Function R(t)

The problem begins by defining a function R(t) as R(t) = sin t. This indicates a standard sine function, which is a fundamental trigonometric function with well-known properties. The task is to find the 14th derivative of R(t) evaluated at t = π/3, written as R(14)(π/3).

Clarifying the Notation

Before proceeding, it’s crucial to interpret the notation correctly:
- R(t) = sin t: The original function.
- R(14)(t): The 14th derivative of R(t) with respect to t.
- R(14)(π/3): The value of the 14th derivative at t = π/3.

This is a common notation in calculus where the n-th derivative of a function is denoted as f^{(n)}(t). Here, R^{(14)}(t) signifies the 14th derivative.

Fundamentals of Derivatives of the Sine Function

First Few Derivatives of sin t

Understanding the derivatives of sine functions is essential. Let's look at the initial derivatives:

- f(t) = sin t
- f'(t) = cos t
- f''(t) = -sin t
- f'''(t) = -cos t
- f^{(4)}(t) = sin t

Notice that after the 4th derivative, the pattern repeats every 4 derivatives:

- 4th derivative: $\sin t$
- 5th derivative: $\cos t$
- 6th derivative: $-\sin t$
- 7th derivative: $-\cos t$
- 8th derivative: $\sin t$
- and so on...

This cyclic pattern is fundamental in computing higher-order derivatives of sine functions.

Pattern Recognition in Higher-Order Derivatives

Given the cyclic nature, the derivatives of $\sin t$ follow a repeating sequence every 4 derivatives:

Derivative Order (n)	Derivative Expression	Pattern Index (n mod 4)	Sign and Function Type
0	sin t	0	sin t (positive)
1	cos t	1	cos t (positive)
2	-sin t	2	-sin t
3	-cos t	3	-cos t
4	sin t	0	sin t (positive)
...	...	...	...

Thus, the n-th derivative of $\sin t$ can be expressed based on $n \bmod 4$.

Calculating the 14th Derivative of sin t

Using the Pattern to Find $R^{(14)}(t)$

Since $R(t) = \sin t$, the 14th derivative $R^{(14)}(t)$ will follow the same cycle:

- Determine $n \bmod 4$: $14 \bmod 4 = 2$

From the pattern:
- When $n \bmod 4 = 0$, derivative = $\sin t$
- When $n \bmod 4 = 1$, derivative = $\cos t$
- When $n \bmod 4 = 2$, derivative = $-\sin t$
- When $n \bmod 4 = 3$, derivative = $-\cos t$

Given that $14 \bmod 4 = 2$, the 14th derivative is:

$R^{(14)}(t) = -\sin t$

This simplifies the problem significantly.

Evaluating at t = π/3

Now, to find $R^{(14)}(\pi/3)$:

$R^{(14)}(\pi/3) = -\sin(\pi/3)$

Recall that:

$\sin(\pi/3) = \sqrt{3} / 2$

Therefore,

$R^{(14)}(\pi/3) = - (\sqrt{3} / 2)$

Final Result:

$R^{(14)}(\pi/3) = - (\sqrt{3} / 2)$

Summary of Key Steps

Steps to Find the n-th Derivative at a Point for sin t

1. Identify the pattern of derivatives:
- Use the cyclic nature of derivatives of sine functions.
2. Determine n mod 4 to find the corresponding derivative pattern.
3. Write the n-th derivative expression based on this pattern.
4. Substitute the specific value of t into the derivative expression.
5. Simplify to find the numerical or simplified exact value.

Generalization to Other Functions

This approach applies broadly to functions with cyclic derivative patterns, such as sine and cosine functions. Recognizing these patterns simplifies the process of computing higher derivatives at specific points.

Additional Insights: Why Recognize Derivative

Patterns?

Efficiency in Calculations

Knowing the cyclic derivatives allows for quick computation of high-order derivatives without repeatedly differentiating step-by-step.

Applications in Fourier Series and Differential Equations

Understanding these derivative patterns is critical in solving differential equations, analyzing oscillatory systems, and working with Fourier series, where higher derivatives of sine and cosine functions are common.

Extensions to Complex Analysis

The pattern also extends into complex analysis, where derivatives of exponential functions involving sine and cosine are studied.

Conclusion

Final Remarks

The problem of finding $R(14)(\pi/3)$ when $R(t) = \sin t$ hinges on understanding the cyclical nature of derivatives of sine functions. Recognizing that the derivatives repeat every four steps simplifies the calculation dramatically. By calculating 14 mod 4, we determined the 14th derivative to be $-\sin t$, and evaluating at $t = \pi/3$ yields $- (\sqrt{3} / 2)$. This process exemplifies the power of identifying patterns in calculus, leading to efficient and elegant solutions to seemingly complex derivative problems.

Summary of the Key Result

- The 14th derivative of sin t: $R^{(14)}(t) = -\sin t$
- Evaluated at $t = \pi/3$: $R^{(14)}(\pi/3) = - (\sqrt{3} / 2)$

This comprehensive understanding not only solves the immediate problem but also enhances the ability to handle similar derivative calculations in advanced mathematics.

Frequently Asked Questions

What is the given function R(t) in the problem?

The given function is R(t) = sin(t).

What does R(14)(/3) represent in the context of the problem?

It appears to be asking for the 14th derivative of R(t) evaluated at t = π/3.

How do we find the nth derivative of R(t) = sin(t)?

The derivatives of sin(t) cycle every four steps: sin(t), cos(t), -sin(t), -cos(t), and then repeat.

What is the 14th derivative of sin(t)?

