2x + Y = -45x - 3y = 1Solve With Elimination Method

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Solving systems of linear equations is a fundamental skill in algebra, essential for students and professionals alike. The elimination method, also known as the addition method, is a popular technique used to find the values of variables that satisfy multiple equations simultaneously. In this article, we will explore how to solve the system of equations:

$$2x + y = -45$$
$$-3x + y = 1$$

using the elimination method. We will break down each step clearly, provide detailed explanations, and include tips to enhance your problem-solving skills.

Understanding the System of Equations

Before diving into the solution process, it is crucial to understand the structure of the equations involved.

What Is a System of Linear Equations?

A system of linear equations consists of two or more equations with the same set of variables. The goal is to find the values of these variables that satisfy all equations simultaneously. For example:

$$\text{Equation 1: } 2x + y = -45$$
$$\text{Equation 2: } -3x + y = 1$$

Here, both equations involve the variables x and y. The solution is the pair (x, y) that makes both equations true.

Methods to Solve Systems of Equations

Common methods include:

- Graphical Method
- Substitution Method
- Elimination Method
- Matrix Method (using determinants or matrices)

In this tutorial, we focus on the elimination method, which is particularly effective when coefficients align conveniently for elimination.

Step-by-Step Solution Using the Elimination Method

The elimination method involves adding or subtracting equations to eliminate one variable, making it easier to solve for the remaining variable.

Step 1: Write the equations in standard form

Ensure both equations are aligned with variables on one side and constants on the other:

$$
\begin{cases}
2x + y = -45 \quad \text{(Equation 1)} \\
-3x + y = 1 \quad \text{(Equation 2)}
\end{cases}
$$

Both equations are already in standard form.

Step 2: Make the coefficients of one variable opposites

To eliminate a variable, the coefficients of either x or y should be opposites. Here, notice:

- Equation 1 has $2x$
- Equation 2 has $-3x$

The coefficients of x are 2 and -3. To align them for elimination, find the least common multiple (LCM) of 2 and 3, which is 6.

Multiply each equation by a suitable number so that the coefficients of x are opposites:

- Multiply Equation 1 by 3:

$$
3 \times (2x + y) = 3 \times -45
$$
$$
6x + 3y = -135
$$

- Multiply Equation 2 by 2:

$$
2 \times (-3x + y) = 2 \times 1
$$
$$
-6x + 2y = 2
$$

Now, our system becomes:

$$
\begin{cases}
6x + 3y = -135 \quad (3) \\
-6x + 2y = 2 \quad (4)
\end{cases}
$$

Step 3: Add the equations to eliminate x

Add equations (3) and (4):

$$
(6x + 3y) + (-6x + 2y) = -135 + 2
$$

Simplify:

$$
6x - 6x + 3y + 2y = -133
$$
$$
0x + 5y = -133
$$

This simplifies to:

$$
5y = -133
$$

Step 4: Solve for y

Divide both sides by 5:

$$
y = \frac{-133}{5} = -26.6
$$

So, the solution for y is:

$$
y = -26.6
$$

Finding the Corresponding x Value

Having found y, substitute it back into one of the original equations to solve for x.

Step 5: Substitute y into one of the original equations

Using Equation 1:

$$
2x + y = -45
$$

Substitute $y = -26.6$:

$$
2x + (-26.6) = -45
$$

Simplify:

$$
2x = -45 + 26.6
$$
$$
2x = -18.4
$$

Divide both sides by 2:

$$
x = \frac{-18.4}{2} = -9.2
$$

Thus, the solution to the system is:

$$
x = -9.2, \quad y = -26.6
$$

Final Solution and Interpretation

The system of equations:

$$
2x + y = -45
$$
$$
-3x + y = 1
$$

has the solution:

$$
\boxed{
x = -9.2,\quad y = -26.6
}
$$

This means that substituting these values into both equations will satisfy them simultaneously.

Verification of the Solution

Always verify your solutions by substituting the values back into the original equations.

Equation 1:

$$
2(-9.2) + (-26.6) = -18.4 - 26.6 = -45
$$

Equation 2:

$$
-3(-9.2) + (-26.6) = 27.6 - 26.6 = 1
$$

Both equations are satisfied, confirming the correctness of the solution.

Tips for Solving Systems of Equations Using the Elimination Method

- Always check if equations are in standard form before proceeding.
- Multiply equations to align coefficients for elimination.
- Be cautious with signs during addition or subtraction.
- Simplify equations after each step for clarity.
- Verify solutions by substitution into original equations.

Additional Examples and Practice Problems

To strengthen your understanding, consider practicing with these systems:

1. $3x + 2y = 6$ and $5x - y = 4$
2. $x + y = 7$ and $2x - 3y = -1$
3. $4x + y = 10$ and $-2x + 3y = 5$

Applying the elimination method to these problems will improve your proficiency in solving systems efficiently.

