4x + 6x = 12 Solve By Completing Square

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When faced with algebraic equations, solving for the unknown variable can sometimes be straightforward, but other times it requires more advanced techniques. One such method is completing the square, a powerful tool especially useful for quadratic equations. Although the equation 4x + 6x = 12 appears simple at first glance, understanding how to approach it step-by-step provides a strong foundation for tackling more complex problems. This article will explore how to solve equations like 4x + 6x = 12 by completing the square, providing a comprehensive guide suitable for learners at various levels.

Understanding the Equation: 4x + 6x = 12

Before diving into the solution process, it’s essential to understand the nature of the equation:

- The equation involves linear terms: 4x and 6x.
- The sum of these terms equals a constant: 12.
- Combining like terms simplifies the equation.

Step 1: Simplify the Equation

Start by combining the like terms:

$4x + 6x = (4 + 6)x = 10x$

So, the simplified equation is:

$10x = 12$

This equation is linear and straightforward to solve by division:

$x = 12 / 10 = 6 / 5$

However, since the goal is to demonstrate completing the square, we'll consider a more complex quadratic version of the equation, such as:

$4x^2 + 6x = 12$

This quadratic form is more suitable for applying the completing the square method.

Solving a Quadratic Equation Using Completing the Square

Equation for Practice:

Let's now assume the original problem is:

$4x^2 + 6x = 12$

Our goal is to solve for x by completing the square.

Step 1: Bring all terms to one side

Rewrite the equation:

$4x^2 + 6x - 12 = 0$

But for completing the square, it's often easiest to work with the quadratic in the form:

$4x^2 + 6x = 12$

Step 2: Make the coefficient of x^2 equal to 1

Divide the entire equation by 4:

$x^2 + (6/4)x = 12 / 4$

Simplify:

$x^2 + (3/2)x = 3$

Step 3: Complete the square

To complete the square, take half of the coefficient of x, square it, and add it to both sides:

- Coefficient of x: 3/2
- Half of this: (3/4)
- Square of this: $(3/4)^2 = 9/16$

Add 9/16 to both sides:

$x^2 + (3/2)x + 9/16 = 3 + 9/16$

Express the right side with common denominator:

$3 = 48/16$

So,

$x^2 + (3/2)x + 9/16 = (48/16) + (9/16) = 57/16$

Step 4: Write the left side as a perfect square

The left side is a perfect square trinomial:

$(x + 3/4)^2 = 57/16$

Step 5: Solve for x

Take the square root of both sides:

$x + 3/4 = \pm\sqrt{(57/16)}$

Simplify the radical:

$x + 3/4 = \pm (\sqrt{57}) / 4$

Subtract 3/4 from both sides:

$x = -3/4 \pm (\sqrt{}57) / 4$

Expressed as a single fraction:

$x = (-3 \pm \sqrt{}57) / 4$

Summary of the Solution Process

The steps involved in solving quadratic equations by completing the square include:

1. Rearranging the equation to set it to the form suitable for completing the square.
2. Dividing through to make the coefficient of x^2 equal to 1.
3. Calculating half the coefficient of x, then squaring it.
4. Adding this square to both sides to complete the square.
5. Expressing the left side as a perfect square.
6. Taking the square root of both sides.
7. Isolating x to find the solutions.

Visual Example: Step-by-Step Breakdown

Let’s summarize with a clear step-by-step list:

1. Start with the quadratic equation: $4x^2 + 6x = 12$

2. Divide all terms by 4: $x^2 + (3/2)x = 3$

3. Calculate half of the coefficient of x: $(3/2) \div 2 = 3/4$

4. Square this value: $(3/4)^2 = 9/16$

5. Add 9/16 to both sides: $x^2 + (3/2)x + 9/16 = 3 + 9/16$

6. Express right side as a fraction: $3 = 48/16$, so right side becomes 57/16

7. Rewrite left side as a perfect square: $(x + 3/4)^2 = 57/16$

8. Take the square root of both sides: $x + 3/4 = \pm\sqrt{(57/16)}$

9. Simplify the radical: $x + 3/4 = \pm (\sqrt{57})/4$

10. Subtract 3/4 from both sides: $x = -3/4 \pm (\sqrt{57})/4$

Applications and Importance of Completing the Square

Completing the square is not just an academic exercise; it has practical applications in various fields:

- Quadratic Formula Derivation: Completing the square is essential in deriving the quadratic formula.
- Graphing Parabolas: Identifies the vertex form of quadratic functions, useful for graphing.
- Solving Inequalities: Helps in analyzing the nature of solutions.
- Physics and Engineering: Used in trajectory calculations, optimization problems, and more.

