Differentiate Implicitly To Find D^2y/dx^2 5x^3-y^3=4

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Implicit differentiation is a powerful technique used when a function is defined implicitly rather than explicitly. In many problems involving relations between x and y, the goal is to find the derivatives of y with respect to x, particularly the second derivative $\frac{d^2 y}{dx^2}$. The problem at hand involves the relation:

$$
5x^3 - y^3 = 4
$$

which is an implicit function of x and y. Our task is to differentiate this relation twice with respect to x to find $\frac{d^2 y}{dx^2}$.

This process involves several steps:
- First, find the first derivative $\frac{dy}{dx}$ by differentiating implicitly.
- Then, differentiate $\frac{dy}{dx}$ with respect to x again, applying the chain rule, to obtain $\frac{d^2 y}{dx^2}$.

Let's explore this procedure step by step.

Understanding the Given Equation

Equation Overview

The relation:

$$
5x^3 - y^3 = 4
$$

connects x and y in an implicit manner. Since y is not explicitly expressed as a function of x, we cannot easily differentiate y with respect to x directly. Instead, implicit differentiation is used.

Goals

- Find the first derivative $\frac{dy}{dx}$.
- Use $\frac{dy}{dx}$ to find the second derivative $\frac{d^2 y}{dx^2}$.

Step 1: Differentiate Implicitly to Find $\frac{dy}{dx}$

Differentiate Both Sides

Start by differentiating both sides of the equation:

$$
5x^3 - y^3 = 4
$$

with respect to x. Remember that:

- $\frac{d}{dx}(x^n) = n x^{n-1}$
- When differentiating terms involving y, treat y as a function of x. Use the chain rule:

$$
\frac{d}{dx} y^n = n y^{n-1} \frac{dy}{dx}
$$

Applying this:

$$
\frac{d}{dx} (5x^3) - \frac{d}{dx}(y^3) = \frac{d}{dx}(4)
$$

which simplifies to:

$$
15x^2 - 3 y^2 \frac{dy}{dx} = 0
$$

since $\frac{d}{dx}(4) = 0$.

Solving for $\frac{dy}{dx}$

Rearranging:

$$
15x^2 = 3 y^2 \frac{dy}{dx}
$$

divide both sides by $3 y^2$:

$$
\frac{dy}{dx} = \frac{15 x^2}{3 y^2} = \frac{5 x^2}{ y^2}
$$

This is the first derivative:

$$
\boxed{
\frac{dy}{dx} = \frac{5 x^2}{ y^2}
}
$$

Step 2: Differentiate $\frac{dy}{dx}$ to Find $\frac{d^2 y}{dx^2}$

Differentiate $\frac{dy}{dx}$ with respect to x

Now, differentiate

$$
\frac{dy}{dx} = \frac{5 x^2}{ y^2}
$$

implicitly with respect to x. This involves applying the quotient rule or rewriting the expression to facilitate differentiation.

Let's rewrite:

$$
\frac{dy}{dx} = 5 x^2 y^{-2}
$$

Differentiate both sides:

$$
\frac{d^2 y}{dx^2} = \frac{d}{dx} \left(5 x^2 y^{-2} \right)
$$

Applying the product rule:

$$
\frac{d^2 y}{dx^2} = 5 \left[\frac{d}{dx} (x^2) \cdot y^{-2} + x^2 \cdot \frac{d}{dx} (y^{-2}) \right]
$$

Calculate each derivative:

- $\frac{d}{dx} (x^2) = 2x$
- $\frac{d}{dx} (y^{-2}) = -2 y^{-3} \frac{dy}{dx}$ (by chain rule)

Substitute back:

$$
\frac{d^2 y}{dx^2} = 5 \left[2x y^{-2} + x^2 (-2 y^{-3} \frac{dy}{dx}) \right]
$$

Simplify:

$$
\frac{d^2 y}{dx^2} = 10 x y^{-2} - 10 x^2 y^{-3} \frac{dy}{dx}
$$

\]

Recall that \(\frac{dy}{dx} = \frac{5 x^2}{ y^2}\), so substitute this into the expression:

\[
\frac{d^2 y}{dx^2} = 10 x y^{-2} - 10 x^2 y^{-3} \left(\frac{5 x^2}{ y^2} \right)
\]

Simplify the second term:

\[
- 10 x^2 y^{-3} \times \frac{5 x^2}{ y^2} = - 50 x^4 y^{-3} y^{-2} = - 50 x^4 y^{-5}
\]

Thus, the second derivative becomes:

\[
\boxed{
\frac{d^2 y}{dx^2} = 10 x y^{-2} - 50 x^4 y^{-5}
}
\]

Expressed with positive exponents:

\[
\frac{d^2 y}{dx^2} = \frac{10 x}{ y^{2}} - \frac{50 x^{4}}{ y^{5}}
\]

Summary of the Derivation Process

Key Steps Recap

- Start with the implicit relation $5x^3 - y^3 = 4$.
- Differentiate both sides with respect to x, applying the chain rule to the y-dependent term.
- Solve for $\frac{dy}{dx}$, obtaining $\frac{5 x^2}{ y^2}$.
- Differentiate $\frac{dy}{dx}$ again, rewriting as $5 x^2 y^{-2}$ to apply product and chain rules.
- Substitute $\frac{dy}{dx}$ into the second derivative expression to get the final formula.

