

A Line That Passes Through (4,12) and (-7,-3)

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Understanding how to find the equation of a line that passes through two specific points is a fundamental skill in algebra and coordinate geometry. In this article, we delve into the process of determining the line that passes through the points (4,12) and (-7,-3). Whether you're a student tackling your math homework or a teacher preparing lessons, mastering this concept is essential for developing a solid foundation in coordinate geometry.

How to Find the Equation of a Line Passing Through Two Points

To find the equation of a line passing through the points (4,12) and (-7,-3), you need to follow a series of steps involving calculating the slope and then using point-slope or slope-intercept form. Understanding each step ensures accurate and efficient problem-solving.

1. Calculating the Slope of the Line

The slope of a line indicates its steepness and is represented as "m". It is calculated using the formula:

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

where (x_1, y_1) and (x_2, y_2) are the coordinates of the two points.

Applying this to our points:

- Point 1: (4,12)

- Point 2: (-7,-3)

$$m = \frac{-3 - 12}{-7 - 4} = \frac{-15}{-11} = \frac{15}{11}$$

So, the slope of the line passing through (4,12) and (-7,-3) is $\frac{15}{11}$.

2. Choosing the Equation Form

Once the slope is known, the next step is to write the equation of the line. The two common forms are:

- Point-Slope Form: $(y - y_1 = m(x - x_1))$

- Slope-Intercept Form: $(y = mx + b)$

Using the point-slope form often makes it easier to find the equation when you know a point and the slope.

3. Deriving the Equation of the Line

Using the point (4,12):

$$\begin{aligned} & \\ y - 12 &= \frac{15}{11} (x - 4) \\ & \end{aligned}$$

Expanding this:

$$\begin{aligned} & \\ y - 12 &= \frac{15}{11}x - \frac{15}{11} \times 4 \\ & \end{aligned}$$

$$\begin{aligned} & \\ y - 12 &= \frac{15}{11}x - \frac{60}{11} \\ & \end{aligned}$$

Adding 12 (which is $(\frac{132}{11})$) to both sides:

$$\begin{aligned} & \\ y &= \frac{15}{11}x - \frac{60}{11} + \frac{132}{11} \\ & \end{aligned}$$

$$\begin{aligned} & \\ y &= \frac{15}{11}x + \frac{72}{11} \\ & \end{aligned}$$

Thus, the equation of the line passing through (4,12) and (-7,-3) in slope-intercept form is:

$$\begin{aligned} & \\ \boxed{ y &= \frac{15}{11}x + \frac{72}{11} } \\ & \end{aligned}$$

Understanding the Significance of the Equation

Knowing the equation of the line allows you to analyze and interpret its properties in various contexts.

1. The Slope $(\frac{15}{11})$

- Indicates a positive slope, meaning the line rises as x increases.

- Steepness is approximately 1.36, showing a relatively moderate incline.

2. The Y-Intercept ($\frac{72}{11} \approx 6.55$)

- Represents the point where the line crosses the y-axis.
- Useful for graphing and understanding how the line behaves relative to the y-axis.

Graphing the Line Passing Through (4,12) and (-7,-3)

Graphing the line helps visualize the relationship between the points and the line's slope.

1. Plotting Known Points

- Point 1: (4, 12)
- Point 2: (-7, -3)

2. Using the Slope to Find Additional Points

- Starting from (4, 12), move down 15 units and right 11 units to find another point.
- Alternatively, from (-7, -3), move up 15 and left 11 to reach the same line.

3. Drawing the Line

- Connect the points smoothly to illustrate the line.
- Ensure the line extends beyond the points for accuracy.

Applications of Finding a Line Passing Through Two Points

Knowing how to find the equation of a line through two points has numerous practical applications across various fields.

1. In Mathematics and Education

- Helps students understand linear relationships.
- Foundation for studying linear functions and inequalities.

2. In Engineering and Physics

- Used in modeling real-world phenomena such as motion, forces, and structural analysis.
- Helps in designing and analyzing systems that follow linear patterns.

3. In Business and Economics

- Used in cost analysis, revenue projections, and trend forecasting.
- Assists in creating models based on data points.

Advanced Topics and Variations

Once comfortable with the basic methods, exploring more complex scenarios can deepen your understanding.

1. Line Passing Through a Point with a Given Slope

- Use point-slope form to derive the equation.

2. Line Parallel and Perpendicular to a Given Line

- Parallel lines have the same slope.
- Perpendicular lines have slopes that are negative reciprocals.

3. Finding the Equation of a Line When One Point and the Slope Are Known

- Use the point-slope form directly to find the equation.

