Solve Algebraically (x-3)^4-5=11

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When approaching algebraic equations such as (x-3)^4 - 5 = 11, the goal is to isolate the variable—in this case, x—and find its value(s). This process involves understanding the properties of exponents, applying inverse operations, and carefully handling the algebraic manipulations to arrive at the solution. Solving equations like this can sometimes produce more than one solution, especially when higher powers are involved, so it’s essential to consider all possible roots, including negative ones, when applicable. In this article, we will explore step-by-step methods to solve the equation (x-3)^4 - 5 = 11, discuss potential challenges, and analyze what the solutions tell us about the problem.

Understanding the Equation

Equation Breakdown

The given equation is:

(x - 3)^4 - 5 = 11

This expression involves:

- A binomial expression raised to the fourth power: (x - 3)^4
- A constant subtraction: -5
- An equal sign with a constant on the right side: 11

The first step in solving such an equation is to understand what each part represents and how they relate to each other. The key is to isolate the term with the variable first, then work backwards through the operations.

Goal of the Solution

- Isolate the exponential expression (x - 3)^4.
- Solve for the inner expression (x - 3).
- Find the value(s) of x.

Step-by-Step Solution Process

Step 1: Isolate the Power Term

To start, add 5 to both sides of the equation to isolate the exponential term:

$(x - 3)^4 - 5 + 5 = 11 + 5$

Simplifies to:

$(x - 3)^4 = 16$

Step 2: Take the Fourth Root of Both Sides

Next, to solve for (x - 3), take the fourth root of both sides. Recall that:

- The fourth root is the same as raising to the power of 1/4.
- When taking roots of both sides, consider all possible roots, including positive and negative, because $(x - 3)^4$ is always non-negative, but the fourth root can be both positive and negative.

Applying the fourth root:

$x - 3 = \pm\sqrt[4]{16}$

Since 16 is a perfect fourth power ($2^4 = 16$), the fourth roots are:

$x - 3 = \pm 2$

Note: The ± indicates two solutions: one positive and one negative.

Step 3: Solve for x

Now, solve for x in both cases:

1. $x - 3 = 2$

$x = 2 + 3$

$x = 5$

2. $x - 3 = -2$

$x = -2 + 3$

$x = 1$

Final Solutions and Verification

Solutions Summary

The solutions to the original equation are:

- $x = 5$
- $x = 1$

Verification of Solutions

It’s important to verify these solutions by substituting them back into the original equation:

For $x = 5$:

- Compute $(5 - 3)^4 - 5$

- $(2)^4 - 5 = 16 - 5 = 11$

- Which matches the right side of the original equation.

For $x = 1$:

- Compute $(1 - 3)^4 - 5$

- $(-2)^4 - 5 = 16 - 5 = 11$

- Which also satisfies the original equation.

Since both solutions satisfy the original equation, they are valid.

Discussion of the Solution and Additional Considerations

Impact of Even Powers on Solutions

Because the power involved is even (the fourth power), both positive and negative roots of the intermediate step are valid solutions. This is a common characteristic in equations involving even exponents, where negative inputs can yield positive outputs.

Potential for Extraneous Solutions

In some algebraic equations, operations such as taking even roots can introduce extraneous solutions, which are solutions that satisfy the intermediate steps but not the original equation. However, in this case, verification confirms that both solutions (x = 1 and x = 5) are valid.

Extensions and Related Problems

- What happens if the right side is a different constant? The process remains similar, but the roots and solutions may differ.
- How to handle equations involving higher even powers or different exponents.
- The importance of verifying solutions, especially when dealing with roots and powers.

Summary and Key Takeaways

- Start by isolating the exponential or polynomial expression involving the variable.
- When dealing with even powers, consider both positive and negative roots.
- Take the appropriate root (in this case, the fourth root) to simplify the equation.
- Solve for the variable by isolating it after applying roots.
- Always verify solutions to ensure they satisfy the original equation.
- Recognize that algebraic equations involving even powers often have multiple solutions due to symmetry.

Conclusion

Solving algebraically the equation (x - 3)^4 - 5 = 11 involves a straightforward sequence of algebraic steps: isolating the power term, taking roots, and solving for x. The process highlights the importance of understanding how exponents and roots interact and the necessity of verifying solutions. The solutions $x = 1$ and $x = 5$ demonstrate how symmetry in even powers can produce multiple valid solutions. Mastering these techniques enhances problem-solving skills and deepens understanding of algebraic concepts, providing a foundation for tackling more complex equations in mathematics.

