A Triangle Has Vertices At (-1,5), (4,2), And (1,2).

A Triangle Has Vertices At (-1,5), (4,2), And (1,2). Understanding the properties and characteristics of a triangle based on its vertices is fundamental in geometry. In this article, we explore the triangle with vertices at these specific coordinates, delving into its area, perimeter, type, centroid, and other important geometric features. Whether you're a student, educator, or math enthusiast, this comprehensive guide aims to clarify how to analyze and interpret such a triangle comprehensively.

Introduction to Coordinates and Triangles

Before diving into the specific triangle with vertices at (-1,5), (4,2), and (1,2), it is essential to understand the basics of coordinate geometry and how triangles are represented within this framework.

What Are Coordinates?

- Coordinates are pairs of numbers that define a point's position in the Cartesian plane.
- The first number represents the horizontal position (x-coordinate).
- The second number indicates the vertical position (y-coordinate).

Representing Triangles in the Coordinate Plane

- A triangle can be plotted using three points, called vertices.
- The shape and size depend on the positions of these vertices.
- Analyzing triangles in the coordinate plane allows for precise calculation of area, perimeter, and other properties using algebraic formulas.

The Vertices of the Triangle

Let's examine the specific vertices of the triangle:

- Vertex A: (-1, 5)
- Vertex B: (4, 2)
- Vertex C: (1, 2)

Plotting these points on a coordinate plane provides the initial visual understanding of the triangle's shape and orientation.

Visual Analysis

- Point A is located left and above the other points.
- Points B and C are horizontally aligned at $y=2$, forming a base.
- The triangle appears to be scalene, with no sides necessarily equal, but this needs verification.

Calculating Side Lengths

To analyze the triangle comprehensively, the first step is to compute the lengths of its sides using the distance formula:

$$ d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} $$

Side AB

- Coordinates: A(-1,5), B(4,2)
- Calculation:

$$
AB = \sqrt{(4 - (-1))^2 + (2 - 5)^2} = \sqrt{(5)^2 + (-3)^2} = \sqrt{25 + 9} = \sqrt{34} \approx 5.83
$$

Side BC

- Coordinates: B(4,2), C(1,2)
- Calculation:

$$
BC = \sqrt{(1 - 4)^2 + (2 - 2)^2} = \sqrt{(-3)^2 + 0^2} = \sqrt{9} = 3
$$

Side AC

- Coordinates: A(-1,5), C(1,2)
- Calculation:

$$
AC = \sqrt{(1 - (-1))^2 + (2 - 5)^2} = \sqrt{(2)^2 + (-3)^2} = \sqrt{4 + 9} = \sqrt{13} \approx 3.61
$$

Summary of side lengths:
- AB ≈ 5.83 units
- BC = 3 units
- AC ≈ 3.61 units

This confirms the triangle is scalene because all sides are of different lengths.

Calculating the Area of the Triangle

The area is a crucial property, and it can be computed using the coordinate geometry formula:

$$
\text{Area} = \frac{1}{2} |x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)|
$$

Substituting the vertex coordinates:

- $(x_1, y_1) = (-1, 5)$
- $(x_2, y_2) = (4, 2)$
- $(x_3, y_3) = (1, 2)$

Calculation:

$$
\begin{aligned}
\text{Area} &= \frac{1}{2} |(-1)(2 - 2) + 4(2 - 5) + 1(5 - 2)| \\
&= \frac{1}{2} |(-1)(0) + 4(-3) + 1(3)| \\
&= \frac{1}{2} |0 - 12 + 3| \\
&= \frac{1}{2} | -9 | = \frac{1}{2} \times 9 = 4.5
\end{aligned}
$$

Result:

- The area of the triangle is 4.5 square units.

Perimeter and Semiperimeter

Knowing the side lengths, the perimeter is straightforward:

$$
P = AB + BC + AC \approx 5.83 + 3 + 3.61 = 12.44
$$

The semi-perimeter, often used in Heron's formula, is:

$$
s = \frac{P}{2} \approx \frac{12.44}{2} = 6.22
$$

Classifying the Triangle

Based on side lengths:

- Scalene: No sides are equal.
- Acute, Right, or Obtuse?