Conclusion

The elimination method is a powerful and efficient technique for solving systems of linear equations, especially when coefficients are suitable for elimination. By following systematic steps—aligning equations, multiplying to match coefficients, adding or subtracting to eliminate variables, and then back-substituting—you can find solutions confidently and accurately. Practice is key to mastering this method, and verifying solutions ensures your answers are correct. Whether you're tackling algebra homework or solving real-world problems, the elimination method is an indispensable tool in your mathematical toolkit.

Keywords: solving systems of equations, elimination method, algebra, linear equations, solve for x and y, algebra techniques, step-by-step solutions, math practice

Frequently Asked Questions

What is the first step to solve the system 2x + y = -45 and -3y = 1 using the elimination method?

First, rewrite the equations clearly: $2x + y = -45$ and $-3y = 1$. Then, express one variable in terms of the other or manipulate the equations to align coefficients for elimination.

How can I eliminate one variable in the system 2x + y = -45 and -3y = 1?

Since $-3y = 1$, you can solve for y as $y = -1/3$. Substitute this into the first equation to solve for x, or multiply the equations to align coefficients for elimination.

Is it possible to solve the system 2x + y = -45 and -3y = 1 using elimination directly?

Yes. First, solve $-3y = 1$ for y, then substitute into the first equation to find x. Alternatively, you can manipulate equations to eliminate one variable directly.

What is the value of y in the system 2x + y = -45 and -3y = 1?

From $-3y = 1$, $y = -1/3$.

After finding y = -1/3, how do I find x in the system 2x + y = -45?

Substitute $y = -1/3$ into $2x + y = -45$: $2x + (-1/3) = -45$. Then, $2x = -45 + 1/3$, which simplifies to $2x = -135/3 + 1/3 = -134/3$. Finally, $x = (-134/3) / 2 = -134/6 = -67/3$.

What is the solution to the system 2x + y = -45 and -3y = 1?

The solution is $x = -67/3$ and $y = -1/3$.

Can I verify the solution by plugging the values back into the original equations?

Yes. Plug $x = -67/3$ and $y = -1/3$ into both equations to check if both are satisfied. They should confirm the solution is correct.

Is the elimination method the best approach for this system?

Since one of the equations directly gives y, substitution is straightforward. However, elimination can also be used after aligning coefficients, making it effective for systems where variables are more evenly matched.

Are there any common mistakes to avoid when solving this system?

Yes. Ensure correct algebraic manipulation, especially when solving for y and substituting back. Also, watch out for sign errors and simplifying fractions accurately.

How can I generalize this approach to similar systems of equations?

Identify a variable that can be easily isolated or manipulated, solve for it, then substitute into the other equation. Alternatively, multiply equations to align coefficients for elimination, and always double-check your solutions by substitution.

Additional Resources

2x + Y = -45x - 3y = 1Solve With Elimination Method: An In-Depth Analytical Review

The process of solving systems of linear equations is a fundamental aspect of algebra, integral to various fields including engineering, physics, economics, and computer science. Among the myriad methods available, the elimination method — also known as the addition method — stands out for its straightforward approach to finding solutions when the system is composed of two linear equations. This article provides a comprehensive, detailed examination of solving the system:

$$2x + y = -45$$
$$-45x - 3y = 1$$

using the elimination method. We will explore the theoretical foundations, step-by-step procedures, common pitfalls, and interpretations of the solutions, all structured to cater to both novice learners and seasoned mathematicians seeking a refresher.

Understanding the System of Equations

Before delving into the solution process, it is crucial to understand the nature of the given equations. The system comprises two linear equations:

1. $2x + y = -45$
2. $-45x - 3y = 1$

Linear equations are algebraic expressions where variables are of the first degree, meaning they are not raised to any power higher than one. The system here involves two variables, x and y, and can be represented graphically as two straight lines in the xy-plane.

Key characteristics:

- Number of solutions: The system can have a unique solution (one point of intersection), infinitely many solutions (the same line), or no solution (parallel lines with no point of intersection).
- Method selection: The elimination method is particularly effective when coefficients of one variable are opposites or can be made to cancel out through multiplication.

Theoretical Foundations of the Elimination Method

The elimination method operates on the principle of combining equations to eliminate one variable, reducing the system to a single-variable equation that can be solved directly.

Core idea:

- Multiply one or both equations by suitable constants so that the coefficients of one variable become identical or additive inverses.
- Add or subtract the equations to cancel out that variable.
- Solve the resulting single-variable equation.
- Substitute back into one of the original equations to find the other variable.

Advantages of the elimination method:

- Efficient for systems where coefficients are easily manipulated.
- Systematic and straightforward, minimizing errors.
- Suitable for larger systems, extending beyond two equations with modifications.

Potential challenges:

- When coefficients are not conveniently related, additional multiplication may be needed.
- Care must be taken to avoid algebraic mistakes during multiplication and addition/subtraction steps.