Conclusion

Finding the equation of a line passing through two points, such as (4,12) and (-7,-3), involves calculating the slope and then applying the point-slope or slope-intercept form. The process not only aids in graphing and visualization but also plays a crucial role in various scientific and practical fields. Mastering this skill enhances your understanding of linear relationships and prepares you for more advanced topics in mathematics and related disciplines. Whether for academic purposes or real-world applications, knowing how to determine the line passing through two points is an invaluable tool in your mathematical toolkit.

Frequently Asked Questions

What is the slope of the line passing through the points (4, 12) and (-7, -3)?

The slope is $(-3 - 12) / (-7 - 4) = (-15) / (-11) = 15/11$.

What is the equation of the line passing through (4, 12) and (-7, -3)?

Using point-slope form: $y - 12 = (15/11)(x - 4)$. In slope-intercept form: $y = (15/11)x + (132/11)$.

How do you find the y-intercept of the line passing through (4, 12) and (-7, -3)?

Using the slope $15/11$ and point $(4, 12)$: $y - 12 = (15/11)(x - 4)$. When $x = 0$, $y = (15/11)0 + b$; solving for b gives y-intercept as $132/11$, or 12 .

Are the points (4, 12) and (-7, -3) collinear with the origin?

No, since they do not lie on the same line passing through the origin; the line passing through these points does not include $(0, y)$ unless calculated specifically.

What is the midpoint of the segment connecting (4, 12) and (-7, -3)?

The midpoint is $((4 + (-7))/2, (12 + (-3))/2) = (-3/2, 9/2)$ or $(-1.5, 4.5)$.

How long is the line segment between (4, 12) and (-7, -3)?

Distance = $\sqrt{(-7 - 4)^2 + (-3 - 12)^2} = \sqrt{(-11)^2 + (-15)^2} = \sqrt{121 + 225} = \sqrt{346} \approx 18.6$ units.

Is the line passing through (4, 12) and (-7, -3) increasing or decreasing?

Since the slope is positive $(15/11)$, the line is increasing.

Can the line passing through (4, 12) and (-7, -3) be written in standard form?

Yes. Starting from the point-slope form: $y - 12 = (15/11)(x - 4)$. Multiply both sides by 11 to clear fractions: $11(y - 12) = 15(x - 4)$. Simplify to get $11y - 132 = 15x - 60$, or rearranged as $15x - 11y = -72$.

Additional Resources

A Line That Passes Through (4,12) and (-7,-3): An Analytical Perspective

Understanding the characteristics and equations of lines passing through specific points forms a fundamental part of coordinate geometry. When a line is known to pass through two distinct points, such as $(4, 12)$ and $(-7, -3)$, it opens avenues for exploring its slope, equation, and various properties. This article delves into a comprehensive analysis of such a line, offering insights into its mathematical underpinnings, derivations, and broader applications.

Introduction to Coordinate Geometry and the Significance of Points

Coordinate geometry, also known as analytic geometry, bridges algebra and geometry by representing geometric figures through algebraic equations in a coordinate plane. Points are designated by ordered pairs (x, y) , where 'x' indicates the position along the horizontal axis and 'y' along the vertical axis.

Knowing two points on a line is instrumental because:

- It allows the calculation of the line's slope, indicating its steepness and direction.
- It facilitates the derivation of the line's equation in various forms.
- It enables the analysis of the line's behavior, intersections, and applications in real-world contexts.

The points under consideration, $(4, 12)$ and $(-7, -3)$, are chosen to exemplify how to systematically analyze such a line.

Calculating the Slope of the Line

Definition of Slope

The slope (m) of a line describes how much the y-coordinate changes for a unit change in the x-coordinate. It quantifies the line's inclination and is calculated as:

$$m = \frac{\Delta y}{\Delta x} = \frac{y_2 - y_1}{x_2 - x_1}$$

where:

- (x_1, y_1) and (x_2, y_2) are the two known points.

Applying the Formula

Given:

$$(x_1, y_1) = (4, 12)$$

$$\begin{aligned} & \backslash \\ & \backslash \\ (x_2, y_2) &= (-7, -3) \\ & \backslash \end{aligned}$$

Calculating the differences:

$$\begin{aligned} & \backslash \\ \Delta y &= y_2 - y_1 = -3 - 12 = -15 \\ & \backslash \\ & \backslash \\ \Delta x &= x_2 - x_1 = -7 - 4 = -11 \\ & \backslash \end{aligned}$$

Thus,

$$\begin{aligned} & \backslash \\ m &= \frac{-15}{-11} = \frac{15}{11} \\ & \backslash \end{aligned}$$

Result: The slope of the line passing through (4, 12) and (-7, -3) is $\frac{15}{11}$.