To determine whether the triangle is acute, right, or obtuse, compare the squares of the sides:

$$
AB^2 \approx 34, \quad BC^2 = 9, \quad AC^2 \approx 13
$$

Order the squares:

$$
AC^2 \approx 13, \quad BC^2 = 9, \quad AB^2 \approx 34

\]

Check the Pythagorean theorem:

- For a right triangle: The sum of the squares of the two shorter sides equals the square of the longest side.

Test:

$$
AC^2 + BC^2 \approx 13 + 9 = 22 \neq 34
$$

Since $22 < 34$, the triangle is obtuse, with the obtuse angle opposite side AB.

Finding the Centroid

The centroid (the point where medians intersect) can be found by averaging the x and y coordinates:

$$
\text{Centroid} (G) = \left(\frac{x_1 + x_2 + x_3}{3}, \frac{y_1 + y_2 + y_3}{3} \right)
$$

Calculations:

$$
G_x = \frac{-1 + 4 + 1}{3} = \frac{4}{3} \approx 1.33
$$
$$
G_y = \frac{5 + 2 + 2}{3} = \frac{9}{3} = 3
$$

Centroid coordinates: (1.33, 3)

Other Key Properties

Circumcenter and Circumcircle

- The circumcenter is the point equidistant from all vertices.
- Finding it involves solving perpendicular bisectors of sides.
- For this triangle, the circumcenter lies approximately at a specific point, which can be computed algebraically or graphically.

Incenter and Incircle

- The incenter is the intersection of angle bisectors.
- It is the center of the inscribed circle touching all sides.
- Coordinates can be calculated using the formula involving side lengths and vertices.

Orthocenter

- The intersection point of altitudes.
- Its position varies based on the triangle's shape.

Applications of Coordinate Geometry in Triangle Analysis

Understanding how to analyze a triangle with given vertices using coordinate geometry has numerous practical applications:

- Engineering: Designing structures where precise measurements are critical.
- Computer Graphics: Rendering shapes based on coordinate data.
- Navigation and GIS: Calculating areas and distances on maps.
- Education: Teaching fundamental concepts of geometry and algebra.

Step-by-Step Summary for Analyzing a Triangle with Given Vertices

1. Plot the vertices on the coordinate plane for visual understanding.
2. Calculate side lengths using the distance formula.
3. Determine the area with the coordinate geometry formula.

4. Compute the perimeter by summing side lengths.
5. Classify the triangle based on side lengths and angles.
6. Find key centers such as centroid, circumcenter, and incenter.
7. Interpret properties like angles, type, and special points.

Conclusion

Analyzing a triangle with vertices at (-1,5), (4,2), and (1,2) demonstrates the power of coordinate geometry in understanding geometric shapes. From calculating side lengths and area to classifying the triangle as scalene and obtuse, each step reveals insights

Frequently Asked Questions

What is the length of the side between the points (-1, 5) and (4, 2)?

Using the distance formula, the length is $\sqrt{[(4 - (-1))^2 + (2 - 5)^2]} = \sqrt{[(5)^2 + (-3)^2]} = \sqrt{[25 + 9]} = \sqrt{34} \approx 5.83$.

What is the length of the side between points (4, 2) and (1, 2)?

Since both points have the same y-coordinate, the length is $|4 - 1| = 3$ units.

What is the length of the side between points (-1, 5) and (1, 2)?

Using the distance formula, $\sqrt{[(1 - (-1))^2 + (2 - 5)^2]} = \sqrt{[(2)^2 + (-3)^2]} = \sqrt{[4 + 9]} = \sqrt{13} \approx 3.61$.

What type of triangle is formed by the points (-1, 5), (4, 2), and (1, 2)?

Since two sides are of different lengths and no sides are equal, it is a scalene triangle.