Since derivatives repeat every 4 steps, 14 mod 4 = 2, so the 14th derivative is the same as the 2nd derivative, which is -sin(t).

How do we evaluate the 14th derivative at t = π/3?

Substitute t = π/3 into the 14th derivative: -sin(π/3) = - (√3/2) = -√3/2.

Additional Resources

If R(t) = Sin t, Then Find R(14)(π/3)

Introduction

Mathematics is a profound language that helps us decode the universe's intricate patterns. Among its many branches, calculus stands out as a powerful tool for understanding change and motion. When dealing with functions like R(t) = sin t, a common challenge is finding higher-order derivatives at specific points. In this context, the problem "If R(t) = sin t, then find R(14)(π/3)" requires us to determine the 14th derivative of R(t) evaluated at t = π/3.

This article delves into the mathematical journey to solve such a problem, exploring derivatives of trigonometric functions, establishing general patterns, and applying these insights to find the desired value. From foundational concepts to detailed computations, this review aims to provide a comprehensive understanding suitable for students, educators, and enthusiasts interested in advanced calculus.

Understanding the Problem

The notation R(14) signifies the 14th derivative of the function R(t) with respect to t, denoted as $R^{(14)}(t)$. The problem asks us to evaluate this 14th derivative at t = π/3, given that R(t) = sin t.

This task involves two main steps:

1. Determining the general pattern of derivatives of sin t.
2. Applying this pattern to find the 14th derivative and then evaluating it at t = π/3.

Derivatives of the Sine Function: A Pattern Recognition

The sine function is periodic in its derivatives, with a repeating cycle every four derivatives.

Basic derivatives:

- First derivative: $R'(t) = \cos t$
- Second derivative: $R''(t) = -\sin t$
- Third derivative: $R'''(t) = -\cos t$
- Fourth derivative: $R^{(4)}(t) = \sin t$

Notice the pattern repeats every four derivatives:

Derivative Order	Expression	Simplification
0 (original)	sin t	sin t
1	cos t	cos t
2	-sin t	-sin t
3	-cos t	-cos t
4	sin t	sin t (cycle repeats)
5	cos t	cos t
...	...	...

This periodicity suggests that derivatives of sin t follow a cycle of length 4:

- $n \equiv 0 \mod 4$: $R^{(n)}(t) = \sin t$
- $n \equiv 1 \mod 4$: $R^{(n)}(t) = \cos t$
- $n \equiv 2 \mod 4$: $R^{(n)}(t) = -\sin t$
- $n \equiv 3 \mod 4$: $R^{(n)}(t) = -\cos t$

Deriving the 14th Derivative

Given the pattern, to find $R^{(14)}(t)$:

1. Calculate 14 mod 4:
$14 \div 4 = 3$ remainder 2
So, $14 \equiv 2 \pmod 4$

2. According to the pattern, when $n \equiv 2 \bmod 4$, the derivative is:

$R^{(n)}(t) = -\sin t$

3. Therefore:

$R^{(14)}(t) = -\sin t$

Evaluating at $t = \pi/3$

Having established that:

$R^{(14)}(t) = -\sin t$

We now substitute $t = \pi/3$:

$R^{(14)}(\pi/3) = -\sin(\pi/3)$

Recall that $\sin(\pi/3) = \sqrt{3}/2$

Thus,

$R^{(14)}(\pi/3) = -(\sqrt{3}/2) = -\sqrt{3}/2$

Summary of the Calculation

Step	Result
Derivative pattern (n mod 4)	$R^{(n)}(t) = \sin t, \cos t, -\sin t, -\cos t$
For n=14	$14 \equiv 2 \pmod 4$
Derivative expression	$-\sin t$
Substitute $t = \pi/3$	$-\sin(\pi/3) = -(\sqrt{3}/2)$
Final answer	$-\sqrt{3}/2$

Additional Considerations

Significance of the Pattern

The periodicity of derivatives of sine and cosine functions exemplifies the elegance of trigonometric calculus. Recognizing such patterns simplifies otherwise complex differentiation tasks, especially for higher-order derivatives.

Extensions to Other Functions

The methodology used here extends to other functions with cyclical derivatives, such as tangent, cotangent, or exponential functions with sinusoidal components, where understanding the fundamental derivative cycles can streamline calculations.

Practical Applications

- Signal Processing: High-order derivatives of sinusoidal signals are foundational in Fourier analysis.
- Physics: Understanding the derivatives of sinusoidal functions relates to motion, waves, and oscillations.
- Engineering: Control systems often analyze higher derivatives for system stability.

Final Thoughts

The problem "If R(t) = sin t, then find R(14)(π/3)" exemplifies the power of pattern recognition in calculus. By understanding the cyclical nature of derivatives of sine functions, complex-looking tasks become manageable. The key takeaway is that higher-order derivatives of fundamental functions often follow predictable patterns, and leveraging these patterns can lead to elegant solutions.

In this case, the 14th derivative of sin t evaluated at π/3 is $-\sqrt{3}/2$. This result not only showcases a specific calculation but also highlights the beauty of mathematical symmetry and periodicity inherent in trigonometric functions.

References

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Conclusion

Understanding the derivatives of sine functions and their periodic nature allows for quick and accurate calculations of higher-order derivatives at specific points. Recognizing patterns in derivatives is a valuable skill in both theoretical and applied mathematics, simplifying complex problems into straightforward evaluations. Whether for academic purposes or real-world applications, mastering these concepts enhances analytical capabilities and deepens appreciation for the elegance of calculus.

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