Additional Tips for Solving Quadratics by Completing the Square

- Always aim to make the coefficient of x^2 equal to 1 before completing the square.
- Keep careful track of fractions; they are common in these calculations.
- Remember to consider both positive and negative roots when taking square roots.
- Practice with different coefficients and constants to build confidence.

Common Mistakes to Avoid

- Forgetting to add the square of half the coefficient of x to both sides.
- Mixing up the signs when taking square roots.
- Not simplifying radical expressions fully.
- Skipping steps, leading to errors in the solution.

Conclusion

While the equation $4x + 6x = 12$ is simple in its linear form, transforming it into quadratic form and solving by completing the square provides valuable insight into the mechanics of algebra. This method is especially powerful when dealing with quadratic equations that are not easily factorable. Mastering completing the square enhances problem-solving skills, deepens understanding of quadratic functions, and opens doors to advanced topics in mathematics. Whether you’re a student preparing for exams or a professional applying math in real-world scenarios, understanding how to solve equations like $4x + 6x = 12$ by completing the square is an essential mathematical skill.

Frequently Asked Questions

How do you solve the equation $4x + 6x = 12$ by completing the square?

First, combine like terms to get $10x = 12$, then rewrite the equation to isolate x, or convert it into a quadratic form if needed. However, since it's linear, completing the square isn't necessary here—it's more applicable for quadratic equations.

Is completing the square applicable for solving linear equations like $4x + 6x = 12$?

No, completing the square is a method used for quadratic equations. Since $4x + 6x = 12$ simplifies to a linear equation, it's best solved by combining like terms and isolating x.

What is the first step in solving $4x + 6x = 12$?

Combine like terms to simplify the equation: $4x + 6x = 10x$, so the equation becomes $10x = 12$.

Can we transform $10x = 12$ into a quadratic form to complete the square?

No, $10x = 12$ is a linear equation. To complete the square, you need a quadratic equation, such as $ax^2 + bx + c = 0$.

What is the solution for x in the equation 10x = 12?

Divide both sides by 10 to find x: x = 12 / 10 = 1.2.

When should you use completing the square in solving equations?

You should use completing the square when solving quadratic equations in the form ax^2 + bx + c = 0 to find the roots of the equation.

Additional Resources

4x + 6x = 12 Solve By Completing Square

Mathematics often challenges learners with its diverse array of problem-solving techniques. Among these, the method of completing the square stands out as a powerful tool, especially when dealing with quadratic equations. While the equation 4x + 6x = 12 appears straightforward at first glance, it offers an excellent opportunity to explore the process of solving by completing the square—an approach that deepens understanding of quadratic expressions and their properties. This article delves into this method with clarity and precision, providing a comprehensive guide for students and enthusiasts eager to master the technique.

Understanding the Equation: From Simplification to Quadratic Form

Before diving into the completing the square method, it's essential to analyze the given equation:

4x + 6x = 12

Step 1: Simplify the Equation

Combine like terms:

$4x + 6x = (4 + 6) x = 10x$

So, the simplified form is:

$10x = 12$

At this stage, the solution is straightforward:

$x = 12 / 10 = 6 / 5$

However, the goal here is to illustrate the process of solving quadratic equations via completing the square, which requires transforming the equation into a quadratic form.

Step 2: Recognize the Need for a Quadratic Equation

The initial linear form does not lend itself to completing the square directly. To demonstrate the method, we modify the problem into a quadratic equation. For instance, consider the following adjusted problem:

$4x^2 + 6x = 12$

Now, this is a quadratic equation with the standard form:

$a x^2 + b x + c = 0$

which is suitable for completing the square.

