Sin(tan^1x)=cos(sin^1(x/2))

Sin(tan^1x)=cos(sin^1(x/2)) is an intriguing trigonometric equation that combines inverse trigonometric functions with basic sine and cosine functions. Exploring such equations helps deepen our understanding of the relationships between different trigonometric functions and their inverses, which are fundamental in calculus, geometry, and mathematical analysis. This article aims to analyze the equation thoroughly, understand its solutions, and explore related concepts, all while optimizing for SEO to help students, educators, and math enthusiasts find comprehensive information on this topic.

Understanding the Equation: Sin(tan^1x)=cos(sin^1(x/2))

Before delving into solving the equation, it is essential to understand the individual components and their properties.

Inverse Trigonometric Functions

Inverse trigonometric functions such as *tan^(-1)x* (arctangent) and *sin^(-1)x* (arcsine) are functions that reverse the effects of the basic trigonometric functions. They are crucial in solving equations involving angles and ratios.

- **tan^(-1)x (arctangent)** gives the angle whose tangent is x.
- **sin^(-1)x (arcsine)** provides the angle whose sine is x.

Both functions have specific domains and ranges:
- *tan^(-1)x* is defined for all real x, with a range of \(-\frac{\pi}{2}\) to \(\frac{\pi}{2}\).
- *sin^(-1)x* is defined for x in \([-1,1]\), with a range of \(-\frac{\pi}{2}\) to \(\frac{\pi}{2}\).

Basic Sine and Cosine Functions

The sine and cosine functions are fundamental in trigonometry:
- *sin θ* measures the y-coordinate of a point on the unit circle.
- *cos θ* measures the x-coordinate of a point on the unit circle.

In our equation:
- The left side is *sin(tan^(-1)x)*.
- The right side is *cos(sin^(-1)(x/2))*.

Understanding how these components interact helps in solving the equation.

Analyzing the Equation Step-by-Step

To comprehend and solve the equation *Sin(tan^1x)=cos(sin^1(x/2))*, we analyze each side separately and look for potential relationships.

Evaluating the Left Side: sin(tan^(-1)x)

Let's consider:
- *θ = tan^(-1)x*
- Then, *tan θ = x*.

Using the right triangle representation:
- Opposite side = x
- Adjacent side = 1
- Hypotenuse = $\sqrt{1 + x^2}$

Thus:
- *$\sin \theta$ = opposite / hypotenuse = $x / \sqrt{1 + x^2}$.*

Therefore:

$$\sin(\tan^{-1}x) = x / \sqrt{1 + x^2}$$

Evaluating the Right Side: cos(sin^(-1)(x/2))

Let:
- *$\phi = \sin^{-1}(x/2)$*
- Then, *$\sin \phi = x/2$.*

Representing this with a right triangle:
- Opposite side = $x/2$
- Hypotenuse = 1

The adjacent side:
- *adjacent = $\sqrt{1 - (x/2)^2} = \sqrt{1 - x^2/4} = \sqrt{(4 - x^2)/4} = (1/2)\sqrt{4 - x^2}$*

Now:
- *$\cos \phi$ = adjacent / hypotenuse = $(1/2)\sqrt{4 - x^2} / 1 = (1/2)\sqrt{4 - x^2}$*

So:

$$\cos(\sin^{-1}(x/2)) = (1/2)\sqrt{4 - x^2}$$

Rewriting the Equation in Simplified Form

Putting it all together, the original equation becomes:

$$x / \sqrt{1 + x^2} = (1/2)\sqrt{4 - x^2}$$

To verify the solutions and understand the behavior, we analyze this equality further.

Solving the Equation: Algebraic Approach

Let's explore the algebraic steps to find the solutions for x.

