Solve 3x + Y = 105x - 2y - 2 = 0

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Introduction

Mathematics is a universal language that provides tools to analyze and solve problems across various fields including engineering, physics, economics, and everyday life. One common type of problem involves solving systems of equations, which helps determine the values of unknown variables that satisfy multiple conditions simultaneously.

The equation $3x + y = 105x - 2y - 2 = 0$ is a compound algebraic statement that involves multiple expressions set equal to zero. Understanding how to interpret and solve such equations is fundamental to developing problem-solving skills in algebra. In this article, we will explore methods to solve this specific system, analyze its structure, and interpret the solutions within a broader mathematical context.

Understanding the Equation: Breaking Down the Problem

Before diving into solution strategies, it’s essential to interpret the given expression correctly. The statement:

$3x + y = 105x - 2y - 2 = 0$

can be understood as two separate equations:

1. $3x + y = 0$
2. $105x - 2y - 2 = 0$

This interpretation stems from the common mathematical convention where multiple equalities are listed in sequence, implying each is set equal to zero. Therefore, the problem reduces to solving the system:

$$
\begin{cases}
3x + y = 0 \quad \text{(Equation 1)} \\
105x - 2y - 2 = 0 \quad \text{(Equation 2)}
\end{cases}
$$

Step-by-Step Solution of the System of Equations

1. Express one variable in terms of the other

From Equation 1:

$$
3x + y = 0 \Rightarrow y = -3x
$$

This allows us to substitute y in the second equation.

2. Substitute into the second equation

Substituting $y = -3x$ into Equation 2:

$$
105x - 2(-3x) - 2 = 0
$$

Simplify:

$$
105x + 6x - 2 = 0
$$

Combine like terms:

$$
(105x + 6x) - 2 = 0 \Rightarrow 111x - 2 = 0
$$

3. Solve for x

Add 2 to both sides:

$$
111x = 2
$$

Divide both sides by 111:

$$
x = \frac{2}{111}
$$

4. Find y using the value of x

Recall:

$$
y = -3x
$$

Substitute x = $\frac{2}{111}$:

$$
y = -3 \times \frac{2}{111} = -\frac{6}{111}
$$

Simplify the fraction:

$$
y = -\frac{2}{37}
$$

Final Solution

The solutions to the system are:

$$
\boxed{
x = \frac{2}{111}, \quad y = -\frac{2}{37}
}
$$

This pair (x, y) satisfies both equations simultaneously.

Verifying the Solution

It’s vital to verify the solution by substituting the values back into the original equations.

Check Equation 1:

$$
3x + y = 3 \times \frac{2}{111} + \left(-\frac{2}{37}\right)
$$

Calculate:

$$
\frac{6}{111} - \frac{2}{37}
$$

Express both fractions with a common denominator:

- Denominator for $\frac{6}{111}$: 111
- Denominator for $\frac{2}{37}$: 37

Note that 111 = 3 × 37, so:

$$
\frac{6}{111} = \frac{6}{3 \times 37} = \frac{2}{37}
$$

Thus,

$$
\frac{2}{37} - \frac{2}{37} = 0
$$

which confirms that Equation 1 holds true.

Check Equation 2:

$$
105x - 2y - 2 = 105 \times \frac{2}{111} - 2 \times \left(-\frac{2}{37}\right) - 2
$$

Calculate:

$$
\frac{210}{111} + \frac{4}{37} - 2
$$

Simplify $\frac{210}{111}$:

$$
\frac{210}{111} = \frac{70}{37}
$$

Now, combine:

$$
\frac{70}{37} + \frac{4}{37} - 2 = \frac{74}{37} - 2
$$

Express 2 as $\frac{74}{37}$:

$$
\frac{74}{37} - \frac{74}{37} = 0
$$

Hence, Equation 2 is also satisfied, confirming our solution.

Broader Context: Solving Linear Systems

The example above demonstrates a standard methodology for solving linear systems:

- Express one variable in terms of the other.
- Substitute into the second equation.
- Solve for one variable.
- Back-substitute to find the other.

Types of solutions:

- Unique solution: as in this case, where the system is consistent and the equations are independent.
- Infinite solutions: when equations are dependent (multiple equations describing the same line).
- No solution: when equations are inconsistent (parallel lines, for example).

Understanding these distinctions is crucial for analyzing more complex systems.

Applications of Solving Systems of Equations

Linear systems like the one discussed have numerous practical applications:

- Engineering: designing circuits, analyzing forces.
- Economics: modeling supply and demand.
- Physics: calculating trajectories and forces.
- Computer Graphics: transformations and rendering.

Mastering solving techniques enables professionals and students to model real-world problems efficiently.

Additional Techniques for Solving Systems

While substitution works well for small systems, other methods include:

1. Graphical Method

Plotting each equation on a coordinate plane and identifying the point(s) of intersection.

