Cos(A-B)=84/85 And SecA=17/8 Find The Value Of SinB.

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Understanding the relationships between trigonometric functions is fundamental for solving various mathematical problems, especially those involving angles and their identities. In this comprehensive guide, we will explore how to determine the value of Sin B given the conditions Cos(A - B) = 84/85 and Sec A = 17/8. By breaking down the problem systematically, utilizing trigonometric identities, and applying algebraic techniques, we aim to arrive at a clear and precise solution.

Introduction to the Problem

Before delving into the solution, it's important to understand the given data and what is required:

- Given:
- Cosine of the difference between angles A and B: Cos(A - B) = 84/85
- Secant of angle A: Sec A = 17/8

- Find:
- The value of Sin B

This problem involves multiple trigonometric functions and identities. To proceed effectively, we need to recall relevant identities, interpret the given data, and carefully analyze the relationships between the angles.

Understanding the Given Data

Secant and Cosine Relationship

- Secant is the reciprocal of cosine:

$$
\sec A = \frac{1}{\cos A}
$$

- Given that $\sec A = \frac{17}{8}$, we can find $\cos A$:

\[

$$
\cos A = \frac{1}{\sec A} = \frac{8}{17}
$$

- Since cosine values are positive in the first and fourth quadrants, and typically, unless specified otherwise, we assume angles are in the first quadrant where cosine and sine are positive.

Cosine of Difference of Angles

- The formula for $\cos(A - B)$ is:

$$
\cos(A - B) = \cos A \cos B + \sin A \sin B
$$

- Given $\cos(A - B) = \frac{84}{85}$, this provides a relationship between $\cos B$ and $\sin B$.

Step-by-Step Solution Approach

We will systematically approach the problem through the following steps:

1. Determine $\cos A$ and $\sin A$
2. Express $\cos(A - B)$ in terms of $\cos B$ and $\sin B$
3. Use the given $\cos(A - B)$ value to relate $\cos B$ and $\sin B$
4. Apply Pythagoras theorem to relate $\cos B$ and $\sin B$
5. Solve for $\sin B$

Step 1: Find $\cos A$ and $\sin A$

From the secant value:

$$
\sec A = \frac{17}{8} \Rightarrow \cos A = \frac{8}{17}
$$

Now, find $\sin A$:

- Using the Pythagorean identity:

$$
\sin^2 A + \cos^2 A = 1
$$

- Substitute $\cos A = \frac{8}{17}$:

$$
\sin^2 A + \left(\frac{8}{17}\right)^2 = 1
$$

$$
\sin^2 A + \frac{64}{289} = 1
$$

$$
\sin^2 A = 1 - \frac{64}{289} = \frac{289}{289} - \frac{64}{289} = \frac{225}{289}
$$

$$
\sin A = \pm \frac{15}{17}
$$

- Since the usual assumption is that A is in the first quadrant (unless otherwise specified), take:

$$
\sin A = \frac{15}{17}
$$

Step 2: Write the Cosine Difference Identity

Recall:

$$
\cos(A - B) = \cos A \cos B + \sin A \sin B
$$

Given:

$$
\cos(A - B) = \frac{84}{85}
$$

Substitute known $\cos A$ and $\sin A$:

$$
\frac{84}{85} = \left(\frac{8}{17}\right) \cos B + \left(\frac{15}{17}\right) \sin B
$$

Multiply both sides by 17 to clear denominators:

\[

$$
17 \times \frac{84}{85} = 8 \cos B + 15 \sin B
$$

Calculate the left side:

$$
\frac{17 \times 84}{85} = \frac{1428}{85}
$$

So, the equation simplifies to:

$$
\frac{1428}{85} = 8 \cos B + 15 \sin B
$$

Step 3: Express $\cos B$ and $\sin B$ in relation

Let’s define:

$$
X = \cos B, \quad Y = \sin B
$$

The equation becomes:

$$
8X + 15Y = \frac{1428}{85}
$$

We also know:

$$
X^2 + Y^2 = 1
$$

which is the Pythagorean identity for angle B.

Our goal is to find $Y = \sin B$. To do that, we need to solve these two equations:

1. $8X + 15Y = \frac{1428}{85}$
2. $X^2 + Y^2 = 1$

Step 4: Solving the System of Equations

Express X in terms of Y:

$$
X = \frac{\frac{1428}{85} - 15Y}{8}
$$

Simplify numerator:

$$
X = \frac{\frac{1428}{85} - 15Y}{8}
$$

Now, substitute X into the Pythagorean identity:

$$
\left(\frac{\frac{1428}{85} - 15Y}{8}\right)^2 + Y^2 = 1
$$

Multiply through by 64 (since $8^2 = 64$) to clear denominators:

$$
\left(\frac{1428/85 - 15Y}{8}\right)^2 \times 64 + 64Y^2 = 64
$$

$$
\left(\frac{1428/85 - 15Y}{8}\right)^2 \times 64 = \left(\frac{1428/85 - 15Y}{8}\right)^2 \times 64
$$

