Cot A. Cos (30 - A) - Sin (30 - A) = 3/ 2 Cosec A

Cot A. Cos (30 - A) - Sin (30 - A) = 3/ 2 Cosec A

Understanding and simplifying trigonometric expressions is fundamental in mathematics, especially in solving complex equations involving angles. Among these, the identity and simplification of expressions like Cot A. Cos (30 - A) - Sin (30 - A) = 3/ 2 Cosec A play a crucial role in advancing problem-solving skills in trigonometry. This article delves deeply into this expression, providing a comprehensive explanation, step-by-step simplification, and insights into related concepts.

Introduction to the Trigonometric Expression

The expression under consideration is:

Cot A. Cos (30 - A) - Sin (30 - A) = 3/ 2 Cosec A

This involves multiple trigonometric functions of angle A, specifically cotangent, cosine, sine, and cosecant. To understand and verify this identity, it is essential to revisit the fundamental definitions and properties of these functions.

Key Functions:

- Cot A: Cotangent of angle A = cos A / sin A
- Cos (30 - A): Cosine of the difference between 30° and A
- Sin (30 - A): Sine of the difference between 30° and A

- Cosec A: Cosecant of A = 1 / sin A

Fundamental Trigonometric Identities

Before attempting to simplify the expression, let's review some essential identities:

1. Difference Formulas

- Cos (x - y): cos x cos y + sin x sin y
- Sin (x - y): sin x cos y - cos x sin y

2. Pythagorean Identities

- $\sin^2 A + \cos^2 A = 1$
- csc A = 1 / sin A
- sec A = 1 / cos A
- cot A = cos A / sin A

3. Special Angles

- cos 30° = $\sqrt{3}/2$
- sin 30° = 1/2

Step-by-Step Simplification

Let's approach the expression systematically:

Step 1: Write the original expression

\[\text{Expression} = \cot A \cdot \cos (30 - A) - \sin (30 - A) \]

Step 2: Express cot A in terms of sin and cos

\[\cot A = \frac{\cos A}{\sin A} \]

Step 3: Expand cos(30 - A) and sin(30 - A) using difference formulas

\[
\begin{aligned}
\cos (30 - A) &= \cos 30^\circ \cos A + \sin 30^\circ \sin A \\
&= \left(\frac{\sqrt{3}}{2}\right) \cos A + \left(\frac{1}{2}\right) \sin A \\
\sin (30 - A) &= \sin 30^\circ \cos A - \cos 30^\circ \sin A \\
&= \left(\frac{1}{2}\right) \cos A - \left(\frac{\sqrt{3}}{2}\right) \sin A
\end{aligned}
\]

Step 4: Substitute these into the expression

\[
\begin{aligned}
\text{Expression} &= \left(\frac{\cos A}{\sin A}\right) \left[\left(\frac{\sqrt{3}}{2}\right) \cos A + \left(\frac{1}{2}\right) \sin A \right] - \left[\left(\frac{1}{2}\right) \cos A - \left(\frac{\sqrt{3}}{2}\right) \sin A \right]
\end{aligned}

\]

Step 5: Simplify each part

\[

\begin{aligned}

&= \frac{\cos A}{\sin A} \times \left(\frac{\sqrt{3}}{2} \cos A + \frac{1}{2} \sin A \right) - \frac{1}{2} \cos A + \frac{\sqrt{3}}{2} \sin A

\end{aligned}

\]

Break down the first term:

\[

= \frac{\cos A}{\sin A} \times \frac{\sqrt{3}}{2} \cos A + \frac{\cos A}{\sin A} \times \frac{1}{2} \sin A

\]

which simplifies to:

\[

= \frac{\sqrt{3}}{2} \frac{\cos^2 A}{\sin A} + \frac{1}{2} \frac{\cos A \sin A}{\sin A}

\]

Further simplification:

\[

= \frac{\sqrt{3}}{2} \frac{\cos^2 A}{\sin A} + \frac{1}{2} \cos A

\]

Now, the entire expression becomes:

$$
\boxed{
\frac{\sqrt{3}}{2} \frac{\cos^2 A}{\sin A} + \frac{1}{2} \cos A - \frac{1}{2} \cos A + \frac{\sqrt{3}}{2} \sin A
}
$$

Observe that $\frac{1}{2} \cos A - \frac{1}{2} \cos A = 0$, so they cancel out.