Is the triangle with vertices at (-1, 5), (4, 2), and (1, 2) a right triangle?

No, because the squares of the side lengths are 34, 9, and 13, and they do not satisfy the Pythagorean theorem ($34 \neq 9 + 13$).

What is the area of the triangle with vertices at (-1, 5), (4, 2), and (1, 2)?

Using the shoelace formula, the area is $0.5 \times |(-1)(2 - 2) + 4(2 - 5) + 1(5 - 2)| = 0.5 \times |0 + 4(-3) + 1(3)| = 0.5 \times |0 - 12 + 3| = 0.5 \times 9 = 4.5$ square units.

What are the coordinates of the centroid of the triangle?

The centroid is at the average of the x-coordinates and y-coordinates: ((-1 + 4 + 1)/3, (5 + 2 + 2)/3) = (4/3, 9/3) = (1.33, 3).

Additional Resources

A Triangle Has Vertices At (-1,5), (4,2), And (1,2).

Understanding the geometric properties and characteristics of a triangle defined by specific vertices is fundamental in the field of coordinate geometry. The triangle with vertices at (-1, 5), (4, 2), and (1, 2) offers a compelling case study for exploring various mathematical concepts such as distance calculations, slope analysis, area determination, and classification based on side lengths. This analysis aims to dissect each aspect systematically, providing insights into the triangle's structure, orientation, and properties.

Introduction to the Triangle’s Coordinates

At the core of any geometric analysis lies the precise understanding of the vertices' coordinates. The points in question are:

- Point A: (-1, 5)
- Point B: (4, 2)
- Point C: (1, 2)

These points are situated within the Cartesian plane, a two-dimensional coordinate system where each point is defined by an x-coordinate (horizontal axis) and a y-coordinate (vertical axis).

Significance of the Coordinates

- The x-coordinates (-1, 4, 1) provide horizontal positioning.
- The y-coordinates (5, 2, 2) indicate vertical placement.
- Variations in these coordinates influence the shape, size, and orientation of the triangle.

Calculating the Side Lengths

A fundamental step in understanding a triangle involves computing the lengths of its sides. These lengths are crucial for classifying the triangle and analyzing its properties.

Distance Formula

The distance between two points (x_1, y_1) and (x_2, y_2) is given by the Euclidean distance formula:

$$
d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}
$$

Using this formula, we calculate the three sides:

Side AB: between (-1, 5) and (4, 2)

$$
AB = \sqrt{(4 - (-1))^2 + (2 - 5)^2} = \sqrt{(5)^2 + (-3)^2} = \sqrt{25 + 9} = \sqrt{34} \approx 5.83
$$

Side BC: between (4, 2) and (1, 2)

$$
BC = \sqrt{(1 - 4)^2 + (2 - 2)^2} = \sqrt{(-3)^2 + 0^2} = \sqrt{9 + 0} = 3
$$

Side CA: between (1, 2) and (-1, 5)

\[

CA = \sqrt{(-1 - 1)^2 + (5 - 2)^2} = \sqrt{(-2)^2 + 3^2} = \sqrt{4 + 9} = \sqrt{13} \approx 3.61
\]

Summary of side lengths:

- AB ≈ 5.83 units
- BC = 3 units
- CA ≈ 3.61 units

Analyzing the Triangle’s Classification

Based on the side lengths, the triangle can be classified in multiple ways:

1. By Side Lengths

- Scalene Triangle: All sides have different lengths (which is the case here: approximately 5.83, 3, and 3.61).
- Isosceles Triangle: Two sides are equal in length (not applicable here).
- Equilateral Triangle: All sides are equal (not applicable here).

Thus, the triangle is scalene.

2. By Angles

To classify by angles, we examine the squares of the sides and compare them, based on the Pythagorean theorem:

- If \(c^2 = a^2 + b^2 \), the triangle is right-angled.
- If \(c^2 > a^2 + b^2 \), the triangle is obtuse.
- If \(c^2 < a^2 + b^2 \), the triangle is acute.

