Solve For X. (x+6)(-x+1)=0

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Solving equations where factors are multiplied together and set equal to zero is a fundamental skill in algebra. The given equation, $(x + 6)(-x + 1) = 0$, is a product of two binomials, and our goal is to find all values of x that satisfy this equation. The key principle here is the Zero Product Property, which states that if the product of two factors is zero, then at least one of the factors must be zero. This simplifies the process, allowing us to solve each factor separately and then combine the solutions.

In this article, we will explore the step-by-step process to solve this particular equation, analyze the solutions, and understand the underlying algebraic principles. We will also look into how to verify solutions and discuss the importance of solving such equations in broader mathematical contexts.

Understanding the Zero Product Property

Definition and Importance

The Zero Product Property is a cornerstone of algebra. It states:

- If $a \times b = 0$, then $a = 0$ or $b = 0$.

This property allows us to break down complex equations involving factors into simpler, solvable parts. It's applicable whenever the product of two expressions equals zero, regardless of the nature of those expressions.

Application to Binomials

When dealing with binomials or other polynomial factors, the property simplifies solving equations by isolating each factor:

- Set each factor equal to zero.
- Solve each resulting linear or polynomial equation separately.
- Combine all solutions.

In our specific problem, this approach will enable us to find all possible x-values that satisfy the original equation.

Step-by-Step Solution of (x + 6)(-x + 1) = 0

Step 1: Recognize the structure of the equation

The equation $(x + 6)(-x + 1) = 0$ is a factored form of a quadratic expression, which means the solutions can be found by applying the Zero Product Property directly.

Step 2: Set each factor equal to zero

- First factor: $x + 6 = 0$
- Second factor: $-x + 1 = 0$

By solving these two simple equations, we will find the values of x that satisfy the original equation.

Step 3: Solve each equation separately

Equation 1: $x + 6 = 0$

- Subtract 6 from both sides:

$x = -6$

Equation 2: $-x + 1 = 0$

- Subtract 1 from both sides:

$-x = -1$

- Multiply both sides by -1 to solve for x:

$x = 1$

Step 4: List all solutions

The solutions are:

- $x = -6$
- $x = 1$

These are the x-values that make the original equation true.

Verification of Solutions

Why verify solutions?

While the algebraic steps are straightforward, verifying solutions ensures that no mistakes were made and that the solutions indeed satisfy the original equation.

Verification process

Verify $x = -6$:

- Substitute into the original equation:

$(x + 6)(-x + 1) = 0$

$(-6 + 6)(-(-6) + 1) = (0)(6 + 1) = 0 \times 7 = 0$

- The result is 0, confirming $x = -6$ is a solution.

Verify $x = 1$:

- Substitute into the original equation:

$(1 + 6)(-1 + 1) = (7)(0) = 0$

- The result is 0, confirming $x = 1$ is a solution.

Thus, both solutions are valid.

Graphical Interpretation of the Equation

Plotting the factors as functions

The original equation involves two factors:

- $f(x) = x + 6$
- $g(x) = -x + 1$

The product is zero where either $f(x) = 0$ or $g(x) = 0$, which corresponds to the x-intercepts of the graphs of these functions.

Visualizing solutions

- The line $y = x + 6$ crosses the x-axis at $x = -6$.
- The line $y = -x + 1$ crosses the x-axis at $x = 1$.

The solutions we found correspond to these points, confirming the algebraic solution's correctness.

Broader Implications and Applications

Real-world applications of solving equations like $(x + 6)(-x + 1) = 0$

Equations of this form appear in various fields:

- Physics: To find points where certain conditions are met, like equilibrium points.
- Engineering: For stress analysis where multiple factors contribute to failure thresholds.
- Economics: To determine break-even points where profit functions equal zero.

Extending to higher-degree polynomials

The principle used here extends to solving more complex equations:

- Polynomial equations of degree higher than two are factored into simpler parts.
- The Zero Product Property applies recursively or through factorization techniques.
- Numerical methods may be employed when algebraic solutions are complex.

