Solve: Log2(x-1)+log2(x+5)=4

Solve: Log2(x-1)+log2(x+5)=4 is a common logarithmic equation that students and math enthusiasts often encounter. Understanding how to approach and solve such equations is fundamental in algebra and helps build a strong foundation for more advanced mathematical concepts. In this article, we will explore the step-by-step process of solving the equation, clarify the key concepts involved, and provide tips to master similar problems.

Understanding the Equation

Before diving into the solution, it is important to understand the structure of the equation:

Logarithmic equations involving sums of logs are often simplified using properties of logarithms. The given equation:

Log2(x-1) + log2(x+5) = 4

involves two logarithmic expressions with the same base, which suggests that we can combine them into a single logarithm.

Key Concepts in Solving Logarithmic Equations

1. Logarithm Properties

To effectively solve the equation, recall the key properties of logarithms:

- **Product Property:** log_b(M) + log_b(N) = log_b(M N)
- **Change of Base:** log_b(a) = ln(a) / ln(b), useful for conversions but not necessary here since base 2 is consistent
- **Exponential Form:** If log_b(x) = y, then x = b^y

In our case, the sum of logs can be combined into a single logarithm:

log2(x-1) + log2(x+5) = log2[(x-1)(x+5)]

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2. Domain Restrictions

Logarithmic functions are only defined for positive arguments:

- $x - 1 > 0 \rightarrow x > 1$
- $x + 5 > 0 \rightarrow x > -5$

Since the more restrictive condition is $x > 1$, the domain of the solution must satisfy $x > 1$.

Step-by-Step Solution

Step 1: Combine the Logarithms

Utilize the product property of logs:

$\log_2(x-1) + \log_2(x+5) = \log_2[(x-1)(x+5)]$

So, the original equation becomes:

$\log_2[(x-1)(x+5)] = 4$

Step 2: Convert Logarithmic Equation to Exponential Form

Recall that if $\log_b(A) = C$, then $A = b^C$. Applying this:

$(x-1)(x+5) = 2^4$

Calculate 2^4:

$(x-1)(x+5) = 16$

Step 3: Expand and Form a Quadratic Equation

Expand the left side:

$x(x+5) - 1(x+5) = 16$
$x^2 + 5x - x - 5 = 16$
$x^2 + 4x - 5 = 16$

Bring all to one side to set the quadratic to zero:

$x^2 + 4x - 21 = 0$

Solving the Quadratic Equation

Step 4: Use the Quadratic Formula

The quadratic formula:

$x = [-b \pm \sqrt{(b^2 - 4ac)}] / 2a$

For the quadratic $x^2 + 4x - 21 = 0$:

$a = 1, b = 4, c = -21$

Calculate the discriminant:

$D = b^2 - 4ac = 4^2 - 4(1)(-21) = 16 + 84 = 100$

Find the roots:

$x = [-4 \pm \sqrt{100}] / 2 = [-4 \pm 10] / 2$

Calculate both solutions:

- $x = (-4 + 10) / 2 = 6 / 2 = 3$
- $x = (-4 - 10) / 2 = -14 / 2 = -7$

Checking the Domain Restrictions

Recall that the original Logarithmic functions require $x > 1$.

- For $x = 3$: Since $3 > 1$, it is within the domain.

- For x = -7: Since -7 is not > 1, it is outside the domain.

Therefore, the only valid solution is:

x = 3

Final Answer and Verification

To verify, substitute x = 3 back into the original equation:

Log2(3-1) + log2(3+5) = ?

Calculate:

log2(2) + log2(8) = ?

Recall that:

log2(2) = 1 (since 2^1 = 2)

log2(8) = 3 (since 2^3 = 8)

Sum:

1 + 3 = 4

which matches the right side of the original equation.

Thus, x = 3 is the correct solution.

Summary of the Solution Process

1. Combine the logs using the product property.
2. Convert the logarithmic equation into an exponential form.
3. Form a quadratic equation and solve using the quadratic formula.
4. Check the solutions against the domain restrictions.
5. Verify the solutions by substitution.

Additional Tips for Solving Logarithmic Equations

- Always analyze the domain restrictions before solving.
- Use logarithmic properties to simplify expressions.
- Convert logarithmic equations to exponential form for easier solving.
- Check all solutions in the original equation to avoid extraneous roots.
- Practice with different bases and more complex equations to strengthen your skills.

Conclusion

The process of solving log2(x-1) + log2(x+5) = 4 illustrates fundamental principles of logarithms and algebra. By combining logs, converting to exponential form, solving the resulting quadratic, and verifying solutions, you can confidently tackle similar equations. Remember to always consider the domain restrictions imposed by the logarithmic functions to ensure your solutions are valid.

