Solve Dy/dx = (1-x)(1-y)

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The differential equation *Dy/dx = (1 - x)(1 - y)* is a first-order nonlinear ordinary differential equation that appears frequently in mathematical modeling and various scientific fields. Solving this equation involves understanding the structure of the differential equation, recognizing its type, and applying appropriate methods such as separation of variables or substitution techniques. In this article, we will explore comprehensive strategies to solve *Dy/dx = (1 - x)(1 - y)*, analyze its solutions, and discuss its applications.

Understanding the Differential Equation Dy/dx = (1 - x)(1 - y)

Before diving into solution methods, it is essential to understand the form and properties of the differential equation.

Type and Nature of the Equation

The equation *Dy/dx = (1 - x)(1 - y)* is a first-order nonlinear ordinary differential equation because of the product of (1 - x) and (1 - y). Its structure suggests the potential for transformation into a more manageable form, often through substitution.

Rearrangement and Recognizing Patterns

Notice that the right-hand side is a product of two factors, each linear in x and y. This pattern hints at the possibility of applying separation of variables or a substitution that simplifies the multiplicative structure.

Methods to Solve Dy/dx = (1 - x)(1 - y)

There are several approaches to solving this differential equation. The most straightforward involves separation of variables, but other methods like substitution can also be effective.

Method 1: Separation of Variables

Separation of variables is feasible here because the right-hand side factors into functions of x and y separately.

1. Rewrite the equation:

   $$
   \frac{dy}{dx} = (1 - x)(1 - y)
   $$

2. Separate variables:

   $$
   \frac{dy}{1 - y} = (1 - x) dx
   $$

3. Integrate both sides:

$$
\int \frac{1}{1 - y} dy = \int (1 - x) dx
$$

4. Compute the integrals:

$$
- \ln|1 - y| = x - \frac{x^2}{2} + C
$$

where C is the constant of integration.

5. Express y explicitly:

$$
|1 - y| = e^{ - x + \frac{x^2}{2} - C }
$$

or equivalently,

$$
1 - y = \pm e^{ - x + \frac{x^2}{2} + K }
$$

where $K = - C$, which is an arbitrary constant.

Finally,

\[

y(x) = 1 \mp e^{ - x + \frac{x^2}{2} + K }

\]

This is the general solution to the differential equation obtained via separation of variables.

Note: The choice of the sign depends on initial conditions or specific boundary conditions.

Method 2: Substitution Approach

An alternative method involves substitution to simplify the equation.

Step 1: Substitution \(u = 1 - y \)

Rearranged, the differential equation becomes:

\[

\frac{dy}{dx} = (1 - x) u

\]

Since \(y = 1 - u \), then:

\[

\frac{dy}{dx} = - \frac{du}{dx}

\]

Hence,

\[

$$
- \frac{du}{dx} = (1 - x) u
$$

or,

$$
\frac{du}{dx} + (1 - x) u = 0
$$

This is a linear first-order differential equation in u.

Step 2: Solve the linear differential equation

The standard form:

$$
\frac{du}{dx} + P(x) u = 0
$$

where $P(x) = 1 - x$.

The integrating factor:

$$
\mu(x) = e^{\int P(x) dx} = e^{\int (1 - x) dx} = e^{x - \frac{x^2}{2}}
$$

Multiply both sides of the differential equation by $\mu(x)$:

$$
e^{x - \frac{x^2}{2}} \frac{du}{dx} + e^{x - \frac{x^2}{2}} (1 - x) u = 0
$$

\]

which simplifies to:

\[
\frac{d}{dx} \left(u e^{x - \frac{x^2}{2}} \right) = 0
\]

Integrate both sides:

\[
u e^{x - \frac{x^2}{2}} = C
\]

or,

\[
u = C e^{ - x + \frac{x^2}{2} }
\]

Recall \(u = 1 - y \), so:

\[
1 - y = C e^{ - x + \frac{x^2}{2} }
\]

and

\[
\boxed{
y(x) = 1 - C e^{ - x + \frac{x^2}{2} }

}

\]

This matches the solution obtained via separation of variables, confirming the consistency of methods.

Special Solutions and Initial Conditions

In many practical problems, initial conditions are provided to determine the particular solution.

Applying Initial Conditions

Suppose $y(x_0) = y_0$. Then, from the general solution:

$$
y_0 = 1 - C e^{ - x_0 + \frac{x_0^2}{2} }
$$

which gives:

$$
C = \left(1 - y_0 \right) e^{ x_0 - \frac{x_0^2}{2} }
$$

Substituting back, the particular solution is:

$$
y(x) = 1 - \left(1 - y_0 \right) e^{ x_0 - \frac{x_0^2}{2} } e^{ - x + \frac{x^2}{2} }
$$

Applications of Solving Dy/dx = (1 - x)(1 - y)

This differential equation appears in various contexts:

- **Population Dynamics:** Modeling populations with growth rates affected by environmental factors, where the change depends on current population and time.
- **Physics:** Describing systems where the rate of change depends on the difference between a parameter and a variable, such as cooling or heating processes.
- **Economics:** Modeling investment or consumption patterns where the rate depends on current and external factors represented by x and y.
- **Engineering:** Control systems where feedback depends on the state variables, leading to equations similar in structure.