Frequently Asked Questions

How do I solve the equation (x-3)^4 - 5 = 11 algebraically?

First, add 5 to both sides: $(x-3)^4 = 16$. Then, take the fourth root of both sides: $x-3 = \pm\sqrt{\sqrt{16}} = \pm 2$. Finally, solve for x: $x = 3 \pm 2$, resulting in $x = 5$ or $x = 1$.

What are the steps to solve (x-3)^4 - 5 = 11 for x?

Step 1: Add 5 to both sides to get $(x-3)^4 = 16$. Step 2: Take the fourth root of both sides, considering both positive and negative roots: $x-3 = \pm 2$. Step 3: Add 3 to both sides to find x: $x = 3 \pm 2$, so $x = 5$ or $x = 1$.

Are there any restrictions on x when solving (x-3)^4 - 5 = 11?

Since the equation involves a fourth power, which is defined for all real numbers, there are no restrictions on x. Both solutions $x=1$ and $x=5$ are valid.

How do I verify the solutions x=1 and x=5 for the equation?

Plug $x=1$ into the original equation: $(1-3)^4 - 5 = (-2)^4 - 5 = 16 - 5 = 11$, which is correct. For $x=5$: $(5-3)^4 - 5 = (2)^4 - 5 = 16 - 5 = 11$, also correct.

Can the solutions to (x-3)^4 - 5 = 11 be complex numbers?

Since taking the fourth root of 16 yields real solutions, the solutions $x=1$ and $x=5$ are real. There are no complex solutions in this case.

What is the significance of taking the ± root when solving (x-3)^4 = 16?

Taking the ± root accounts for both positive and negative solutions because raising both to the fourth power results in a positive number regardless of the sign. So, x-3 = ±2 captures both possibilities.

Is there an alternative method to solve (x-3)^4 - 5 = 11?

Yes, you could substitute y = x - 3, transforming the equation into y^4 = 16, then solve for y, and finally find x. This method simplifies the process by reducing the original expression.

What is the final answer to the equation (x-3)^4 - 5 = 11?

The solutions are x = 1 and x = 5.

Additional Resources

Solve Algebraically (x - 3)^4 - 5 = 11: A Comprehensive Step-by-Step Guide

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Introduction to the Problem

Algebraic equations are foundational in mathematics, providing a way to find unknown values based on known relationships. The problem (x - 3)^4 - 5 = 11 is a classic example of a polynomial equation that involves a quartic (degree 4) expression. Solving such equations algebraically requires systematic approaches, including substitution, isolating variables, and applying algebraic principles.

This guide aims to walk through the entire process of solving this equation thoroughly, highlighting key concepts and strategies along the way. Whether you're a student brushing up on algebra skills or someone interested in exploring more complex equations, this detailed analysis will serve as an insightful resource.

Understanding the Equation

Before diving into the solution, it’s essential to understand the structure of the equation:

$$
(x - 3)^4 - 5 = 11
$$

- The main variable is x.
- The expression involves a binomial $(x - 3)$ raised to the 4th power.
- There is a constant offset of -5 on the left side.
- The right side is a constant value, 11.

This is a polynomial equation of degree 4 due to the quartic power. The goal is to find all real (and possibly complex) solutions for x.

Step 1: Isolate the Power Term

The first step in solving the equation algebraically is to isolate the quartic expression:

$$
(x - 3)^4 - 5 = 11
$$

Add 5 to both sides:

$$
(x - 3)^4 = 11 + 5
$$

$$
(x - 3)^4 = 16
$$

This step simplifies the problem to finding the values of x such that the fourth power of $(x - 3)$ equals 16.

Step 2: Solve for the Inner Expression

To find x, we need to solve:

$$
(x - 3)^4 = 16
$$

Since the left side is a perfect power, this reduces to considering the 4th root of both sides:

$$
x - 3 = \pm \sqrt[4]{16}
$$

Note that:

$$
\sqrt[4]{16} = \pm \sqrt{\sqrt{16}} = \pm \sqrt{4} = \pm 2
$$

But this step requires caution because when solving equations involving even powers, we must consider both positive and negative roots.

Key point: Because the 4th power eliminates sign information, both positive and negative roots of 16 are valid solutions for the inner expression.