Mastering such techniques not only enhances your problem-solving skills but also prepares you for more advanced topics in mathematics, such as exponential functions, logarithmic inequalities, and calculus. Practice regularly with different types of logarithmic equations to become proficient and confident in solving them efficiently.

Frequently Asked Questions

How do I solve the equation log2(x-1) + log2(x+5) = 4?

Use the logarithm property: log2(a) + log2(b) = log2(ab). So, log2[(x-1)(x+5)] = 4. Then, rewrite as (x-1)(x+5) = 2^4 = 16, and solve the quadratic equation.

What is the first step in solving log2(x-1) + log2(x+5) = 4?

Combine the two logarithms into a single logarithm: log2[(x-1)(x+5)] = 4.

How do I solve for x after combining the logarithms in the equation?

Rewrite the equation as (x-1)(x+5) = 16, then expand and solve the resulting quadratic equation.

What quadratic equation do I get after expanding (x-1)(x+5) = 16?

Expanding gives x^2 + 5x - x - 5 = 16, which simplifies to x^2 + 4x - 5 = 16. Subtract 16 from both sides to get x^2 + 4x - 21 = 0.

How do I solve the quadratic equation x^2 + 4x - 21 = 0?

Use the quadratic formula: x = [-b ± sqrt(b^2 - 4ac)] / 2a, where a=1, b=4, c=-21. Calculate the discriminant and find the two solutions.

What are the solutions for x after solving the quadratic equation?

Calculate discriminant: 4^2 - 41(-21) = 16 + 84 = 100. So, x = [-4 ± sqrt(100)] / 2 = [-4 ± 10] / 2. Therefore, x = (6/2) = 3 and x = (-14/2) = -7.

Are both solutions x=3 and x=-7 valid in the original logarithmic equation?

No. Since the arguments of the logarithms must be positive, check x-1 > 0 and x+5 > 0. For x=3, both are positive; for x=-7, x-1 = -8 (negative), so x=-7 is invalid.

What is the final answer to the equation log2(x-1) + log2(x+5) = 4?

The only valid solution is x=3.

Can the solution x=3 be verified directly in the original equation?

Yes. Substitute x=3: log2(3-1) + log2(3+5) = log2(2) + log2(8) = 1 + 3 = 4. It satisfies the equation.

What is the key takeaway when solving logarithmic equations like this?

Combine logarithms first, convert to algebraic equations, solve the resulting quadratic, and always check the solutions to ensure they satisfy the original domain restrictions.

Additional Resources

Solve: Log2(x-1)+log2(x+5)=4

Understanding the equation $\log_2(x-1) + \log_2(x+5) = 4$ offers a valuable glimpse into the fundamental principles of logarithmic functions and their applications within algebra. This problem encapsulates core concepts such as the properties of logarithms, domain restrictions, and algebraic transformations, serving as an excellent example for both students and enthusiasts seeking to deepen their comprehension of logarithmic equations. In this comprehensive review, we will dissect the problem step-by-step, elucidate the underlying mathematical principles, explore solution strategies, and analyze the implications of various solutions within the context of logarithmic functions.

Understanding the Logarithmic Equation

What is a logarithm?

At its core, a logarithm is the inverse operation of exponentiation. For a positive real number $a \neq 1$, the logarithm $\log_a x$ answers the question: "To what power must a be raised to obtain x?" Mathematically, this is expressed as:

$$
a^y = x \quad \Longleftrightarrow \quad y = \log_a x
$$

In our specific case, the base is 2, which simplifies many calculations, especially considering binary systems and computational contexts.

The structure of the given equation

The equation:

$$
\log_2(x-1) + \log_2(x+5) = 4
$$

involves a sum of two logarithmic expressions. The properties of logarithms allow us to combine these into a single logarithmic term, provided certain conditions are met, which streamlines the solving process.

Key Properties of Logarithms Utilized

Logarithm product rule

One of the fundamental properties of logarithms is the product rule:

$$
\log_a M + \log_a N = \log_a (MN)
$$

This rule states that the sum of two logarithms with the same base equals the logarithm of the product of their arguments.

Domain considerations

Before applying the properties, it's essential to recognize the domain restrictions:

- The arguments of the logarithms must be positive:

$$
x - 1 > 0 \quad \Rightarrow \quad x > 1
$$
$$
x + 5 > 0 \quad \Rightarrow \quad x > -5
$$

Since $x > 1$ is stricter than $x > -5$, the overall domain for the solution is:

$$
x > 1
$$

This domain restriction ensures that the logarithms are defined within the real numbers.