Squaring Both Sides

Squaring both sides eliminates the square roots:

$$[x / \sqrt{1 + x^2}]^2 = [(1/2)\sqrt{4 - x^2}]^2$$

Calculating each side:

$$x^2 / (1 + x^2) = (1/4)(4 - x^2)$$

Multiply through both sides by $4(1 + x^2)$ to clear denominators:

$$4x^2 = (1 + x^2)(4 - x^2)$$

Expand the right side:
- $(1)(4 - x^2) + x^2 (4 - x^2) = 4 - x^2 + 4x^2 - x^4$

Simplify:

$$4x^2 = 4 - x^2 + 4x^2 - x^4$$

Bring all to one side:

$$4x^2 - 4 + x^2 - 4x^2 + x^4 = 0$$

Combine like terms:
- $4x^2 - 4x^2 = 0$
- x^2 remains
- Constant term: -4
- Plus x^4

Resulting in:

$$x^4 + x^2 - 4 = 0$$

This is a quadratic in x^2:

$$\text{Let } y = x^2$$
$$\Rightarrow y^2 + y - 4 = 0$$

Solve for y using quadratic formula:

$$y = [-1 \pm \sqrt{(1^2 - 41(-4))}] / 2$$
$$= [-1 \pm \sqrt{(1 + 16)}] / 2$$
$$= [-1 \pm \sqrt{17}] / 2$$

Since $x^2 = y$, and $x^2 \geq 0$, we only consider positive solutions:

- First root:

$$y = [-1 + \sqrt{17}] / 2$$

which is positive because $\sqrt{17} \approx 4.123$, so

$$y \approx (-1 + 4.123) / 2 \approx 3.123 / 2 \approx 1.5615$$

- Second root:

$$y = [-1 - \sqrt{17}] / 2$$

which is negative:

$$(-1 - 4.123) / 2 \approx -5.123 / 2 \approx -2.5615$$

and since x^2 cannot be negative, discard this solution.

Thus:

$$x^2 \approx 1.5615$$
$$\Rightarrow x \approx \pm\sqrt{1.5615} \approx \pm 1.249$$

Important: These solutions must be checked against the original equation to ensure they are valid, especially because squaring can introduce extraneous solutions.

Verification of Solutions

To verify whether $x \approx \pm 1.249$ satisfy the original equation, substitute back into the simplified form.

Check for $x \approx 1.249$:

- Left side:

$$x / \sqrt{(1 + x^2)} \approx 1.249 / \sqrt{(1 + 1.5615)} \approx 1.249 / \sqrt{2.5615} \approx 1.249 / 1.6 \approx 0.7806$$

- Right side:

$$(1/2)\sqrt{(4 - x^2)} \approx 0.5 \ \ \sqrt{(4 - 1.5615)} \approx 0.5 \ \ \sqrt{2.4385} \approx 0.5 \ \ 1.5615 \approx 0.78075$$

The values are approximately equal, confirming that $x \approx 1.249$ is a valid solution.

Similarly, for $x \approx -1.249$:

- Left side:

$$-1.249 / \sqrt{(1 + 1.5615)} \approx -1.249 / 1.6 \approx -0.7806$$

- Right side:

$$0.5 \ \ \sqrt{(4 - 1.5615)} \approx 0.78075$$

Since the right side

Frequently Asked Questions

What is the main goal when solving the equation Sin(tan^(-1) x) = cos(Sin^(-1)(x/2))?

The main goal is to find all real values of x that satisfy the equation by simplifying and applying inverse trigonometric identities.

How can the expression Sin(tan^(-1) x) be simplified?

Sin(tan^(-1) x) can be simplified using the identity Sin(tan^(-1) x) = x / □ $(1 + x^2)$.

What is the simplified form of cos(Sin^(-1)(x/2))?

cos(Sin^(-1)(x/2)) simplifies to □ $(1 - (x/2)^2)$ = □ $(1 - x^2/4)$.

How do you set up the equation after simplification?

After simplification, set x / □ $(1 + x^2)$ = □ $(1 - x^2/4)$ and solve for x.

What are the key steps to solving the resulting equation?

Key steps include squaring both sides to eliminate radicals, simplifying the resulting algebraic equation, and solving for x, then verifying solutions.

Are there any domain restrictions for x in the original equation?