Interpretation of the Slope

A positive slope of $\frac{15}{11}$ indicates the line ascends from left to right, albeit with a gentle incline, since the ratio is less than 1. This slope encapsulates the rate at which y increases relative to x.

Deriving the Equation of the Line

Having established the slope, the next step involves formulating the line's equation. Multiple forms exist, but the point-slope and slope-intercept forms are most common.

Point-Slope Form

The point-slope form is:

$$\begin{aligned} & \backslash \\ y - y_1 &= m (x - x_1) \\ & \backslash \end{aligned}$$

Choosing the point (4, 12):

$$y - 12 = \frac{15}{11} (x - 4)$$

This form is useful because it directly relates the line's slope to a specific point on the line.

Slope-Intercept Form

Rearranging the point-slope form to solve for y:

$$y = m (x - 4) + 12$$

$$y = \frac{15}{11} (x - 4) + 12$$

Expanding:

$$y = \frac{15}{11}x - \frac{15}{11} \times 4 + 12$$

$$y = \frac{15}{11}x - \frac{60}{11} + 12$$

Express 12 as $\frac{132}{11}$ to combine:

$$y = \frac{15}{11}x + \left(-\frac{60}{11} + \frac{132}{11} \right)$$

$$y = \frac{15}{11}x + \frac{72}{11}$$

Final Equation in Slope-Intercept Form:

$$\boxed{y = \frac{15}{11}x + \frac{72}{11}}$$

This equation succinctly captures the line's behavior, with the slope $\frac{15}{11}$ and y-intercept $\frac{72}{11}$.

Analyzing the Line's Properties

1. Intercepts

- Y-intercept: The point where the line crosses the y-axis occurs at $(x=0)$. Substituting into the equation:

$$y = \frac{15}{11} \times 0 + \frac{72}{11} = \frac{72}{11} \approx 6.545$$

- X-intercept: The point where the line crosses the x-axis occurs at $(y=0)$. Solving for (x) :

$$0 = \frac{15}{11}x + \frac{72}{11}$$

Multiply through by 11 to clear denominators:

$$0 = 15x + 72$$

$$15x = -72$$

$$x = -\frac{72}{15} = -\frac{24}{5} = -4.8$$

Summary:

Intercept	Coordinates	Approximate Value
Y-intercept	$(0, \frac{72}{11})$	$(0, 6.545)$
X-intercept	$(-\frac{24}{5}, 0)$	$(-4.8, 0)$

2. Slope Significance

The positive slope implies the line rises as it moves from left to right. Its ratio $(\frac{15}{11})$ indicates that for every 11 units increase in x, y increases by 15 units.

3. Length of Segment Between Given Points

Using the distance formula:

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} = \sqrt{(-7 - 4)^2 + (-3 - 12)^2}$$

$$d = \sqrt{(-11)^2 + (-15)^2} = \sqrt{121 + 225} = \sqrt{346} \approx 18.6$$

This length measures the segment between the two known points, highlighting the span of data and the line's coverage.

Broader Applications and Contextual Significance

Understanding the line passing through these points offers more than pure mathematical insight; it extends into practical applications across various fields.

1. Engineering and Physics

- Trajectory Analysis: The line can represent the path of an object, with the slope indicating velocity components.
- Structural Design: Engineers use such lines to model stress lines or load distributions.

2. Economics and Business

- Cost-Benefit Models: Linear models often assume such lines to predict revenue, costs, or profit margins based on variables.
- Trend Analysis: The slope can indicate growth rates or decline trends.

3. Computer Graphics and Visualization

- Line equations underpin rendering algorithms, collision detection, and geometric transformations.

4. Educational Significance

- Demonstrates core concepts like slope, intercepts, and equations, reinforcing foundational knowledge in mathematics curricula.

Extensions and Advanced Considerations

The basic analysis provides a foundation, but several advanced topics can deepen understanding.

1. Parallel and Perpendicular Lines

- Parallel lines share the same slope ($\frac{15}{11}$). Any line passing through a different point with this slope remains parallel.
- Perpendicular lines have slopes that are negative reciprocals: $-\frac{11}{15}$. For example, a line with this slope passing through a point on the original line would be perpendicular.

2. Line in Different Forms

- Standard form: Rearranged as $(Ax + By + C = 0)$.

Starting from the slope-intercept form:

$$y = \frac{15}{11}x + \frac{72}{11}$$

Multiply through by 11:

$$11y = 15x + 72$$

Rewrite as:

$$15x - 11y + 72 = 0$$

- Parametric equations: Useful for modeling motion or dynamic systems.

3. Line Equations in Higher Dimensions

While this analysis focuses on 2D,

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