Thus, the solutions for $x - 3$ are:

$$
x - 3 = \pm 2
$$

which yields two separate cases:

1. $x - 3 = 2$
2. $x - 3 = -2$

Step 3: Find the Values of x

Now, solve each case:

- Case 1:

$$
x - 3 = 2
$$
$$
x = 2 + 3

\]
\[
x = 5
\]

- Case 2:

\[
x - 3 = -2
\]
\[
x = -2 + 3
\]
\[
x = 1
\]

These are the candidate solutions for the original equation.

Step 4: Verify the Solutions

Verification ensures that the solutions satisfy the original equation.

- For \(x = 5 \):

\[
(x - 3)^4 - 5 = 11
\]
\[
(5 - 3)^4 - 5 = 11
\]
\[
(2)^4 - 5 = 11
\]
\[
16 - 5 = 11
\]
\[
11 = 11 \quad \text{(True)}
\]

- For \(x = 1 \):

\[
(1 - 3)^4 - 5 = 11
\]
\[

$$
(-2)^4 - 5 = 11
$$
$$
16 - 5 = 11
$$
$$
11 = 11 \quad \text{(True)}
$$

Both solutions satisfy the original equation, confirming their correctness.

Additional Considerations: Complex Solutions

Since the equation involves an even power, it is also worth considering whether complex solutions exist. The equation:

$$
(x - 3)^4 = 16
$$

has four solutions in the complex plane because:

$$
x - 3 = \sqrt[4]{16} \times e^{i \frac{\pi k}{2}}
$$

for $k = 0, 1, 2, 3$, corresponding to the four 4th roots of 16.

The four roots are:

$$
x - 3 = 2 e^{i \frac{\pi k}{2}}
$$

which explicitly are:

- For $k=0$:

$$
x - 3 = 2 e^{i 0} = 2
$$
$$
x= 5
$$

- For $k=1$:

$$
x - 3 = 2 e^{i \frac{\pi}{2}} = 2 (i) = 2i
$$
$$
x= 3 + 2i
$$

- For $k=2$:

$$
x - 3 = 2 e^{i \pi} = 2 (-1) = -2
$$
$$
x= 1
$$

- For $k=3$:

$$
x - 3 = 2 e^{i \frac{3\pi}{2}} = 2 (-i) = -2i
$$
$$
x= 3 - 2i
$$

Thus, the complex solutions are:

$$
x = 5, \quad x=1, \quad x=3 + 2i, \quad x=3 - 2i
$$

In most algebraic contexts, only real solutions are considered unless complex solutions are explicitly sought.

Summary of the Solution Set

Solution Type	Solutions	Explanation
Real solutions	$x = 1, \quad x=5$	Derived from taking the 4th root of 16 and considering both positive and negative roots.
Complex solutions	$x = 3 + 2i, \quad x=3 - 2i$	Derived from complex roots of 16 in the complex plane.

Key Takeaways and Tips for Solving Similar Equations

1. Isolate the highest power term to simplify the problem.
2. Take roots carefully, considering both positive and negative roots for even powers.
3. Remember to verify solutions by substituting back into the original equation.
4. Be aware of complex solutions when dealing with polynomial equations, especially even powers.
5. Use De Moivre's theorem for roots of complex numbers if solving in the complex plane.

Common Pitfalls and How to Avoid Them

- Ignoring the multiple roots of even powers: Failing to consider both positive and negative roots can lead to incomplete solutions.
- Forgetting to verify solutions: Always substitute solutions back into the original to confirm correctness.
- Overlooking complex solutions: While real solutions are often sufficient, in advanced contexts, complex solutions provide a complete picture.
- Misapplication of roots: Remember that $\sqrt[4]{a}$ yields four roots in the complex plane, not just the positive real root.

Conclusion

The algebraic solution to $(x - 3)^4 - 5 = 11$ demonstrates the systematic approach necessary for solving polynomial equations involving higher powers. By isolating the power term, considering both positive and negative roots, and verifying solutions, one can confidently find all solutions in the real and complex domains.

This problem exemplifies how understanding the properties of exponents, roots, and complex numbers is crucial in algebra. Mastery of these concepts not only aids in solving similar equations but also builds a solid foundation for tackling more advanced mathematical problems.

Final Remarks

Whether you are preparing for exams, developing your problem-solving skills, or exploring the fascinating world of algebra, understanding the step-by-step process outlined above enhances your mathematical reasoning. Keep practicing with variations of such equations to deepen your comprehension and become proficient in algebraic manipulations.

Happy solving!

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