Let's identify the longest side:

- Longest side: AB ≈ 5.83 (which is approximately \(\sqrt{34}\))

Now, compare \(AB^2 \) with \(BC^2 + CA^2 \):

$$
AB^2 = 34
$$

$$
BC^2 = 9
$$

$$
CA^2 = 13
$$

$$
BC^2 + CA^2 = 9 + 13 = 22
$$

Since $AB^2 = 34 > 22$, the triangle is obtuse-angled.

Area Calculation

The area of the triangle provides insight into its size and spatial properties.

Method 1: Using the Shoelace Formula

Given the vertices:

- $(x_1, y_1) = (-1, 5)$
- $(x_2, y_2) = (4, 2)$
- $(x_3, y_3) = (1, 2)$

The shoelace formula states:

$$
\text{Area} = \frac{1}{2} |x_1 y_2 + x_2 y_3 + x_3 y_1 - y_1 x_2 - y_2 x_3 - y_3 x_1|
$$

Calculations:

$$
x_1 y_2 = -1 \times 2 = -2
$$

$$
x_2 y_3 = 4 \times 2 = 8
$$

$$
x_3 y_1 = 1 \times 5 = 5
$$

$$
y_1 x_2 = 5 \times 4 = 20
$$

$$
y_2 x_3 = 2 \times 1 = 2
$$

$$
y_3 x_1 = 2 \times (-1) = -2
$$

Sum of the first three terms:

$$
-2 + 8 + 5 = 11
$$

Sum of the second three:

$$
20 + 2 + (-2) = 20
$$

Area:

$$
\frac{1}{2} |11 - 20| = \frac{1}{2} \times 9 = 4.5
$$

Result: The area of the triangle is 4.5 square units.

Coordinate Geometry Insights and Geometric Properties

Beyond basic calculations, analyzing the triangle's orientation, centroid, and other properties provides a richer understanding.

1. Slope of Sides and Orientation

Calculating the slopes of each side reveals the triangle's orientation:

- Slope of AB:

$$
m_{AB} = \frac{2 - 5}{4 - (-1)} = \frac{-3}{5} = -0.6
$$

- Slope of BC:

$$
m_{BC} = \frac{2 - 2}{1 - 4} = \frac{0}{-3} = 0
$$

- Slope of CA:

$$
m_{CA} = \frac{5 - 2}{-1 - 1} = \frac{3}{-2} = -1.5
$$

Interpreting the slopes:

- Side AB has a negative slope, indicating a downward trend from left to right.
- Side BC has zero slope, indicating a horizontal line.
- Side CA has a steep negative slope.

This reveals that side BC is horizontal, lying along $y=2$, which is significant for understanding the triangle's positioning.

2. Midpoints and Centroid

Calculating the centroid, the intersection point of medians, provides insight into the triangle's balance point:

$$
G_x = \frac{x_1 + x_2 + x_3}{3} = \frac{-1 + 4 + 1}{3} = \frac{4}{3} \approx 1.33
$$

$$
G_y = \frac{y_1 + y_2 + y_3}{3} = \frac{5 + 2 + 2}{3} = \frac{9}{3} = 3
$$

Centroid G: approximately at (1.33, 3).

This point divides each median into a 2:1 ratio and acts as the triangle's center of mass.

Applications and Broader Significance

Understanding the properties of this specific triangle extends beyond theoretical mathematics into practical fields such as engineering, computer graphics, navigation, and urban planning.

1. Engineering and Structural Design

Knowing side lengths and angles helps in structural analysis, ensuring stability and optimal material usage.

2. Computer Graphics and CAD

Accurately modeling shapes relies on vertex coordinates, side lengths, and area calculations for rendering and design.

3. Navigation and Geographic Information Systems (GIS)

Coordinate geometry

A Triangle Has Vertices At 1 5 4 2 And 1 2

A Triangle Has Vertices At 1 5 4 2 And 1 2

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