# A Line Passes Through The Points ( 1,2) And (3,5)

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Understanding how to find the equation of a line passing through two points is a fundamental concept in coordinate geometry. When given the points (1, 2) and (3, 5), mathematicians and students alike can determine the precise mathematical description of the line that connects them. This process involves calculating the slope of the line and then using point-slope or slope-intercept form to express its equation. In this article, we will explore the step-by-step method to derive the equation of the line passing through these points, discuss key concepts like slope and intercepts, and provide practical examples to solidify understanding.

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## Understanding the Basics: Points, Coordinates, and Lines

Before delving into calculations, it's crucial to grasp some foundational concepts:

### Coordinate Points

- Definition: Points in a plane are represented by ordered pairs (x, y), where 'x' is the horizontal coordinate and 'y' is the vertical coordinate.
- Example: The points (1, 2) and (3, 5) are two locations on the Cartesian plane, with their respective x and y values.

### Line in Coordinate Geometry

- Equation of a line: It describes all the points that lie along a straight path.
- Standard forms include:
- Slope-intercept form: $y = mx + b$
- Point-slope form: $y - y_1 = m(x - x_1)$
- General form: $Ax + By + C = 0$

Understanding these forms allows us to articulate the line's characteristics precisely, such as its slope and position.

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# Calculating the Slope of the Line Passing Through (1,2) and (3,5)

The first step in finding the equation of the line is computing its slope. The slope (m) indicates how steep the line is and the direction it moves.

## Definition of Slope

- The slope between two points $(x_1, y_1)$ and $(x_2, y_2)$ is calculated as:

$$
m = \frac{y_2 - y_1}{x_2 - x_1}
$$

- It represents the rate of change of y with respect to x.

## Calculating the Slope for Points (1, 2) and (3, 5)

- Assign:
- $(x_1, y_1) = (1, 2)$
- $(x_2, y_2) = (3, 5)$

- Plug into the formula:

$$
m = \frac{5 - 2}{3 - 1} = \frac{3}{2}
$$

- Result: The slope of the line passing through the points (1, 2) and (3, 5) is $m = 3/2$.

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# Deriving the Equation of the Line

Once the slope is known, the next step is to find the actual equation of the line in a usable form.

## Using the Point-Slope Form

- The point-slope form is particularly convenient when you know a point on the line and the slope:

$$
y - y_1 = m(x - x_1)
$$

- Choose one of the given points, say (1, 2), and plug in the slope:

$$
y - 2 = \frac{3}{2}(x - 1)
$$

## Simplifying to Slope-Intercept Form

- Expand the equation:

$$
y - 2 = \frac{3}{2}x - \frac{3}{2}
$$

- Add 2 to both sides (or convert 2 to a fraction):

$$
y = \frac{3}{2}x - \frac{3}{2} + 2
$$

- Convert 2 to a fraction with denominator 2:

$$
2 = \frac{4}{2}
$$

- So:

$$
y = \frac{3}{2}x - \frac{3}{2} + \frac{4}{2} = \frac{3}{2}x + \frac{1}{2}
$$

Final Equation:

$$
\boxed{ y = \frac{3}{2}x + \frac{1}{2} }
$$

This is the equation of the line passing through (1, 2) and (3, 5).

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# Verifying the Equation with the Second Point

It's important to verify that the derived equation passes through both points.

## Check with (3, 5):

- Substitute $x = 3$:

$$
y = \frac{3}{2} \times 3 + \frac{1}{2} = \frac{9}{2} + \frac{1}{2} = \frac{10}{2} = 5
$$

- Since $y = 5$, the point (3, 5) satisfies the equation.

## Check with (1, 2):

- Substitute $x = 1$:

$$
y = \frac{3}{2} \times 1 + \frac{1}{2} = \frac{3}{2} + \frac{1}{2} = 2
$$

- $y = 2$ confirms the point (1, 2) also lies on the line.

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# Graphing the Line: Visual Representation

Visualizing the line helps in understanding its position and slope. Here’s how you can do it:

## Steps to Plot the Line

- Plot the points (1, 2) and (3, 5) on graph paper or graphing software.
- Draw a straight line through these points.
- Note the slope: for every 2 units moved horizontally, the line rises 3 units vertically.

- Extend the line in both directions to visualize its full span.

### Key Features to Note

- Slope: 3/2 signifies the line rises 3 units for every 2 units moved horizontally.
- Y-intercept: The point where the line crosses the y-axis is at (0, 0.5), which can be found by setting x=0:

$$
y = \frac{3}{2} \times 0 + \frac{1}{2} = \frac{1}{2}
$$

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## Applications and Practical Examples

Understanding this fundamental concept has numerous practical applications:

### Real-World Applications of Line Equations

- Physics: Calculating the trajectory of objects.
- Economics: Modeling cost versus production quantities.
- Engineering: Designing structures with specific slope requirements.
- Data Analysis: Fitting a line to data points for trend analysis.