2. Elimination Method

Adding or subtracting equations to eliminate a variable, especially useful for larger systems.

3. Matrix Method (Using Inverse matrices or Cramer's Rule)

Applying linear algebra techniques for systems with more variables and equations.

Handling More Complex or Nonlinear Systems

The system in this article is linear, but real-world problems often involve nonlinear equations. These require different approaches such as:

- Factoring
- Substitution
- Numerical methods (like Newton-Raphson)
- Graphical methods

Understanding the linear case builds a foundation for tackling more advanced problems.

Conclusion

The problem Solve 3x + y = 105x - 2y - 2 = 0 exemplifies fundamental algebraic techniques for solving systems of equations. By interpreting the compound statement correctly, expressing one variable in terms of another, and substituting back, we arrived at a unique solution:

\[
\boxed{
x = \frac{2}{111}, \quad y = -\frac{2}{37}
}
\]

This process highlights the importance of systematic problem-solving strategies in mathematics. Whether for academic purposes or real-world applications, mastering these methods is essential for analyzing and solving a wide range of problems involving multiple variables.

Furthermore, understanding the different types of solutions and the various methods available prepares students and professionals to handle more complex systems efficiently. As you continue exploring algebra and linear systems, keep practicing these techniques to build confidence and proficiency in mathematical problem-solving.

Keywords for SEO Optimization

- Solving systems of equations
- Algebraic methods
- Linear systems solutions
- Substitution method
- Solving for variables
- Mathematical problem-solving
- Equation systems in algebra
- Linear algebra basics
- Practical applications of systems of equations
- How to solve simultaneous equations

Frequently Asked Questions

How can I solve the system of equations $3x + y = 0$ and $105x - 2y - 2 = 0$?

You can solve the system by substitution or elimination. For example, from the first equation, express y as $y = -3x$, then substitute into the second to find x, and subsequently y.

What is the solution to the system $3x + y = 0$ and $105x - 2y - 2 = 0$?

Solving the system yields $x = 0.02$ and $y = -0.06$, which satisfy both equations.

Are the equations 3x + y = 0 and 105x - 2y - 2 = consistent?

Yes, they are consistent. They have a unique solution obtained by solving the system algebraically.

How do I eliminate y to solve the system 3x + y = 0 and 105x - 2y - 2 = 0?

Multiply the first equation by 2 to get 6x + 2y = 0, then add it to the second equation to eliminate y and solve for x.

Can I use matrix methods to solve the equations 3x + y = 0 and 105x - 2y - 2 = 0?

Yes, you can represent the system as a matrix and use methods like Gaussian elimination or Cramer's rule to find the solution.

Additional Resources

In-Depth Analysis of the System of Equations: Solve 3x + Y = 105x - 2y - 2 = 0

Understanding and solving systems of equations is fundamental in algebra, providing insights into how variables interact and how to find values that satisfy multiple conditions simultaneously. The given system, expressed as:

$$
\begin{cases}
3x + y = 0 \\
105x - 2y - 2 = 0
\end{cases}
$$

is a linear system involving two equations with two variables, x and y. This review aims to thoroughly analyze this system, explore various methods of solution, interpret the geometric significance, and discuss potential applications and implications.

Overview of the System

The Equations in Detail

The system comprises:

1. Equation 1: $3x + y = 0$
2. Equation 2: $105x - 2y - 2 = 0$

These are linear equations, representing straight lines in the Cartesian plane. Solving the system involves finding the point(s) (x, y) that lie on both lines simultaneously.

Significance of the System

- Mathematical importance: Solving such systems uncovers intersection points, which are crucial in geometry, optimization, and real-world modeling.
- Practical applications: Systems like these appear in physics (e.g., equilibrium problems), economics (cost-profit analysis), engineering, and computer science.

Analytical Approach to Solving the System

Method 1: Substitution

The substitution method involves solving one equation for one variable and substituting into the other.

Step-by-step:

Step 1: Solve Equation 1 for y:

$$
3x + y = 0 \implies y = -3x
$$

Step 2: Substitute $y = -3x$ into Equation 2:

$$
105x - 2(-3x) - 2 = 0
$$

$$
105x + 6x - 2 = 0
$$

$$
(105 + 6)x - 2 = 0
$$

$$
111x - 2 = 0
$$

Step 3: Solve for x:

$$
111x = 2 \implies x = \frac{2}{111}
$$

Step 4: Find y:

$$
y = -3x = -3 \times \frac{2}{111} = -\frac{6}{111} = -\frac{2}{37}
$$

Result: The solution is:

$$
\boxed{
\left(\frac{2}{111}, -\frac{2}{37} \right)
}
$$

Method 2: Elimination

The elimination method involves aligning coefficients to cancel one variable.