Summary and Key Takeaways

- The equation $(x + 6)(-x + 1) = 0$ relies on the Zero Product Property.
- Solving involves setting each factor equal to zero and solving individually.
- The solutions $x = -6$ and $x = 1$ satisfy the original equation upon verification.
- Graphically, solutions correspond to the x-intercepts of the factors.
- Understanding these concepts aids in solving more complex algebraic equations and applying them in real-world contexts.

Final thoughts

Mastering the process of solving factored equations like $(x + 6)(-x + 1) = 0$ is essential for progressing in algebra and higher mathematics. Recognizing the structure of equations, applying fundamental properties, and verifying solutions form the basis of robust problem-solving skills. Whether in academics, engineering, economics, or sciences, these techniques are invaluable tools for analyzing and understanding complex relationships.

Frequently Asked Questions

What are the solutions to the equation (x+6)(-x+1)=0?

The solutions are x = -6 and x = 1.

How do you solve the equation (x+6)(-x+1)=0 for x?

Set each factor equal to zero: x + 6 = 0 and -x + 1 = 0, then solve for x to find the solutions.

Why do we use the zero product property to solve (x+6)(-x+1)=0?

Because if the product of two factors is zero, then at least one factor must be zero, which allows us to find the solutions for x.

What is the step-by-step process to solve (x+6)(-x+1)=0?

1. Set each factor equal to zero: x + 6 = 0 and -x + 1 = 0. 2. Solve each equation: x = -6 and x = 1. 3. The solutions are x = -6 and x = 1.

Can the solutions to (x+6)(-x+1)=0 be complex numbers?

No, in this case, the solutions are real numbers: x = -6 and x = 1, since the equations are linear.

Additional Resources

Solve For X. (x+6)(-x+1)=0: An Investigative Analysis of a Fundamental Algebraic Equation

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Algebra, often regarded as the gateway to higher mathematics, introduces students and enthusiasts alike to the essential skill of solving equations. Among these, quadratic and linear equations hold a special place due to their universality and foundational importance. One such fundamental equation is:

Solve For X. (x+6)(-x+1)=0

While on the surface this appears straightforward, a deeper exploration reveals layers of algebraic principles, solution strategies, and the importance of understanding the structure of equations. This investigative article aims to dissect this equation comprehensively, exploring its origins, solution methods, implications, and pedagogical significance.

Understanding the Equation: An Initial Overview

The equation in question is a factored form:

\[
(x + 6)(-x + 1) = 0
\]

This form immediately suggests the application of the Zero Product Property, which states that if the product of two factors is zero, then at least one of the factors must be zero. This property is a cornerstone in solving polynomial equations and is frequently used in algebra courses.

Key points to consider:

- The equation is a product of two linear factors.
- The solutions are the roots where each factor equals zero.
- The factors are linear and involve constants and the variable \(x\).

Applying the Zero Product Property: Step-by-Step Solution

The most direct approach to solving this equation involves setting each factor equal to zero:

\[
\begin{cases}
x + 6 = 0 \\
- x + 1 = 0
\end{cases}
\]

Solution for each factor:

1. First factor: $x + 6 = 0$

$$
x = -6
$$

2. Second factor: $-x + 1 = 0$

$$
-x = -1 \Rightarrow x = 1
$$

Solutions:

$$
x = -6, \quad x = 1
$$

These are the roots of the quadratic expression, indicating the points where the original product equals zero.

Deeper Analysis: Structural Insights and Alternative Methods

While the zero product property provides a straightforward solution, it's instructive to verify and understand the roots further by expanding the original expression, analyzing its structure, and exploring alternative solution methods.

Expanding the Expression

Expanding $(x + 6)(-x + 1)$:

$$
x \times (-x + 1) + 6 \times (-x + 1) = -x^2 + x - 6x + 6 = -x^2 - 5x + 6
$$

Rearranged in standard quadratic form:

$$
- x^2 - 5x + 6 = 0
$$

Multiplying through by (-1) to simplify:

$$
x^2 + 5x - 6 = 0
$$

This quadratic can now be solved using factoring, quadratic formula, or completing the square.