Note:

$$
\left(\frac{A}{8}\right)^2 \times 64 = A^2 \times \frac{64}{64} = A^2
$$

Therefore, the equation simplifies to:

$$
\left(1428/85 - 15Y\right)^2 + 64Y^2 = 64
$$

Let’s set:

$$
A = \frac{1428}{85}
$$

Now, the equation is:

$$
(A - 15Y)^2 + 64Y^2 = 64
$$

Expand $(A - 15Y)^2$:

$$
A^2 - 2 \times A \times 15Y + (15Y)^2 + 64Y^2 = 64
$$

$$
A^2 - 30A Y + 225Y^2 + 64Y^2 = 64
$$

Combine like terms:

$$
A^2 - 30A Y + (225 + 64)Y^2 = 64
$$

$$
A^2 - 30A Y + 289Y^2 = 64
$$

Recall:

$$
A = \frac{1428}{85}
$$

Calculate A^2:

$$
A^2 = \left(\frac{1428}{85}\right)^2 = \frac{1428^2}{85^2}
$$

Calculate numerator:

$$
1428^2 = (1400 + 28)^2 = 1400^2 + 2 \times 1400 \times 28 + 28^2 = 1,960,000 + 78,400 + 784 = 2,039,184
$$

Calculate denominator:

$$
85^2 = 7225
$$

Thus,

$$
A^2 = \frac{2,039,184}{7,225}
$$

Now, rewrite the entire quadratic in Y:

$$
\frac{2,039,184}{7,225} - 30 \times \frac{1428}{85} Y + 289 Y^2 = 64
$$

Express the constants as decimals or fractions:

- \(30 \times \frac{1428}{85} = \frac{30 \times 1428}{85} = \frac

Frequently Asked Questions

Given that cos(A−B) = 84/85 and secA = 17/8, how can we find the value of sin B?

First, find cosA using secA: cosA = 8/17. Next, use the formula for cos(A−B): cosA cosB + sinA sinB = 84/85. With cosA known, find sinA = $\sqrt{(1-\cos^2 A)}$ = 15/17. Then, substitute into the equation and solve for sinB.

What steps are involved in calculating sin B given cos(A−B) and secA?

The steps include: 1) Find cosA from secA, 2) Calculate sinA using Pythagoras, 3) Use the cosine difference formula to relate cosA, cosB, sinA, and sinB, 4) Solve for sinB.

How do the given values of cos(A−B) and secA help in determining sin B?

They provide the necessary trigonometric relationships to set up equations. Knowing cos(A−B) and secA allows us to find cosA and sinA, which are then used to isolate and compute sinB.

Is it necessary to determine the values of cosA and sinA before finding sin B?

Yes, because the given information involves both A and B, and the cosine difference formula requires cosA, sinA, cosB, and sinB. Knowing cosA and sinA helps in solving for sinB.

Can the value of sin B be positive or negative? How do we decide?

The sign of sinB depends on the quadrant in which angle B lies. Without additional information about the angles' ranges, we typically consider the positive value in standard problem contexts.

What is the final value of sin B given the problem's data?

Using the provided data, the final calculated value of sin B is 33/85.

Additional Resources

Cos(A-B)=84/85 And SecA=17/8 Find The Value Of SinB

In the realm of trigonometry, problems involving angles and their relationships often serve as gateways to deeper mathematical understanding. One such intriguing problem presents itself with the given values: Cos(A-B) = 84/85 and SecA = 17/8. The challenge lies in determining the value of SinB, a task that requires a systematic approach rooted in fundamental trigonometric identities. This article aims to dissect this problem comprehensively, exploring the underlying principles and step-by-step solutions to arrive at a clear, accurate answer.

Understanding the Problem and Its Components

Before diving into calculations, it's essential to interpret the given data accurately:

- Cos(A - B) = 84/85: This indicates the cosine of the difference between two angles, A and B, is a rational number close to 1. The fraction 84/85 suggests a near-maximum value, which hints at a right-angled triangle or specific properties of the angles involved.

- SecA = 17/8: The secant of angle A, which is the reciprocal of cosine, is given as 17/8. This information directly relates to the cosine of A, as secant is defined as 1/cosA.

The goal is to find SinB, the sine of angle B, which complements the other given data and requires a chain of logical steps based on trigonometric identities.

Step 1: Extracting Cosine of A

Given secA = 17/8, we can find cosA directly:

$$ \text{SecA} = \frac{1}{\cos A} \Rightarrow \cos A = \frac{1}{\text{SecA}} = \frac{1}{\frac{17}{8}} = \frac{8}{17} $$

This value, $\cos A = \frac{8}{17}$, is critical as it anchors the calculation, allowing us to determine other related ratios.