Summary and Key Takeaways

- The differential equation *Dy/dx = (1 - x)(1 - y)* can be effectively solved using separation of variables or substitution methods.
- Applying substitution $u = 1 - y$ transforms the nonlinear equation into a linear first-order differential equation, simplifying the solution process.
- The general solution involves exponential functions and an arbitrary constant, which can be determined using initial conditions.
- Recognizing the structure of the differential equation allows for straightforward solution strategies, making it accessible for applications across scientific disciplines.
- Understanding these solution methods enhances problem-solving skills for similar nonlinear

differential equations.

Final Thoughts

Mastering the solution of *Dy/dx = (1 - x)(1 - y)* equips you with tools to tackle a broad class of nonlinear differential equations. Whether through separation of variables or substitution, these techniques are foundational in mathematical analysis, modeling, and engineering. As you practice solving such equations, you'll develop intuition for recognizing patterns and applying the most effective method, ultimately broadening your analytical capabilities.

Frequently Asked Questions

What type of differential equation is dy/dx = (1 - x)(1 - y)?

It is a first-order, nonlinear ordinary differential equation that can be solved using separation of variables.

How can I solve the differential equation dy/dx = (1 - x)(1 - y)?

Rearrange the equation to separate variables: dy/(1 - y) = (1 - x) dx, then integrate both sides to find the general solution.

What is the integral of 1/(1 - y) with respect to y?

The integral is -ln|1 - y| + C, where C is the constant of integration.

What is the integral of (1 - x) with respect to x?

The integral is x - x^2/2 + C, where C is the constant of integration.

What is the general solution to the differential equation dy/dx = (1 - x)(1 - y)?

The general solution is $(1 - y) e^{x - x^2/2} = C$, where C is an arbitrary constant.

How do initial conditions affect the solution of dy/dx = (1 - x)(1 - y)?

Initial conditions allow you to solve for the constant C in the general solution, giving a particular solution tailored to specific data points.

Can this differential equation be solved explicitly for y in terms of x?

Yes, by integrating and solving for y, you can express y explicitly as a function of x, typically involving exponential functions.

Are there any special solutions to dy/dx = (1 - x)(1 - y)?

Yes, setting y = 1 or x = 1 can lead to particular solutions, such as y = 1 being a constant solution when the derivative is zero.

What are common methods to verify the solution to dy/dx = (1 - x)(1 - y)?

Differentiate the proposed solution and check if it satisfies the original differential equation, or substitute back into the equation to verify correctness.

Additional Resources

Solve Dy/dx = (1 - x)(1 - y): A Comprehensive Guide to Solving the Differential Equation

When tackling differential equations, one of the most common types encountered in calculus and applied mathematics is the first-order nonlinear differential equation. Among these, the equation Dy/dx

= (1 - x)(1 - y) presents an intriguing case due to its structure and potential for various solution methods. Understanding how to solve Dy/dx = (1 - x)(1 - y) not only deepens one's grasp of differential equations but also enhances problem-solving skills applicable in physics, engineering, and other scientific disciplines.

In this guide, we will explore the step-by-step approach to solving this differential equation, emphasizing substitution methods, integrating factors, and the importance of recognizing the structure of the equation. By the end, you'll have a clear, comprehensive understanding of how to handle similar equations and the underlying principles that guide their solutions.

Understanding the Differential Equation

The given differential equation is:

Dy/dx = (1 - x)(1 - y)

This is a first-order ordinary differential equation (ODE). Notice the right-hand side involves the product of two linear expressions, (1 - x) and (1 - y), making it nonlinear in y. Recognizing the structure is crucial because it suggests the possibility of transforming the equation into a separable form or using substitution techniques.

Step 1: Recognize the Type of Equation

Before diving into solution methods, identify the nature of the equation:

- Is it separable?
- Is it linear?

- Does it fit a known form like homogeneous or Bernoulli?

Let's analyze:

- The equation Dy/dx = (1 - x)(1 - y) involves the product of functions in x and y.
- If we can write it in a form where variables are separated, that would be ideal.

Step 2: Attempting to Make the Equation Separable

The equation:

$dy/dx = (1 - x)(1 - y)$

can be rewritten as:

$dy/dx = (1 - x)(1 - y)$

To separate variables, think about expressing all y terms with dy and all x terms with dx:

$dy / (1 - y) = (1 - x)\, dx$

This is a promising step because the variables are now separated, enabling direct integration.

Step 3: Separating Variables

Given:

$$dy / (1 - y) = (1 - x)\, dx$$

This transformation is valid because $dy/dx = (1 - x)(1 - y)$ implies:

$$dy / (1 - y) = (1 - x)\, dx$$

Now, both sides are ready for integration.

