-x+y=-3 and 6x-9y=30

-x + y = -3 and 6x - 9y = 30 are two linear equations that form a system of equations. Understanding how to solve such systems is fundamental in algebra, as it allows us to find the values of variables that satisfy multiple conditions simultaneously. Whether you're a student seeking to improve your math skills or someone interested in practical applications, mastering the methods to solve these equations is essential. In this article, we will explore the nature of these equations, various solving techniques, step-by-step examples, and real-world applications.

Understanding the System of Equations

A system of equations involves two or more equations with the same set of variables. The goal is to find the point(s) where these equations intersect, which corresponds to the solution(s) of the system.

What Are Linear Equations?

Linear equations are algebraic expressions where the highest power of the variable is one. They graph as straight lines on a coordinate plane.

For example:

- -x + y = -3
- 6x - 9y = 30

These are both linear because each variable is to the first power, and no products of variables or exponents are involved.

Representing the System Graphically

Graphing the equations helps visualize their solution:

- Each line represents one equation.
- The point where the lines intersect is the solution to the system.

Understanding the geometric interpretation is vital because it provides insight into the solutions:

- Unique solution: The lines intersect at exactly one point.
- No solution: The lines are parallel and do not intersect.
- Infinite solutions: The lines coincide (are the same line).

Methods to Solve the System of Equations

There are multiple methods to solve systems of linear equations:

1. Substitution Method
2. Elimination Method (Addition or Subtraction Method)
3. Graphical Method
4. Matrix Method (using determinants or matrices)

In this article, we will focus mainly on substitution and elimination, as they are most straightforward for a system like this.

Substitution Method

This involves solving one equation for one variable and substituting into the other.

Elimination Method

This involves adding or subtracting equations to eliminate one variable, making it easier to solve for the remaining variable.

Step-by-Step Solution of the System

Let's now solve the system:

- $-x + y = -3$ ...(1)
- $6x - 9y = 30$ ...(2)

Step 1: Simplify the equations if possible

Equation (2) can be simplified by dividing all terms by 3:

$6x - 9y = 30$
□ $(6x / 3) - (9y / 3) = 30 / 3$
□ $2x - 3y = 10$ ...(3)

Now, our simplified system is:

- $-x + y = -3$ ...(1)
- $2x - 3y = 10$ ...(3)

Step 2: Solve one equation for one variable

From equation (1):

$-x + y = -3$
□ $y = x - 3$

Step 3: Substitute into the other equation

Substitute $y = x - 3$ into equation (3):

$2x - 3(x - 3) = 10$

Expand:

$2x - 3x + 9 = 10$

Combine like terms:

$- x + 9 = 10$

Solve for x:

$-x = 10 - 9$

$-x = 1$

$x = -1$

Step 4: Find y

Using $y = x - 3$:

$y = -1 - 3 = -4$

Step 5: Write the solution

The solution to the system is:

$x = -1, y = -4$

This point (-1, -4) is where the two lines intersect on the coordinate plane.

Verifying the Solution

Always verify your solution by substituting the values back into original equations:

- Equation (1):

$-x + y = -3$

$-(-1) + (-4) = 1 - 4 = -3$ ✓

- Equation (2):

$6x - 9y = 30$

$6(-1) - 9(-4) = -6 + 36 = 30$ ✓

Since both equations are satisfied, the solution is correct.

Graphical Representation

Plotting the two lines:

- For $-x + y = -3$, rewrite as $y = x - 3$
- For $6x - 9y = 30$ (or $2x - 3y = 10$), as $y = (2/3)x - (10/3)$

The point of intersection (-1, -4) confirms the solution graphically.

Applications of Solving Linear Systems

Understanding systems of equations extends beyond mathematics into various fields:

- Physics: Calculating forces, velocities, or trajectories where multiple conditions are involved.
- Economics: Finding equilibrium points in supply and demand models.
- Engineering: Circuit analysis involving multiple voltage and current equations.
- Computer Graphics: Calculating intersections of lines and shapes.

Additional Techniques and Tips

- Use substitution when one equation is easily solved for a variable.
- Use elimination when the coefficients of a variable are opposites or easily made to cancel out.
- Always simplify equations before solving.
- Verify solutions to avoid mistakes.
- Graphically, solutions are points where lines intersect; this is especially useful for visual learners.

Practice Problems for Mastery

To reinforce your understanding, try solving these systems:

1. $3x + 2y = 12$ and $x - y = 3$
2. $4x - y = 7$ and $2x + y = 8$
3. $-2x + 4y = 10$ and $x + y = 3$

Work through each problem using substitution or elimination, and verify your solutions.

Conclusion

The system of equations $-x + y = -3$ and $6x - 9y = 30$ provides a clear example of solving linear systems through algebraic methods. By simplifying equations, choosing an appropriate solving method, and verifying solutions, you can find the intersection point of the lines represented by these equations. Mastering these techniques is essential not only for academic success but also for practical problem-solving in various scientific and engineering disciplines.

Remember, practice is key. The more systems you solve, the more intuitive these methods become, enabling you to tackle even more complex problems with confidence.

Frequently Asked Questions

How can I solve the system of equations $-x + y = -3$ and $6x - 9y = 30$?

You can solve the system using substitution or elimination. For example, from the first equation, $y = x - 3$. Substitute into the second: $6x - 9(x - 3) = 30$, then solve for x and find y accordingly.

What is the solution to the system $-x + y = -3$ and $6x - 9y = 30$?