Factoring the Quadratic

Find two numbers that multiply to -6 and add to 5:

$$
\text{Factors of } -6: (1, -6), (-1, 6), (2, -3), (-2, 3)
$$

Sum analysis:

- $1 + (-6) = -5$
- $-1 + 6 = 5$ ← matches the middle term
- $2 + (-3) = -1$
- $-2 + 3 = 1$

Thus, the factors are $(x - 1)(x + 6)$. So,

$$
x^2 + 5x - 6 = (x - 1)(x + 6)
$$

Setting each factor to zero yields:

$$
x - 1 = 0 \Rightarrow x = 1
$$
$$
x + 6 = 0 \Rightarrow x = -6
$$

This confirms the earlier solutions, illustrating the consistency of algebraic methods.

Quadratic Formula Approach

Alternatively, applying the quadratic formula:

$$
x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}
$$

where $a=1$, $b=5$, $c=-6$:

$$
x = \frac{-5 \pm \sqrt{25 - 4 \times 1 \times (-6)}}{2} = \frac{-5 \pm \sqrt{25 + 24}}{2} = \frac{-5 \pm \sqrt{49}}{2}
$$

$$
x = \frac{-5 \pm 7}{2}
$$

Solutions:

- $x = \frac{-5 + 7}{2} = \frac{2}{2} = 1$
- $x = \frac{-5 - 7}{2} = \frac{-12}{2} = -6$

Again, the solutions are consistent, demonstrating the robustness of algebraic solution methods.

Graphical Interpretation and Significance

Understanding solutions graphically enhances comprehension:

- The original expression corresponds to a parabola opening downward (since leading coefficient is negative).
- The roots -6 and 1 are the x-intercepts where the parabola crosses the x-axis.
- Visualizing this provides insight into the nature of quadratic roots and their relationship to the factors.

Plotting the quadratic $-x^2 - 5x + 6$ or the factored form confirms the roots visually, reinforcing the algebraic solution's validity.

Implications and Broader Context

While this specific equation appears simple, it exemplifies core principles of algebra that underpin advanced mathematical concepts:

- Factorization Techniques: Recognizing when and how to factor quadratic expressions is essential.
- Zero Product Property: A fundamental rule enabling straightforward solutions to polynomial equations.
- Equation Transformation: Moving between factored, expanded, and quadratic forms

deepens understanding.
- Solution Verification: Cross-checking solutions via different methods ensures accuracy.

In educational contexts, this exercise demonstrates how multiple pathways lead to the same solution, fostering critical thinking and problem-solving skills.

Common Pitfalls and Misconceptions

Despite the simplicity, learners should be aware of potential errors:

- Ignoring extraneous solutions: Not applicable here, but important in more complex equations involving roots.
- Incorrect factorization: Misidentifying factors can lead to wrong solutions.
- Sign errors: Careful handling of negatives, especially in factors like $-x + 1$, is crucial.
- Overlooking multiple solutions: Failing to consider all roots can result in incomplete solutions.

Awareness of these pitfalls ensures rigorous problem-solving and reinforces good mathematical habits.

Conclusion: The Value of Investigating a Simple Equation

The exploration of Solve For X. (x+6)(-x+1)=0 reveals that even seemingly straightforward algebraic equations encompass rich mathematical structures and principles. From the zero product property to quadratic expansion and graphing, each step enhances understanding and appreciation of algebra's elegance and power.

Moreover, this analysis underscores the importance of multiple solution methods, verification techniques, and conceptual visualization, skills that serve students and practitioners across mathematical disciplines. As a foundational example, this equation exemplifies the core strategies that underpin more complex algebraic problem-solving and highlights the enduring relevance of fundamental principles in mathematics education and research.

In summary, solving for x in $(x+6)(-x+1)=0$ involves recognizing the factored form, applying the zero product property, and verifying solutions through expansion or quadratic methods. The solutions $x = -6$ and $x = 1$ are consistent across different approaches, illustrating the interconnected nature of algebraic techniques. This investigation not only clarifies the solution process but also reinforces essential mathematical concepts applicable across various levels of education and research.

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