Step 4: Integrate Both Sides

Left Side:

$$\int [1 / (1 - y)]\, dy$$

Let’s compute:

- The integral of $1 / (1 - y)$ with respect to y:

$$\int 1 / (1 - y)\, dy = -\ln|1 - y| + C$$

Right Side:

$$\int (1 - x)\, dx$$

Compute:

$$\int (1 - x)\, dx = \int 1\, dx - \int x\, dx = x - (1/2) x^2 + C$$

Step 5: Write Down the General Solution

Putting it all together:

$$-\ln|1 - y| = x - (1/2) x^2 + C$$

Where C is an arbitrary constant. Solving for y:

$$\ln|1 - y| = -x + (1/2) x^2 - C$$

Exponentiating both sides:

$$|1 - y| = e^{-x + (1/2) x^2 - C}$$

Since e^{-C} is just another constant, denote it as $K > 0$:

$$|1 - y| = K e^{-x + (1/2) x^2}$$

Finally, express y explicitly:

$$1 - y = \pm K e^{-x + (1/2) x^2}$$

or

$$y = 1 \mp K e^{-x + (1/2) x^2}$$

where K is an arbitrary positive constant that can be adjusted based on initial conditions.

Summary of the General Solution

The general solution to the differential equation $Dy/dx = (1 - x)(1 - y)$ is:

$$y(x) = 1 \pm C e^{-x + (1/2) x^2}$$

with C being an arbitrary real constant.

Additional Insights and Special Cases

1. Initial Conditions and Particular Solutions

If an initial condition, say $y(x_0) = y_0$, is provided, you can determine C:

$$C = \pm (1 - y_0) e^{x_0 - (1/2) x_0^2}$$

2. Behavior of Solutions

- As x □ □, the exponential term may tend to zero or grow, depending on the sign and the value of C.
- The solutions describe a family of curves, each corresponding to different initial conditions.

3. Invariant Solutions

Check if constant solutions exist by setting $dy/dx = 0$:

$$(1 - x)(1 - y) = 0$$

which yields:

- $1 - x = 0$ □ $x = 1$
- or $1 - y = 0$ □ $y = 1$

Thus, $x = 1$ and $y = 1$ are equilibrium solutions or invariant solutions, indicating specific lines where the solutions are constant.

Alternative Methods and Considerations

While the separation of variables method works neatly here, it's instructive to explore other potential solution techniques:

Method 1: Substitution for a Bernoulli or Homogeneous Equation

The original differential equation can be viewed as separable; however, recognizing it as a different type might lead to other solution paths:

- For instance, if we set $u = 1 - y$, then the differential equation becomes:

$$dy/dx = (1 - x) u$$

but since $dy/dx = - du/dx$, then:

$$- du/dx = (1 - x) u$$

which simplifies to:

$$du/dx + (1 - x) u = 0$$

a linear first-order ODE in u that can be solved using integrating factors.

Method 2: Solving Using Integrating Factors

The linear form:

$$du/dx + (1 - x) u = 0$$

has integrating factor:

$$\mu(x) = e^{\int (1 - x) dx} = e^{x - (1/2) x^2}$$

Multiplying through:

$$\mu(x) du/dx + \mu(x) (1 - x) u = 0$$

which simplifies to:

$$d/dx [\mu(x) u] = 0$$

Integrating:

$$\mu(x) u = \text{constant}$$

and solving for u:

$$u = C / \mu(x) = C e^{-(x - (1/2) x^2)}$$

Recall that $u = 1 - y$, so:

$$1 - y = C e^{-(x - (1/2) x^2)}$$

which matches the previous solution form.

Final Remarks and Practical Applications

The differential equation Dy/dx = (1 - x)(1 - y) exemplifies how recognizing the structure of an equation enables an effective solution approach, often involving substitution and separation of variables. This problem demonstrates the importance of transforming the equation into a manageable form, applying direct integration, and understanding the role of arbitrary constants.

Applications

- Physics: Modeling processes where rates depend on linear combinations of variables.
- Engineering: Analyzing systems with feedback mechanisms modeled by nonlinear ODEs.
- Biology: Describing population dynamics with nonlinear interaction terms.

Summary Checklist for Solving Similar Equations

- Identify the structure: Is it separable, linear, homogeneous, Bernoulli?
- Attempt to separate variables if possible.
- Use substitution when the equation is nonlinear but can be transformed into a linear form.
- Integrate both sides carefully, noting constants.
- Express the solution explicitly and consider initial conditions for particular solutions.
- Analyze behavior for different values of constants and initial conditions.

This comprehensive guide provides a detailed pathway to solving Dy/dx = (1 - x)(1 - y), illustrating key techniques in differential equations and emphasizing the importance of recognizing structure and choosing appropriate solution methods. With this knowledge, tackling a broad class of nonlinear differential equations becomes more approachable and systematic.

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