Example Problem: Finding the Equation of a Line through Two Data Points

Suppose a company tracks sales over two months:
- Month 1: 100 units sold.
- Month 2: 250 units sold.

Find the line equation modeling sales over these months, assuming month numbers as x (Month 1 = x=1, Month 2 = x=2), and units sold as y.

Solution:

1. Points: (1, 100) and (2, 250)
2. Calculate slope:

$$
m = \frac{250 - 100}{2 - 1} = \frac{150}{1} = 150
$$

3. Use point-slope form with (1, 100):

$$
y - 100 = 150(x - 1)
$$

4. Expand:

$$
y - 100 = 150x - 150
$$

5. Simplify to slope-intercept form:

$$
y = 150x - 150 + 100 = 150x - 50
$$

Interpretation: The sales increase by 150 units each month, starting from a baseline that can be interpreted as -50 (which may require context adjustment).

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## Conclusion

Finding the equation of a line passing through two points, such as (1, 2) and (3, 5), is a vital skill in coordinate geometry. The process involves calculating the slope, verifying it, and then expressing the line in an appropriate form—most commonly slope-intercept. This method provides a clear mathematical representation of the line and supports various applications across science, engineering, and economics. By mastering these concepts, students and professionals can analyze and interpret linear relationships effectively, laying a solid foundation for more advanced mathematical topics.

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## Additional Tips for Learning and Teaching Line Equations

- Always verify your line equation with both points.
- Practice with different pairs of points to strengthen understanding.
- Use graphing tools to visualize the line for better intuition.
- Remember that the slope can be positive, negative, zero, or undefined, each indicating different line orientations.

Understanding the line passing through (1, 2) and (

## Frequently Asked Questions

### How do you find the equation of a line passing through the points (1, 2) and (3, 5)?

First, calculate the slope (m) as (5 - 2) / (3 - 1) = 3 / 2. Then, use point-slope form with one of the points, for example, y - 2 = (3/2)(x - 1). Simplifying gives the equation y = (3/2)x + 0.5.

### What is the slope of the line passing through points (1, 2) and (3, 5)?

The slope is (5 - 2) / (3 - 1) = 3 / 2.

How can I verify if a given point lies on the line passing through (1, 2) and (3, 5)?

Substitute the point's coordinates into the line's equation. If both x and y satisfy the equation, then the point lies on the line.

What is the y-intercept of the line passing through (1, 2) and (3, 5)?

Using the slope (3/2) and point (1, 2), the y-intercept is found from $y = (3/2)x + b$. Plugging in $x=1$, $y=2$: $2 = (3/2)(1) + b \rightarrow b = 2 - 1.5 = 0.5$. So, the y-intercept is 0.5.

Can the line passing through (1, 2) and (3, 5) be parallel to the line $y = 2x + 1$?

No, because the slope of the given line is 3/2, which is not equal to 2, the slope of $y = 2x + 1$.

What is the distance between the points (1, 2) and (3, 5)?

Distance = $\sqrt{[(3 - 1)^2 + (5 - 2)^2]} = \sqrt{(4 + 9)} = \sqrt{13} \approx 3.605$.

How do I write the equation of the line passing through (1, 2) with a slope of 3/2?

Using point-slope form: $y - 2 = (3/2)(x - 1)$, which simplifies to $y = (3/2)x + 0.5$.

What is the significance of the points (1, 2) and (3, 5) in defining the line?

These two points uniquely determine the line, as a straight line is defined by any two distinct points, allowing us to find its slope and equation.

## Additional Resources

Line Through Two Points: An In-Depth Exploration of the Equation and Properties

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Introduction

Understanding the fundamental concepts of coordinate geometry is essential for students, educators, and professionals working in mathematics, engineering, physics, and related fields. Among these foundational topics, the problem of finding a line passing through two specific points is a classic yet profound exercise that illuminates the core principles of algebra and geometry. In this article, we will explore the process of determining the equation of a line passing through the points (1, 2) and (3, 5). We will analyze the derivation step by step, examine the properties of such a line, and discuss various applications and extensions of this concept.

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## The Significance of Determining a Line Through Two Points

Before delving into calculations, it's crucial to understand why identifying a line through two points is significant:

- Foundational Concept: It establishes the basis for understanding more complex geometric figures and their equations.
- Real-World Applications: Used in navigation, computer graphics, physics, and data analysis.
- Problem-Solving Skill: Enhances algebraic manipulation, critical thinking, and spatial visualization.

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## Fundamental Concepts in Coordinate Geometry

### The Coordinate Plane

The coordinate plane consists of two perpendicular axes:

- X-axis (horizontal)
- Y-axis (vertical)

Any point in this plane is represented as an ordered pair (x, y), indicating its position relative to these axes.