Yes, since sin^(-1) and tan^(-1) have specific domains, x must satisfy -□ < x < □ for tan^(-1) x, and $-2 \leq x \leq 2$ for sin^(-1)(x/2).

What are the solutions to the equation?

The solutions are x = 0 and x = ±□ 2, after solving the algebraic equation and checking the domain restrictions.

How can you verify the solutions for the original equation?

Substitute each solution back into the original equation to ensure both sides are equal, confirming their validity.

Additional Resources

Sin(tan^1x) = cos(sin^1(x/2)): Unraveling an Intricate Trigonometric Identity

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Introduction: The Fascinating World of Trigonometric Equations

In the vast universe of mathematics, trigonometry stands out as a field rich with identities, transformations, and relationships that often seem complex yet reveal elegant simplicity upon closer examination. At the heart of this exploration lies the intriguing equation:

Sin(tan^1x) = cos(sin^1(x/2))

This equation interweaves inverse trigonometric functions—namely, arctangent and arcsine—with their direct counterparts, sine and cosine. For mathematicians, students, and enthusiasts alike, such identities not only deepen understanding of trigonometric functions but also serve as gateways to advanced topics like calculus, complex analysis, and geometric interpretations.

The purpose of this article is to dissect, analyze, and understand this equation thoroughly, providing clarity through detailed explanations, algebraic manipulations, and geometric insights.

Understanding the Components: A Primer on Inverse Trigonometric Functions

Before delving into the equation itself, it is crucial to revisit the fundamental definitions and properties of the involved functions.

The Inverse Sine (Arcsine) Function: sin^(-1) or arcsin

- Definition: For a real number y in the interval [-1, 1], arcsin(y) returns an angle θ in the range [-π/2, π/2] such that sin(θ) = y.
- Properties:
- Monotonically increasing on its domain.
- Inverse relationship with sine within its principal range.

The Inverse Tangent (Arctangent) Function: tan^(-1) or arctan

- Definition: For any real number x, arctan(x) returns an angle θ in (-π/2, π/2) such that tan(θ) = x.
- Properties:
- Covers the entire real line.
- Monotonically increasing.

Understanding these functions' ranges and behaviors lays the foundation for analyzing the given identity.

Deciphering the Equation: Breaking Down the Identity

The equation involves two parts:

- Left Side: sin(tan^(-1) x)
- Right Side: cos(sin^(-1)(x/2))

By examining each component, we can attempt to express both sides in terms of x and see if they are equivalent.

Evaluating the Left Side: sin(tan^(-1) x)

Let's denote:

$$
\theta = \tan^{-1} x
$$

This means:

$$
x = \tan \theta
$$

Since θ is the angle whose tangent is x, and $\theta \in (-\pi/2, \pi/2)$, we can imagine a right triangle with:

- Opposite side = x
- Adjacent side = 1
- Hypotenuse = $\sqrt{1 + x^2}$

Therefore,

$$
\sin \theta = \frac{\text{opposite}}{\text{hypotenuse}} = \frac{x}{\sqrt{1 + x^2}}
$$

This directly gives:

$$
\sin (\tan^{-1} x) = \frac{x}{\sqrt{1 + x^2}}
$$

Conclusion: The left side simplifies to:

$$
\boxed{\sin (\tan^{-1} x) = \frac{x}{\sqrt{1 + x^2}}}
$$

Evaluating the Right Side: cos(sin^(-1)(x/2))

Let's denote:

$$
\phi = \sin^{-1} \left(\frac{x}{2} \right)
$$

which implies:

$$
\sin \phi = \frac{x}{2}
$$

with $\phi \in [-\pi/2, \pi/2]$. For this angle:

- Opposite side = x/2
- Hypotenuse = 1 (since $\sin \phi = \frac{\text{opposite}}{\text{hypotenuse}}$)

Constructing a right triangle with these ratios:

- Opposite side = x/2
- Hypotenuse = 1
- Adjacent side = $\sqrt{1 - \left(\frac{x}{2}\right)^2} = \sqrt{1 - \frac{x^2}{4}} = \frac{\sqrt{4 - x^2}}{2}$

Now, the cosine of ϕ is:

$$
\cos \phi = \frac{\text{adjacent}}{\text{hypotenuse}} = \frac{\sqrt{4 - x^2}/2}{1} = \frac{\sqrt{4 - x^2}}{2}
$$

Conclusion: The right side simplifies to:

$$
\boxed{\cos (\sin^{-1} (x/2)) = \frac{\sqrt{4 - x^2}}{2}}
$$

The Simplified Identity: Comparing Both Sides

Now, the original equation can be rewritten using the simplified forms:

$$
\frac{x}{\sqrt{1 + x^2}} = \frac{\sqrt{4 - x^2}}{2}
$$

To verify whether the identity holds universally or under specific conditions, we analyze the equality:

$$
\frac{x}{\sqrt{1 + x^2}} = \frac{\sqrt{4 - x^2}}{2}
$$

Validity and Domain Considerations

Step 1: Cross-multiplied form:

$$
2x = \sqrt{1 + x^2} \times \sqrt{4 - x^2}
$$

Step 2: Simplify the right side:

$$
2x = \sqrt{(1 + x^2)(4 - x^2)}
$$

Step 3: Square both sides to eliminate the square root:

$$
(2x)^2 = (1 + x^2)(4 - x^2)
$$

$$
4x^2 = (1 + x^2)(4 - x^2)
$$

Step 4: Expand the right side:

$$
4x^2 = (1)(4 - x^2) + x^2 (4 - x^2) = 4 - x^2 + 4x^2 - x^4
$$

$$
4x^2 = 4 + 3x^2 - x^4
$$

Step 5: Rearranged as:

\[

$$
4x^2 - 3x^2 + x^4 = 4
$$

$$
x^2 + x^4 = 4
$$

or equivalently,

$$
x^4 + x^2 - 4 = 0
$$

Solving the Resulting Quartic Equation

Let's set:

$$
y = x^2
$$

The equation becomes:

$$
y^2 + y - 4 = 0
$$

which is a quadratic in y.

Using the quadratic formula:

$$
y = \frac{-1 \pm \sqrt{1^2 - 4 \times 1 \times (-4)}}{2} = \frac{-1 \pm \sqrt{1 + 16}}{2} = \frac{-1 \pm \sqrt{17}}{2}
$$

Since $y = x^2 \geq 0$, only positive solutions are valid.

- For the positive root:

$$
y = \frac{-1 + \sqrt{17}}{2} \approx \frac{-1 + 4.1231}{2} \approx \frac{3.1231}{2} \approx 1.56155
$$

- For the negative root:

$$
y = \frac{-1 - \sqrt{17}}{2} \approx \frac{-1 - 4.1231}{2} \approx \frac{-5.1231}{2} \approx -2.56155
$$

Since y cannot be negative (as $y = x^2$), only:

$$
x^2 = \frac{-1 + \sqrt{17}}{2} \approx 1.56155
$$

is valid.

Thus,

$$
x = \pm \sqrt{\frac{-1 + \sqrt{17}}{2}}
$$

which are real solutions where the original identity holds.

Interpretation and Geometric Significance

The derivation reveals that the initial trigonometric identity holds only at specific points determined by the solutions to the quartic equation. Essentially, the identity

$$
\sin(\tan^{-1} x) = \cos(\sin^{-1} (x/2))
$$

is not universally valid for all real x, but rather at particular values where the algebraic condition derived above is satisfied.

Geometric intuition:

- The left side, $\sin(\arctan x)$, corresponds to the sine of an angle with tangent x, which can be visualized as a point on a right triangle with sides x and 1.
- The right side, $\cos(\arcsin(x/2))$, corresponds to the cosine of an angle with sine $x/2$, which can be represented similarly.

The points where the two

Sin Tan 1x Cos Sin 1 X 2

Sin Tan 1x Cos Sin 1 X 2

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