Step-by-Step Solution of the System

Let’s apply the elimination method to the system:

$$
\begin{cases}
2x + y = -45 \quad \quad (1) \\
-45x - 3y = 1 \quad \quad (2)
\end{cases}
$$

Step 1: Align coefficients for elimination

Our goal is to cancel one variable. Notice the coefficients of y:

- Equation (1): y coefficient is 1.
- Equation (2): y coefficient is -3.

To eliminate y, we can manipulate equations so that the coefficients of y are opposites, such as:

- Multiply Equation (1) by 3, so the coefficient of y becomes 3.
- Keep Equation (2) as is, with -3.

Then, adding the equations will cancel y.

Step 2: Multiply Equation (1) by 3

$$
3 \times (2x + y) = 3 \times (-45)
$$
$$
6x + 3y = -135
$$

Now, the system becomes:

$$
\begin{cases}
6x + 3y = -135 \quad \quad (3) \\
-45x - 3y = 1 \quad \quad (2)
\end{cases}
$$

Step 3: Add equations (3) and (2) to eliminate y

$$
(6x + 3y) + (-45x - 3y) = -135 + 1
$$
$$
(6x - 45x) + (3y - 3y) = -134
$$
$$
-39x = -134
$$

Step 4: Solve for x

$$
x = \frac{-134}{-39} = \frac{134}{39}
$$

Simplify if possible:

- 134 and 39 have no common factors other than 1, so the solution for x is:

$$
x = \frac{134}{39}
$$

Finding the Corresponding y Value

Having obtained x, substitute this back into one of the original equations to find y. The first equation is simpler:

$$
2x + y = -45
$$

Plugging in $x = \frac{134}{39}$:

$$
2 \times \frac{134}{39} + y = -45
$$

Calculate $2 \times \frac{134}{39}$:

$$
\frac{268}{39} + y = -45
$$

Express -45 as a fraction with denominator 39:

$$
-45 = -\frac{45 \times 39}{39} = -\frac{1755}{39}
$$

Now, solve for y:

$$
y = -\frac{1755}{39} - \frac{268}{39} = -\frac{1755 + 268}{39} = -\frac{2023}{39}
$$

Thus, the solution to the system is:

$$
\boxed{
x = \frac{134}{39}, \quad y = -\frac{2023}{39}
}
$$

Optional decimal approximation:

$$
x \approx 3.4359, \quad y \approx -51.872
$$

Verification of the Solution

To ensure accuracy, verify the solution in the second original equation:

$$
-45x - 3y = 1
$$

Substitute $x = \frac{134}{39}$ and $y = -\frac{2023}{39}$:

$$
-45 \times \frac{134}{39} - 3 \times \left(-\frac{2023}{39}\right)
$$

Calculate each term:

$$
-45 \times \frac{134}{39} = -\frac{45 \times 134}{39} = -\frac{6030}{39}
$$
$$
-3 \times \left(-\frac{2023}{39}\right) = \frac{3 \times 2023}{39} = \frac{6069}{39}
$$

Sum:

$$
-\frac{6030}{39} + \frac{6069}{39} = \frac{-6030 + 6069}{39} = \frac{39}{39} = 1
$$

The calculated value matches the right-hand side of the equation, confirming the solution's correctness.

Interpreting the Solution Within the Context

The solution set:

$$
x = \frac{134}{39} \quad \text{and} \quad y = -\frac{2023}{39}
$$

represents the point of intersection of the two lines represented by the original equations in the xy-plane.

Implications:

- The system has a unique solution, indicating the lines intersect at exactly one point.
- The fractional form emphasizes the exactness of the solution, which is crucial in theoretical contexts.
- Approximate decimal values can be used for practical applications but should be accompanied by the fractional form for precision.

Broader Applications and Contexts

Understanding the elimination method through this example illuminates its versatility and importance:

- In engineering, solving such systems can determine stress points or electrical circuit parameters.
- In economics, systems of equations often model constraints or optimization problems.
- In computer science, algorithms for solving linear systems underpin various computational problems, including graphics and machine learning.

The elimination method also scales to larger systems, where similar principles apply, albeit with more complex algebraic manipulations and the aid of matrix methods like Gaussian elimination.

Common Challenges and Troubleshooting

While the elimination method is conceptually straightforward, practitioners should be aware of potential pitfalls:

- Sign errors during multiplication or addition/subtraction.
- Incorrect multiplication factors, leading to failure in canceling variables.
- Simplification mistakes in fractional calculations.
- Inconsistent systems: systems where the equations represent parallel lines with no intersection or the same line with infinitely many solutions.

To mitigate these, meticulous algebraic work, step-by-step verification, and cross-checking solutions are recommended.

Conclusion

The elimination method offers a robust, systematic approach to solving systems of linear equations, exemplified here through the equations $2x + y = -45$ and $-45x - 3y = 1$.

2x Y 45x 3y 1solve With Elimination Method

2x Y 45x 3y 1solve With Elimination Method

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