Thus, the simplified form is:

$$
\frac{\sqrt{3}}{2} \frac{\cos^2 A}{\sin A} + \frac{\sqrt{3}}{2} \sin A
$$

Step 6: Factor common terms

Factor $\frac{\sqrt{3}}{2}$:

$$
\frac{\sqrt{3}}{2} \left(\frac{\cos^2 A}{\sin A} + \sin A \right)
$$

Express $\frac{\cos^2 A}{\sin A}$ as:

$$
\frac{\cos^2 A}{\sin A} = \frac{\cos^2 A}{\sin A}
$$

Recall that $\cos^2 A = 1 - \sin^2 A$.

So,

$$
\frac{\cos^2 A}{\sin A} = \frac{1 - \sin^2 A}{\sin A} = \frac{1}{\sin A} - \sin A
$$

Now, substitute back:

$$
\frac{\sqrt{3}}{2} \left(\frac{1}{\sin A} - \sin A + \sin A \right) = \frac{\sqrt{3}}{2} \times \frac{1}{\sin A}
$$

The $- \sin A + \sin A$ cancels out, leaving:

$$
\boxed{
\frac{\sqrt{3}}{2} \times \frac{1}{\sin A}
}
$$

Step 7: Express in terms of cosecant

Recall that:

$$
\csc A = \frac{1}{\sin A}
$$

Therefore, the expression simplifies to:

$$
\frac{\sqrt{3}}{2} \csc A
$$

Final Verification of the Identity

Recall the original statement:

$$
\text{Expression} = \cot A \cdot \cos (30 - A) - \sin (30 - A) = \frac{3}{2} \csc A
$$

From our derivation, the simplified form is:

$$
\frac{\sqrt{3}}{2} \csc A
$$

Notice that:

$$
\frac{\sqrt{3}}{2} = \frac{3}{2} \times \frac{1}{\sqrt{3}} \neq \frac{3}{2}
$$

But the original expression asserts the equality:

$$
\cot A \cdot \cos (30 - A) - \sin (30 - A) = \frac{3}{2} \csc A
$$

Our derivation yields:

$$
\frac{\sqrt{3}}{2} \csc A
$$

which implies that the initial identity holds true only under specific conditions or at specific angles where the equality is valid.

Alternatively, if the original problem is to prove the identity, then the derivation confirms that:

$$
\cot A \cdot \cos (30 - A) - \sin (30 - A) = \frac{\sqrt{3}}{2} \csc A
$$

which is consistent with the expressions derived.

Understanding the Significance of the Identity

This derivation demonstrates the importance of trigonometric identities and how they can be manipulated to simplify complex expressions. Such identities are crucial in various applications:

- Solving trigonometric equations
- Simplification in calculus and analytical geometry
- Signal processing and wave analysis
- Engineering problems involving angles and periodic functions

---

Related Concepts and Tips for Trigonometric Simplification

1

Frequently Asked Questions

What is the primary trigonometric identity involved in the equation Cot A · Cos (30° - A) - Sin (30° - A) = 3/2 Cosec A?

The primary identities involved are the definitions of cotangent, sine, cosine, and cosecant functions, along with the subtraction formulas for sine and cosine.

How can the equation Cot A · Cos (30° - A) - Sin (30° - A) = 3/2 Cosec A be simplified using standard identities?

By expressing cotangent as cos A/sin A, and applying angle difference formulas for sine and cosine, the equation can be simplified to relate A directly, often leading to an algebraic or trigonometric quadratic in terms of sin A or cos A.

What are the possible solutions for angle A in the given equation?

Solutions for A can be found by simplifying the equation to a solvable form, then determining A in the interval 0° to 180°, considering the domain restrictions of cot A and cosec A.

Is the equation valid for all A in the domain of the involved functions?

No, the equation is valid only where all functions are defined; specifically, A cannot be 0°, 180°, or angles where sin A = 0, as cosec A and cot A become undefined.

How does the angle 30° influence the structure of the equation?

The fixed angle 30° introduces specific sine and cosine values (sin 30° = 1/2, cos 30° = □3/2), which can be substituted to simplify the equation into algebraic form for A.

Can the equation be solved graphically, and if so, how?

Yes, by plotting both sides of the equation as functions of A and finding their intersection points within the valid domain, solutions can be approximated graphically.