Step-by-step:

Step 1: Write equations in standard form:

$$
\begin{cases}
3x + y = 0 \quad (1) \\
105x - 2y = 2 \quad (2)
\end{cases}
$$

(Note: Equation 2 is rewritten as $105x - 2y = 2$ for clarity.)

Step 2: Multiply Equation (1) by 2 to align y coefficients:

$$
2 \times (3x + y) = 2 \times 0 \implies 6x + 2y = 0
$$

Step 3: Add this new equation to Equation (2):

$$
(105x - 2y) + (6x + 2y) = 2 + 0
$$

$$
(105x + 6x) + (-2y + 2y) = 2
$$

$$
111x = 2
$$

Step 4: Solve for x:

$$
x = \frac{2}{111}
$$

Step 5: Substitute x back into Equation (1):

$$
3 \times \frac{2}{111} + y = 0 \implies \frac{6}{111} + y = 0
$$

$$
y = -\frac{6}{111} = -\frac{2}{37}
$$

$$

Result: Same as with substitution.

Geometric Interpretation

Graphical Representation

- Equation 1: $y = -3x$

- This line passes through the origin with a slope of -3, indicating a steep decline from left to right.

- Equation 2: $105x - 2y = 2$

- Rearranged as $y = \frac{105}{2}x - 1$
- This line has a slope of $\frac{105}{2} = 52.5$, extremely steep and positive, passing through $(0, -1)$.

Intersection Point

The solution $\left(\frac{2}{111}, -\frac{2}{37} \right)$ represents the point where both lines intersect, approximately:

$$
x \approx 0.01802, \quad y \approx -0.05405
$$

Graphically, these lines intersect very close to the origin, with the second line being much steeper than the first.

Significance of the Intersection

- The unique intersection point indicates the system has a single solution, typical for two non-parallel lines.
- The solution point can be plotted to visually verify the intersection.

Theoretical Insights and Implications

System Consistency and Uniqueness

- The lines are not parallel because their slopes differ:
- Slope of Equation 1: -3
- Slope of Equation 2: 52.5

- Since slopes differ, the lines intersect at exactly one point, confirming the system’s consistency and uniqueness of solution.

Linear Independence

- The equations are linearly independent, as evident from their different slopes.
- Independence ensures the system is solvable with a unique solution.

Algebraic vs. Geometric Perspectives

- Algebraically, solutions are derived through substitution or elimination.
- Geometrically, the solution corresponds to the intersection point of two lines in the plane.

Alternative Methods and Advanced Considerations

Matrix Method (Using Cramer's Rule)

- Express the system as a matrix:

$$
A = \begin{bmatrix}
3 & 1 \\
105 & -2
\end{bmatrix},
\quad
\mathbf{x} = \begin{bmatrix}
x \\
y
\end{bmatrix},
\quad
\mathbf{b} = \begin{bmatrix}
0 \\
2
\end{bmatrix}
$$

- Determinant of A:

$$
\det(A) = (3)(-2) - (1)(105) = -6 - 105 = -111 \neq 0
$$

- Solution:

\[
x = \frac{\det(A_x)}{\det(A)}, \quad y = \frac{\det(A_y)}{\det(A)}
\]

where:

\[
A_x = \begin{bmatrix}
0 & 1 \\
2 & -2
\end{bmatrix}, \quad
A_y = \begin{bmatrix}
3 & 0 \\
105 & 2
\end{bmatrix}
\]

- Compute determinants:

\[
\det(A_x) = (0)(-2) - (1)(2) = -2
\]
\[
\det(A_y) = (3)(2) - (0)(105) = 6
\]

- Final solutions:

\[
x = \frac{-2}{-111} = \frac{2}{111}
\]

$$
y = \frac{6}{-111} = -\frac{6}{111} = -\frac{2}{37}
$$

which matches previous solutions, confirming consistency.

Practical Applications and Examples

Engineering and Physics

- Statics: Finding equilibrium points where forces or moments balance.
- Kinematics: Determining intersection points of motion paths.

Economics and Business

- Cost Analysis: Solving for break-even points where two cost or revenue functions intersect.
- Resource Allocation: Optimizing variables subject to multiple linear constraints.

Computer Science

- Graphics: Calculating intersections of lines for rendering or collision detection.
- Algorithms: Solving systems efficiently within larger computational problems.

Limitations and Special Cases

Degenerate Cases

- If the two equations were to be scalar multiples of each other, the system would have infinitely many solutions.
- If the lines were parallel (same slope but different intercepts), the system would be inconsistent (no solution).

Nonlinear Extensions

- Real-world problems often involve nonlinear systems, adding complexity beyond linear algebra.
- The current system demonstrates fundamentals that serve as building blocks for more complex models.

Summary and Key Takeaways

- The system $3x + y = 0$ and $105x - 2y - 2 = 0$ has a single unique solution.
- The solution point is $\left(\frac{2}{111}, -\frac{2}{37} \right)$, approximately \((0.018, -0

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