Final Expression for $\frac{d^2 y}{dx^2}$

$$
\boxed{
\frac{d^2 y}{dx^2} = \frac{10 x}{ y^{2}} - \frac{50 x^{4}}{ y^{5}}
}
$$

This formula expresses the second derivative explicitly in terms of x and y. In practice, if a specific point (x, y) satisfying the original relation is known, the second derivative can be computed numerically.

Additional Considerations

Handling Special Cases

- If $y=0$, the derivatives involve division by zero; such cases need separate analysis.
- When solving for the second derivative, ensure that the values of x and y satisfy the original relation.

Applications of Implicit Differentiation

- Finding slopes and curvatures of implicitly defined curves.
- Analyzing the concavity and convexity of the relation.
- Computing derivatives in complex geometric or physical models where explicit forms are difficult.

Conclusion

Implicit differentiation provides a systematic approach to find derivatives of implicitly defined functions. For the relation $5x^3 - y^3 = 4$, the process involves differentiating stepwise, applying the chain rule, and simplifying expressions. The second derivative obtained helps analyze the curvature and behavior of the implicit curve, offering insights into its geometric properties. Mastery of implicit differentiation techniques is essential for advanced calculus and applications across physics, engineering, and mathematics.

Frequently Asked Questions

How do you differentiate the equation $5x^3 - y^3 = 4$ implicitly to

find the second derivative d^2y/dx^2?

First, differentiate both sides with respect to x, applying implicit differentiation: $d/dx(5x^3) - d/dx(y^3) = 0$. This gives $15x^2 - 3y^2(dy/dx) = 0$. Solve for dy/dx. Then differentiate again, applying the chain rule to dy/dx to find d^2y/dx^2.

What is the first step in differentiating $5x^3 - y^3 = 4$ implicitly to find the second derivative?

Differentiate both sides with respect to x, resulting in $15x^2 - 3y^2(dy/dx) = 0$, which allows you to solve for dy/dx in terms of x and y.

How do you express dy/dx from the implicit equation $5x^3 - y^3 = 4$?

Rearranged, $dy/dx = (15x^2) / (3y^2) = 5x^2 / y^2$.

What is the process to find d^2y/dx^2 after obtaining dy/dx in this implicit differentiation problem?

Differentiate $dy/dx = 5x^2 / y^2$ with respect to x, applying the quotient rule and chain rule, then substitute dy/dx as needed to express d^2y/dx^2 entirely in terms of x and y.

Can you provide the final expression for d^2y/dx^2 for the equation $5x^3 - y^3 = 4$?

Yes. Differentiating twice and simplifying yields: $d^2y/dx^2 = [10x y^2 + 10x^3 y (dy/dx)] / y^4$, where $dy/dx = 5x^2 / y^2$. Substituting dy/dx back into this expression provides the second derivative in terms of x and y.

Additional Resources

Differentiate Implicitly To Find D^2y/dx^2 5x^3 - y^3 = 4

In the realm of calculus, the process of differentiation often involves straightforward functions where explicit formulas are available. However, many real-world problems involve relationships between variables that are intertwined in a way that makes explicit solutions cumbersome or impossible to derive directly. This is where implicit differentiation becomes a powerful tool. Today, we'll delve into how to differentiate an implicit relation—specifically, the equation $5x^3 - y^3 = 4$—to find the second derivative $\frac{d^2 y}{dx^2}$. This technique not only broadens our understanding of calculus but also equips us with methods to analyze more complex systems where variables are connected implicitly.

Understanding Implicit Differentiation

What Is Implicit Differentiation?

Implicit differentiation is a technique used when a function y is defined implicitly with respect to x, meaning the relationship between x and y is given in a form that cannot be easily rewritten as $y = f(x)$. Instead of explicitly solving for y, we differentiate both sides of the equation with respect to x, applying the chain rule to account for y being a function of x.

Example:
Given the relation $F(x, y) = 0$, to find $\frac{dy}{dx}$, differentiate both sides:

$$
\frac{d}{dx} F(x, y) = 0
$$

which involves differentiating with respect to x, treating y as a function of x:

$$
\frac{\partial F}{\partial x} + \frac{\partial F}{\partial y} \frac{dy}{dx} = 0
$$

From this, solve for $\frac{dy}{dx}$:

$$
\frac{dy}{dx} = - \frac{\frac{\partial F}{\partial x}}{\frac{\partial F}{\partial y}}
$$

This approach allows us to find the first derivative without explicitly solving for y.

Applying Implicit Differentiation to the Given Equation

Let's consider the specific relation:

$$
5x^3 - y^3 = 4
$$

Our goal is to find the second derivative $\frac{d^2 y}{dx^2}$. To do this systematically, we'll proceed step by step.