Reframing the Equation: Setting the Stage for Completing the Square

Suppose we have:

$4x^2 + 6x = 12$

Step 1: Bring all terms to one side

$4x^2 + 6x - 12 = 0$

Dividing through by 4 to simplify coefficients:

$x^2 + (6/4)x - 3 = 0$

Simplify the fraction:

$x^2 + (3/2)x - 3 = 0$

Now, the equation is in a form ready for completing the square:

$x^2 + (3/2)x = 3$

Completing the Square: The Step-by-Step Process

Step 1: Isolate the quadratic and linear terms

$x^2 + (3/2)x = 3$

Step 2: Find the value to complete the square

To complete the square, take half of the coefficient of x, then square it:

- Coefficient of x: $3/2$
- Half of this: $(3/4)$
- Square of half: $(3/4)^2 = 9/16$

Step 3: Add this square to both sides

$$x^2 + (3/2) x + 9/16 = 3 + 9/16$$

This keeps the equation balanced.

Step 4: Express the left side as a perfect square

The left side becomes a perfect square trinomial:

$$(x + 3/4)^2 = 3 + 9/16$$

Step 5: Simplify the right side

Convert 3 to sixteenths:

$$3 = 48/16$$

Now, add:

$$48/16 + 9/16 = 57/16$$

So, the equation is:

$$(x + 3/4)^2 = 57/16$$

Solving the Equation: Extracting the Roots

Step 1: Take the square root of both sides

$x + 3/4 = \pm \sqrt{(57/16)}$

Since $\sqrt{(a/b)} = \sqrt{a} / \sqrt{b}$, we get:

$x + 3/4 = \pm \sqrt{57} / 4$

Step 2: Isolate x

$x = -3/4 \pm \sqrt{57} / 4$

Step 3: Write in simplified form

$x = (-3 \pm \sqrt{57}) / 4$

This provides the solutions to the original quadratic equation.

Key Takeaways and Practical Applications

The process of completing the square is fundamental in algebra, especially for solving quadratic equations that are not easily factorable. Here are some key insights:

- Universal applicability: Not all quadratics factor neatly, but completing the square provides a reliable alternative.

- Deriving the quadratic formula: The method directly leads to the quadratic formula, which is invaluable in various fields, including physics, engineering, and economics.
- Understanding the graph: Completing the square reveals the vertex form of a quadratic function, guiding graphing and analysis.

Step-by-Step Guide to Completing the Square

For learners new to the method, here's a concise step-by-step process:

1. Write the quadratic in the form $ax^2 + bx + c = 0$ or $ax^2 + bx = d$.
2. Divide through by a (if the coefficient of x^2 is not 1) to normalize.
3. Isolate the quadratic and linear terms on one side.
4. Calculate half of the linear coefficient (b/2a), then square it.
5. Add this square to both sides of the equation.
6. Express the left side as a perfect square trinomial.
7. Simplify the right side.
8. Take the square root of both sides, remembering the ± symbol.
9. Solve for the variable to find the roots.

Practical Example: Solving $4x^2 + 6x = 12$

Applying all the steps:

- Start with $4x^2 + 6x = 12$
- Divide both sides by 4: $x^2 + (3/2) x = 3$
- Half of (3/2) is (3/4), square is 9/16
- Add 9/16 to both sides: $x^2 + (3/2) x + 9/16 = 3 + 9/16$

- Left side: $(x + 3/4)^2$

- Right side: 48/16 + 9/16 = 57/16

- Take square root: $x + 3/4 = \pm \sqrt{57} / 4$

- Final solutions: $x = -3/4 \pm \sqrt{57} / 4$

These solutions demonstrate the power of completing the square in transforming quadratic equations into solvable forms.

Conclusion: Mastering the Technique for Broader Mathematical Success

While the initial equation 4x + 6x = 12 might seem simple in its linear form, transforming it into a quadratic and solving through completing the square exemplifies the depth of algebraic methods. This technique not only aids in solving complex equations but also enhances conceptual understanding of quadratic functions, their graphs, and their properties.

Whether tackling quadratic equations in academic settings or applying them to real-world problems, mastering completing the square is a vital skill for any serious student of mathematics. Through systematic practice and application, learners can unlock new levels of problem-solving confidence and mathematical insight, paving the way for success across diverse scientific and analytical disciplines.

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