What is the significance of the ratio 3/2 in this equation?

The ratio 3/2 acts as a constant on the right side of the equation, serving as a benchmark to compare the combined trigonometric expression on the left, aiding in determining possible A values.

Are there specific angles A for which the equation simplifies further?

Yes, for special angles where sine or cosine take known values (like 0°, 30°, 45°, 60°, 90°), the equation simplifies, which can help verify solutions or find specific A values.

What methods are recommended to solve this type of trigonometric equation?

Recommended methods include substitution of known identities, algebraic manipulation, use of angle difference formulas, and graphical or iterative approaches to approximate solutions.

What are common pitfalls when solving this equation?

Common pitfalls include ignoring domain restrictions, mishandling the angle difference formulas, and neglecting the undefined points where sin A or cos A are zero, leading to incorrect solutions.

Additional Resources

Understanding the Trigonometric Equation: $\text{Cot } A \cdot \text{Cos } (30° - A) - \text{Sin } (30° - A) = 3/2 \text{ Cosec } A$

In the realm of trigonometry, equations involving multiple functions and angles often present intriguing challenges that require a strategic approach to solve. One such equation is $\text{Cot } A \cdot \text{Cos } (30° - A) - \text{Sin } (30° - A) = 3/2 \text{ Cosec } A$. This equation combines cotangent, cosine, sine, and cosecant functions, with an angle A that is common to several terms, making it a compelling problem for students and enthusiasts alike. In this detailed guide, we'll break down the problem step-by-step, explore relevant identities, and provide a comprehensive pathway to solutions.

Understanding the Equation

Let's begin by examining the structure of the given equation:

$\text{Cot } A \cdot \text{Cos } (30° - A) - \text{Sin } (30° - A) = 3/2 \text{ Cosec } A$

Key components:

- Cot A: Cotangent of angle A
- Cos (30° - A): Cosine of the difference between 30° and A
- Sin (30° - A): Sine of the difference between 30° and A
- Cosec A: Cosecant of A

The goal is often to find the values of A that satisfy this equation within a certain domain (typically 0° to 180° or 0 to 2π radians), or to simplify the equation to relate these functions more directly.

Step 1: Recall and Use Basic Trigonometric Identities

Before attempting to manipulate the given equation, let's revisit some fundamental identities:

- Cot A: cot A = cos A / sin A
- Cosec A: cosec A = 1 / sin A
- Cos (□ - □): cos □ cos □ + sin □ sin □
- Sin (□ - □): sin □ cos □ - cos □ sin □

These identities serve as the building blocks for transformations and simplifications.

Step 2: Express All Functions in Terms of Sin A and Cos A

To simplify the equation, rewrite cot A and cosec A as ratios involving sin A and cos A:

- Cot A = cos A / sin A
- Cosec A = 1 / sin A

Similarly, expand Cos (30° - A) and Sin (30° - A):

- Cos (30° - A) = cos 30° cos A + sin 30° sin A
- Sin (30° - A) = sin 30° cos A - cos 30° sin A

Recall that:

- cos 30° = □3 / 2
- sin 30° = 1 / 2

Now, write these explicitly:

- Cos (30° - A) = (□3 / 2) cos A + (1 / 2) sin A

- Sin (30° - A) = (1 / 2) cos A - ($\sqrt{3}$ / 2) sin A

Step 3: Rewrite the Equation Using These Expressions

Substitute these into the original equation:

Cot A · Cos (30° - A) - Sin (30° - A) = 3/2 Cosec A

becomes

(cos A / sin A) [($\sqrt{3}$ / 2) cos A + (1 / 2) sin A] - [(1 / 2) cos A - ($\sqrt{3}$ / 2) sin A] = (3/2) (1 / sin A)

Now, express everything explicitly:

(cos A / sin A) (($\sqrt{3}$ / 2) cos A + (1 / 2) sin A) - ((1 / 2) cos A - ($\sqrt{3}$ / 2) sin A) = (3/2) (1 / sin A)

Step 4: Simplify the Left Side

First, handle the product:

(cos A / sin A) (($\sqrt{3}$ / 2) cos A + (1 / 2) sin A)

Distribute:

= (cos A / sin A) ($\sqrt{3}$ / 2) cos A + (cos A / sin A) (1 / 2) sin A

Simplify each term:

- First term:

$(\cos A / \sin A) (\sqrt{3} / 2) \cos A = (\sqrt{3} / 2) (\cos A \cos A) / \sin A = (\sqrt{3} / 2) \cos^2 A / \sin A$

- Second term:

$(\cos A / \sin A) (1 / 2) \sin A = (1 / 2) \cos A (\sin A / \sin A) = (1 / 2) \cos A$

Now, rewrite the entire left side:

$[(\sqrt{3} / 2) \cos^2 A / \sin A] + (1 / 2) \cos A - [(1 / 2) \cos A - (\sqrt{3} / 2) \sin A] = (3/2) (1 / \sin A)$

Note that we have a negative sign before the second bracket, so expand:

$= [(\sqrt{3} / 2) \cos^2 A / \sin A] + (1 / 2) \cos A - (1 / 2) \cos A + (\sqrt{3} / 2) \sin A$

Notice that (1 / 2) cos A cancels out:

$(1 / 2) \cos A - (1 / 2) \cos A = 0$

Thus, the left side simplifies to:

$(\sqrt{3} / 2) \cos^2 A / \sin A + (\sqrt{3} / 2) \sin A$

Step 5: Factor the Simplified Left Side

Expressed as:

$(\sqrt{3} / 2) [\cos^2 A / \sin A + \sin A]$

Combine into a single fraction:

$(\sqrt{3} / 2) [(\cos^2 A + \sin^2 A) / \sin A] = (\sqrt{3} / 2) [1 / \sin A]$

Note that $\cos^2 A + \sin^2 A = 1$, so the numerator simplifies to 1.

Thus, the entire left side reduces to:

$(\sqrt{3} / 2) (1 / \sin A)$

Step 6: Write the Equation in Simplified Form

Recall the right side:

$(3/2) (1 / \sin A)$

Now, our simplified equation is:

$(\sqrt{3} / 2) (1 / \sin A) = (3/2) (1 / \sin A)$

Since both sides share the factor $(1 / \sin A)$, and assuming $\sin A \neq 0$ (to avoid undefined expressions), we can divide both sides by $(1 / \sin A)$:

$\sqrt{3} / 2 = 3/2$

Step 7: Solve for the Variable

Now, solve for the constant:

$\sqrt{3} / 2 = 3/2$

Multiply both sides by 2:

$\sqrt{3} = 3$

Now, check whether this is valid:

$\sqrt{3} \approx 1.732$, but $3 \neq 1.732$

Therefore, the equality $\sqrt{3} = 3$ does not hold.

Implication:

Since the simplified step leads to a false statement, it indicates that the original equation has no solution under the assumptions made (specifically, that $\sin A \neq 0$).

Conclusion

Key Takeaways:

- The given equation simplifies to a statement that cannot be true unless certain conditions are met.
- The critical step involved expressing all functions in terms of sine and cosine and simplifying systematically.
- The final contradiction ($\sqrt{3} \neq 3$) indicates no solutions exist for the equation when considering standard real angles where the functions are defined.

Additional Insights and Tips

When approaching similar trigonometric equations, consider these strategies:

- Express all functions in terms of sine and cosine for easier algebraic manipulation.
- Use angle difference identities to expand composite functions.
- Simplify step-by-step, watching for cancellations or common factors.
- Check boundary conditions, such as where functions are undefined (e.g., sin A = 0 for cosecant or cotangent functions).
- Be attentive to the possibility of extraneous solutions or contradictions, as in this problem.

Final Remarks

The analysis of Cot A · Cos (30° - A) - Sin (30° - A) = 3/2 Cosec A demonstrates the power of trigonometric identities and algebraic manipulation. Although the equation leads to a contradiction, understanding each step enhances problem-solving skills and deepens comprehension of trigonometric functions.

By mastering these techniques, students can confidently approach complex and seemingly intimidating equations, turning potential obstacles into opportunities for learning and discovery.

Cot A Cos 30 A Sin 30 A 3 2 Cosec A

Cot A Cos 30 A Sin 30 A 3 2 Cosec A

Related Articles

- Solve For The Interior Of Angle, X.90130130130120%Xo

- Help Please, I Dont Know How To Solve Number 7
- 5 - X = 1/ 2 X + 4 What Is The Answer?

Back to Home