Step 1: Differentiate to Find $\frac{dy}{dx}$

Start by differentiating both sides of the equation with respect to x:

$$
\frac{d}{dx} (5x^3) - \frac{d}{dx} (y^3) = \frac{d}{dx} (4)
$$

\]

Applying differentiation rules:

- The derivative of \(5x^3 \) with respect to \(x \) is \(15x^2 \).
- The derivative of \(y^3 \) with respect to \(x \) involves the chain rule: \(3 y^2 \frac{dy}{dx} \).
- The derivative of a constant is zero.

Putting it together:

\[
15x^2 - 3 y^2 \frac{dy}{dx} = 0
\]

Solve for \(\frac{dy}{dx} \):

\[
3 y^2 \frac{dy}{dx} = 15x^2
\]

\[
\frac{dy}{dx} = \frac{15x^2}{3 y^2} = \frac{5x^2}{y^2}
\]

This expression gives the first derivative in terms of \(x \) and \(y \).

Computing the Second Derivative \(\frac{d^2 y}{dx^2} \)

The second derivative involves differentiating \(\frac{dy}{dx} \) once more with respect to \(x \):

$$
\frac{d^2 y}{dx^2} = \frac{d}{dx} \left(\frac{dy}{dx} \right) = \frac{d}{dx} \left(\frac{5x^2}{y^2} \right)
$$

This derivative is complex because y is a function of x. To differentiate this quotient, we'll use the quotient rule:

$$
\frac{d}{dx} \left(\frac{u}{v} \right) = \frac{v \frac{du}{dx} - u \frac{dv}{dx}}{v^2}
$$

where:

- $u = 5x^2$
- $v = y^2$

Applying the Quotient Rule

Calculate $\frac{du}{dx}$ and $\frac{dv}{dx}$:

- $\frac{du}{dx} = 10x$
- $v = y^2$, so $\frac{dv}{dx} = 2 y \frac{dy}{dx}$

Recall from earlier:

$$
\frac{dy}{dx} = \frac{5x^2}{y^2}
$$

Substitute into $\frac{dv}{dx}$:

$$
\frac{dv}{dx} = 2 y \times \frac{5x^2}{y^2} = 2 y \times \frac{5x^2}{y^2} = \frac{10 x^2}{y}
$$

Now, applying the quotient rule:

$$
\frac{d^2 y}{dx^2} = \frac{ y^2 \times 10x - 5x^2 \times \frac{10 x^2}{y} }{ y^4 }
$$

Simplify numerator step-by-step:

$$
\text{Numerator} = y^2 \times 10x - 5x^2 \times \frac{10 x^2}{y}
$$

$$
= 10 x y^2 - \frac{50 x^4}{y}
$$

Express both terms with a common denominator y:

$$
= \frac{10 x y^3 - 50 x^4}{ y }
$$

Thus,

\[

$$
\frac{d^2 y}{dx^2} = \frac{ \frac{10 x y^3 - 50 x^4}{ y } }{ y^4 } = \frac{10 x y^3 - 50 x^4}{ y \times y^4 }
= \frac{10 x y^3 - 50 x^4}{ y^5 }
$$

Factor numerator:

$$
= \frac{10 x (y^3 - 5 x^3) }{ y^5 }
$$

Expressing the Second Derivative Fully

The second derivative simplifies to:

$$
\boxed{
\frac{d^2 y}{dx^2} = \frac{10 x (y^3 - 5 x^3) }{ y^5 }
}
$$

This formula expresses $\frac{d^2 y}{dx^2}$ explicitly in terms of x and y. To evaluate it at specific points, one would first determine y corresponding to x from the original relation $5x^3 - y^3 = 4$.

Practical Implications and Applications

Understanding how to differentiate implicitly and compute second derivatives is essential in various fields such as physics, engineering, and economics. For example:

- Physics: Analyzing the curvature of a trajectory where position and velocity are related implicitly.
- Engineering: Studying stress-strain relationships where variables are interconnected.
- Economics: Optimizing functions where certain constraints link variables implicitly.

The ability to compute $\frac{d^2 y}{dx^2}$ enables analysts to determine concavity, identify inflection points, and understand the behavior of complex systems without solving explicitly for y.

Summary and Key Takeaways

- Implicit differentiation involves differentiating both sides of an equation with respect to x, treating y as a function of x.
- The first derivative $\frac{dy}{dx}$ can be found by applying the chain rule and algebraic manipulation.
- Computing the second derivative $\frac{d^2 y}{dx^2}$ requires differentiating $\frac{dy}{dx}$, often using the quotient rule and the chain rule.
- The process reveals the curvature and concavity of the implicit relation, providing insights into the geometric nature of the curve.

By mastering implicit differentiation and its application to higher derivatives, mathematicians and scientists can analyze intricate relationships that govern the natural and engineered world, paving the way for deeper understanding and innovation.

In conclusion, differentiating implicitly to find $\frac{d^2 y}{dx^2}$ from the relation $5x^3 - y^3 = 4$ exemplifies the elegance and power of calculus. It demonstrates how, with systematic steps and fundamental rules, complex implicit relationships can be unraveled to reveal their underlying geometric and analytical properties.

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