The solution is $x = 3$ and $y = 0$. Substituting $x = 3$ into the first equation yields $y = 0$, which satisfies both equations.

Are the two equations $-x + y = -3$ and $6x - 9y = 30$ consistent or inconsistent?

They are consistent and have a unique solution, which can be found by solving the system. In this case, the solution is $x=3$, $y=0$.

Can the equations -x + y = -3 and 6x - 9y = 30 be represented graphically?

Yes, graphing both equations will show they intersect at a single point, (3, 0), indicating a unique solution.

Is there a relationship between the equations -x + y = -3 and 6x - 9y = 30?

Yes, the second equation is a multiple of the first. Multiplying the first equation by 6 results in the second, indicating the equations are dependent and represent the same line.

Additional Resources

Understanding and Solving the System of Equations: -x + y = -3 and 6x - 9y = 30

When working with systems of equations, one of the fundamental skills in algebra is learning how to solve for the variables involved. The system of equations consisting of -x + y = -3 and 6x - 9y = 30 presents an interesting case that involves techniques such as substitution, elimination, and analysis of the relationships between the equations. This guide provides a comprehensive breakdown of these two equations, explores their solutions, and offers step-by-step instructions for solving them effectively.

Understanding the System of Equations

Before diving into the solution process, it’s important to understand what these equations represent and how they relate to each other.

The Equations Overview

1. Equation 1: $-x + y = -3$
2. Equation 2: $6x - 9y = 30$

These are linear equations, each representing a straight line in the xy-plane. The solution to the system is the point(s) where these lines intersect, which can be a single point, infinitely many points, or no points at all, depending on their relationship.

Recognizing the Nature of the System

- Consistent and Independent: The lines intersect at exactly one point.
- Consistent and Dependent: The lines are coincident (the same line), meaning infinitely many solutions.
- Inconsistent: The lines are parallel and do not intersect, meaning no solution.

As we analyze the equations, our goal is to determine which of these cases applies.

Step-by-Step Approach to Solving the System

Step 1: Simplify and Rearrange the Equations

Start by rewriting the equations in a more manageable form, ideally solving for y in terms of x or vice versa.

Equation 1:

$-x + y = -3$

$\Rightarrow y = x - 3$

This form makes it straightforward to substitute into the second equation.

Step 2: Substitute and Find the Solution

Plug the expression for y into the second equation:

$6x - 9(y) = 30$

$\Rightarrow 6x - 9(x - 3) = 30$

Now, expand the second term:

$6x - 9x + 27 = 30$

Combine like terms:

$(6x - 9x) + 27 = 30$

$\Rightarrow -3x + 27 = 30$

Subtract 27 from both sides:

$-3x = 3$

Divide both sides by -3:

$x = -1$

Now that we have x, substitute back into the expression for y:

$y = x - 3$

$\Rightarrow y = -1 - 3 = -4$

Solution Point:

$(-1, -4)$

This is the unique point where both lines intersect, indicating the system has one solution.

Additional Analysis: Confirming the Nature of the System

Check for Consistency

To confirm the solution, plug $x = -1$ and $y = -4$ into the second equation:

$6(-1) - 9(-4) = ?$

$\Rightarrow -6 + 36 = 30$

Indeed, $-6 + 36 = 30$, which matches the right side of the second equation.

Graphical Interpretation

Graphing both equations:

- Equation 1: $y = x - 3$ (a line with slope 1, y-intercept at -3)
- Equation 2: $6x - 9y = 30$, which simplifies to:

Divide the entire equation by 3:

$2x - 3y = 10$

$\Rightarrow -3y = 10 - 2x$

$\Rightarrow y = (2x - 10)/3$

Plotting these lines reveals they intersect at the point (-1, -4), confirming the algebraic solution.

Special Cases and What They Signify

While in this particular case, we found a single intersection point, it's important to recognize other possible situations in systems of equations:

1. Infinitely Many Solutions (Dependent System)

Occurs when the two equations are essentially the same line, just scaled differently.

Example:
If the second equation was $2x - 3y = 10$, and the first was $-x + y = -3$, then both equations represent the same line, and all points on that line are solutions.

2. No Solution (Inconsistent System)

Happens when the lines are parallel but not coincident, meaning they never intersect.

Example:
Suppose the second equation was $6x - 9y = 35$; this would be parallel to the original second equation but shifted, resulting in no common solution.

Summary: Key Takeaways

- To solve systems of linear equations, substitution and elimination are powerful methods.
- Recognizing the form of equations helps streamline the solution process.
- Confirming solutions by substitution ensures accuracy.
- The relationship between lines (intersecting at a point, coincident, or parallel) determines the nature of solutions.

Practical Tips for Solving Similar Systems

- Always start by simplifying equations as much as possible.
- Use substitution when one equation is easily solved for a variable.
- Use elimination when coefficients are multiples, making addition/subtraction straightforward.
- Graphically, visualize the lines to understand their relationship.
- Check solutions by substituting back into original equations.

Final Thoughts

The system of equations $-x + y = -3$ and $6x - 9y = 30$ demonstrates the core principles of linear algebra and system solving. By methodically substituting and simplifying, we identified the unique solution at (-1, -4). Mastering these techniques not only enhances problem-solving skills in algebra but also lays the foundation for more advanced topics like matrices, calculus, and differential equations. Whether you're a student, educator, or enthusiast, understanding the relationships between equations and their solutions is an essential step toward mathematical fluency.

X Y 3 And 6x 9y 30

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