Step-by-Step Solution Strategy

Step 1: Combine the logarithmic terms

Applying the product rule:

$$
\log_2(x-1) + \log_2(x+5) = \log_2[(x-1)(x+5)]
$$

The equation simplifies to:

$$
\log_2[(x-1)(x+5)] = 4
$$

Step 2: Rewrite the equation in exponential form

Using the definition of the logarithm:

$$
\log_b y = c \quad \Longleftrightarrow \quad y = b^c
$$

we get:

$$
(x-1)(x+5) = 2^4
$$

Since $2^4 = 16$:

$$
(x-1)(x+5) = 16
$$

Step 3: Expand the quadratic expression

Multiplying out:

$$
x(x+5) - 1(x+5) = 16
$$

$$
x^2 + 5x - x - 5 = 16

\]

$$
x^2 + 4x - 5 = 16
$$

Step 4: Formulate a quadratic equation

Bring all terms to one side:

$$
x^2 + 4x - 5 - 16 = 0
$$

$$
x^2 + 4x - 21 = 0
$$

Step 5: Solve the quadratic equation

Apply the quadratic formula:

$$
x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}
$$

where $a=1$, $b=4$, and $c=-21$:

$$
x = \frac{-4 \pm \sqrt{(4)^2 - 4 \times 1 \times (-21)}}{2}
$$

$$
x = \frac{-4 \pm \sqrt{16 + 84}}{2}
$$

$$
x = \frac{-4 \pm \sqrt{100}}{2}
$$

$$
x = \frac{-4 \pm 10}{2}
$$

Thus, the two solutions are:

$$
x = \frac{-4 + 10}{2} = \frac{6}{2} = 3
$$

$$
x = \frac{-4 - 10}{2} = \frac{-14}{2} = -7
$$

Verification of Solutions within the Domain

Recall that the domain for x is $x > 1$. Therefore:

- The solution $x=3$ is within the domain.
- The solution $x=-7$ is outside the domain because it is less than 1.

Next, verify the validity of the solutions by substituting back into the original equation.

Verification of $x=3$:

$$
\log_2(3-1) + \log_2(3+5) = \log_2(2) + \log_2(8) = 1 + 3 = 4
$$

which matches the right side, confirming that $x=3$ is a valid solution.

Verification of $x=-7$:

$$
\log_2(-7-1) + \log_2(-7+5) = \log_2(-8) + \log_2(-2)
$$

Since the arguments are negative, the logarithms are undefined within the real numbers, so $x=-7$ is invalid.

Final Answer:

$$
\boxed{x=3}
$$

Implications and Broader Context

Significance of the solution

The solution process exemplifies how logarithmic properties facilitate solving complex equations. The ability to reduce a sum of logs to a single log of a product simplifies algebraic manipulations. Furthermore, recognizing domain restrictions is crucial in ensuring the solutions are meaningful within the context of the problem.

Real-world applications

Logarithmic equations like this are prevalent in fields such as engineering, computer science, and finance. For example:

- Signal processing: Logarithmic scales measure sound intensity (decibels).
- Data analysis: Logarithms help manage exponential growth or decay.
- Information theory: Logarithms quantify information content and entropy.

Understanding how to solve these equations enhances analytical skills applicable across numerous disciplines.

Common pitfalls and how to avoid them

- Ignoring domain restrictions: Always verify solutions within the original domain to avoid extraneous solutions.
- Misapplication of logarithm properties: Ensure the properties are applied correctly, especially when dealing with sums or differences.
- Overlooking multiple solutions: Quadratic equations often produce extraneous solutions; verify each.

Conclusion

The problem $\log_2(x-1) + \log_2(x+5) = 4$ serves as a quintessential example of logarithmic algebra. By leveraging properties such as the product rule and understanding the importance of domain restrictions, we found that the only valid solution is $x=3$. This solution not only satisfies the original equation but also aligns with the constraints imposed by the properties of logarithms.

This exercise underscores the elegance and utility of logarithms in solving equations involving exponential relationships. Mastery of these techniques opens doors to tackling more advanced mathematical challenges and applying them effectively in real-world scenarios, from data analysis to engineering. As with many algebraic problems, a methodical approach—combining properties, algebraic manipulation, and verification—ensures accurate and insightful solutions.

References

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- Larson, R., & Edwards, B. H. (2017). Precalculus with Limits. Cengage Learning.
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Note: Always consider the context and domain constraints when solving logarithmic equations to avoid extraneous solutions and ensure meaningful results.

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