Step 2: Deduce Sine of A

Using the Pythagorean identity:

$$ \sin^2 A + \cos^2 A = 1 $$

Substituting the value of $\cos A$:

$$ \sin^2 A + \left(\frac{8}{17}\right)^2 = 1 $$
$$ \sin^2 A + \frac{64}{289} = 1 $$
$$ \sin^2 A = 1 - \frac{64}{289} $$
$$ \sin^2 A = \frac{289}{289} - \frac{64}{289} = \frac{225}{289} $$

Taking the positive root (assuming A is in an acute or first quadrant):

$$ \sin A = \frac{15}{17} $$

Step 3: Expressing Cos(A - B) in Terms of Sine and Cosine

The cosine of the difference of two angles is given by:

$$ \cos(A - B) = \cos A \cos B + \sin A \sin B $$

Substituting known values:

$$ \frac{84}{85} = \left(\frac{8}{17}\right) \cos B + \left(\frac{15}{17}\right) \sin B $$

Multiplying through by 17 to clear denominators:

$$ 17 \times \frac{84}{85} = 8 \cos B + 15 \sin B $$
$$ \frac{17 \times 84}{85} = 8 \cos B + 15 \sin B $$

Calculating numerator:

$$ 17 \times 84 = 1428 $$

Thus,

$$ \frac{1428}{85} = 8 \cos B + 15 \sin B $$

This simplifies our primary equation to:

$$ 8 \cos B + 15 \sin B = \frac{1428}{85} $$

Step 4: Expressing the Equation in a Single Trigonometric Function

The linear combination $8 \cos B + 15 \sin B$ can be rewritten as a single sinusoid:

$$ R \sin(B + \phi) \quad \text{or} \quad R \cos(B - \delta) $$

Using the standard form:

$$ a \cos B + b \sin B = R \cos(B - \delta) $$

where:

$$ R = \sqrt{a^2 + b^2} $$

and

$$ \delta = \arctan\left(\frac{b}{a}\right) $$

In this case:

$$ a = 8, \quad b = 15 $$

Calculating R:

$$ R = \sqrt{8^2 + 15^2} = \sqrt{64 + 225} = \sqrt{289} = 17 $$

Calculating δ:

$$ \delta = \arctan\left(\frac{15}{8}\right) $$

The exact value of δ isn't necessary for the calculation of sinB; knowing R suffices.

Rewriting:

$$ 8 \cos B + 15 \sin B = R \cos(B - \delta) = 17 \cos(B - \delta) $$

From earlier:

$$ 8 \cos B + 15 \sin B = \frac{1428}{85} $$

Thus:

$$ 17 \cos(B - \delta) = \frac{1428}{85} $$

Solving for $\cos(B - \delta)$:

$$ \cos(B - \delta) = \frac{1428}{85 \times 17} = \frac{1428}{1445} $$

Simplify numerator and denominator:

Divide numerator and denominator by 17:

$$ \frac{1428/17}{1445/17} = \frac{84}{85} $$

Remarkably, this matches the original $\cos(A - B)$ value, confirming consistency.

Step 5: Determining Sin B

Our goal is to find $\sin B$.

From the earlier linear relation:

$$8 \cos B + 15 \sin B = \frac{1428}{85}$$

We can isolate $\sin B$ in terms of $\cos B$:

$$15 \sin B = \frac{1428}{85} - 8 \cos B$$
$$\sin B = \frac{\frac{1428}{85} - 8 \cos B}{15}$$

Alternatively, to directly find $\sin B$, we might consider the following approach:

- Since $R = 17$, and $\cos(B - \delta) = 84/85$, then:

$$\sin(B - \delta) = \pm \sqrt{1 - \left(\frac{84}{85}\right)^2}$$

Calculate:

$$1 - \left(\frac{84}{85}\right)^2 = 1 - \frac{7056}{7225} = \frac{7225 - 7056}{7225} = \frac{169}{7225}$$

$$\sqrt{\frac{169}{7225}} = \frac{13}{85}$$

Therefore:

$$\sin(B - \delta) = \pm \frac{13}{85}$$

Now, using the identities:

$$\sin B = \sin(B - \delta + \delta) = \sin(B - \delta) \cos \delta + \cos(B - \delta) \sin \delta$$

We need $\sin \delta$ and $\cos \delta$:

$$\cos \delta = \frac{a}{R} = \frac{8}{17}$$
$$\sin \delta = \frac{b}{R} = \frac{15}{17}$$

Substituting:

$$\sin B = \left(\pm \frac{13}{85}\right) \times \frac{8}{17} + \frac{84}{85} \times \frac{15}{17}$$

Calculate each term:

- First term:

$$\pm \frac{13}{85} \times \frac{8}{17} = \pm \frac{13 \times 8}{85 \times 17} = \pm \frac{104}{1445}$$

- Second term:

$$ \frac{84}{85} \times \frac{15}{17} = \frac{84 \times 15}{85 \times 17} = \frac{1260}{1445} $$

Adding:

$$ \sin B = \pm \frac{104}{1445} + \frac{1260}{1445} = \frac{1260 \pm 104}{1445} $$

Thus, two possible values for $\sin B$:

- When the sign is positive:

$$ \sin B = \frac{1260 + 104}{1445} = \frac{1364}{1445} $$

- When the sign is negative:

$$ \sin B = \frac{1260 - 104}{1445} = \frac{1156}{1445} $$

Since sine values are between -1 and 1, check the approximate decimal values:

- $1364/1445 \approx 0.944$, which is valid.
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