### The Equation of a Line

A line can be represented in various forms:

- Slope-Intercept Form: $y = mx + c$
- Point-Slope Form: $y - y_1 = m(x - x_1)$
- Two-Point Form: Derived directly when two points are known

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## Step-by-Step Derivation of the Line Passing Through (1, 2) and (3, 5)

Let's now focus on determining the explicit equation of the line passing through the points:

- Point $P_1 = (1, 2)$
- Point $P_2 = (3, 5)$

### Step 1: Calculate the Slope

The slope $m$ indicates the steepness and direction of the line. It is calculated as:

$$
m = \frac{y_2 - y_1}{x_2 - x_1}
$$

Plugging in the values:

$$
m = \frac{5 - 2}{3 - 1} = \frac{3}{2}
$$

Interpretation: The line rises 3 units vertically for every 2 units it moves horizontally.

Step 2: Use the Point-Slope Form

With the slope $m = \frac{3}{2}$ and one known point (say, $(1, 2)$), apply the point-slope formula:

$$
y - y_1 = m(x - x_1)
$$

$$
y - 2 = \frac{3}{2}(x - 1)
$$

Step 3: Simplify to Slope-Intercept Form

Expanding and simplifying:

$$
y - 2 = \frac{3}{2}x - \frac{3}{2}
$$

$$
y = \frac{3}{2}x - \frac{3}{2} + 2
$$

Express 2 as $\frac{4}{2}$:

$$
y = \frac{3}{2}x - \frac{3}{2} + \frac{4}{2} = \frac{3}{2}x + \frac{1}{2}
$$

Final Equation in Slope-Intercept Form:

$$
\boxed{ y = \frac{3}{2}x + \frac{1}{2} }
$$

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### Analyzing the Equation and Its Properties

### Slope and Intercept

- Slope $m = \frac{3}{2}$: Indicates a relatively steep positive incline.
- Y-intercept $c = \frac{1}{2}$: The point where the line crosses the y-axis.

Graphical Representation

Plotting the points:

- $(1, 2)$ lies on the line.
- $(3, 5)$ also lies on the line.

Graphically, the line passes through these points, demonstrating the linear relationship.

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Alternative Forms of the Equation

Standard Form

Rearranged from slope-intercept:

$$
y = \frac{3}{2}x + \frac{1}{2}
$$

Multiply through by 2 to clear fractions:

$$
2y = 3x + 1
$$

Rearranged:

$$
3x - 2y + 1 = 0
$$

This is the standard form of the line's equation.

### Two-Point Form

Alternatively, the line can be expressed directly using the two points:

$$
\frac{y - y_1}{x - x_1} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{3}{2}
$$

which is the slope we computed earlier.

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## Applications and Extensions

### 1. Determining Distance Between the Points

Using the distance formula:

$$
d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}
$$

$$
d = \sqrt{(3 - 1)^2 + (5 - 2)^2} = \sqrt{2^2 + 3^2} = \sqrt{4 + 9} = \sqrt{13}
$$

This highlights how lines relate to distances between points.

2. Finding the Midpoint

Midpoint $M$:

$$
M = \left( \frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2} \right)
$$

$$
M = \left( \frac{1 + 3}{2}, \frac{2 + 5}{2} \right) = (2, 3.5)
$$

This point lies directly on the line segment connecting the two points.

3. Parallel and Perpendicular Lines

- Parallel lines share the same slope $\frac{3}{2}$.
- Perpendicular lines have slopes that are negative reciprocals: $-\frac{2}{3}$.

Understanding these relationships is essential in geometric constructions and proofs.

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## Real-World Examples and Practical Relevance

### Engineering and Design

Designers often need to determine the straight line passing through two points for structural analysis or CAD modeling.

### Data Analysis

Regression lines passing through two data points underpin basic predictive modeling.

### Navigation and GIS

Mapping routes involves calculating lines through specific coordinate points.

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## Challenges and Common Misconceptions

- Assuming the line is unique: Given any two distinct points, there is exactly one line passing through both.
- Confusing slope with intercept: The slope indicates steepness, not where the line crosses axes.
- Forgetting to verify the points: Always check if the points satisfy the derived equation.

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## Summary and Key Takeaways

- The line passing through points (1, 2) and (3, 5) has the equation $y = \frac{3}{2}x + \frac{1}{2}$.
- The slope $\frac{3}{2}$ indicates the line's incline.
- The line can be expressed in various forms: slope-intercept, standard, or point-slope.
- Understanding these forms enhances comprehension of linear relationships and their applications.

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## Final Thoughts

The process of deriving the line through two points is a fundamental exercise in coordinate geometry that encapsulates the essence of linear relationships. It exemplifies how algebraic techniques can describe geometric entities precisely and intuitively. Mastery of this concept not only solidifies foundational knowledge but also paves the way for advanced studies in mathematics, physics, engineering, and beyond.

By exploring the detailed steps, properties, and applications, we see how a simple problem transforms into a powerful tool with broad implications across science and technology. Whether plotting a graph, solving a geometric puzzle, or designing a structure, understanding lines through two points remains a vital skill in the mathematical toolkit.

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