Dydx=(x-1)(x+3) At X=2 First Principal

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Understanding how to evaluate the derivative of a function at a specific point is a fundamental concept in calculus. When given a function like $y = (x - 1)(x + 3)$, finding the derivative at a particular point, such as $x = 2$, using the first principles (also known as the definition of the derivative), provides deep insight into the behavior of the function. This article explores the process of calculating $\frac{dy}{dx}$ at $x = 2$ for the function $y = (x-1)(x+3)$ using the first principle, along with related concepts to enhance your understanding of derivatives and their applications.

Understanding the First Principle of Derivatives

The first principle of derivatives, also called the definition of the derivative, is a method to compute the instantaneous rate of change of a function at a particular point. It is based on the limit process and is expressed as:

$$
f'(a) = \lim_{h \to 0} \frac{f(a+h) - f(a)}{h}
$$

where:
- $f'(a)$ is the derivative of the function f at the point a,
- h is a small increment approaching zero.

This approach involves:
- Calculating the function value at $a + h$,
- Subtracting the function value at a,
- Dividing by h,
- Taking the limit as $h \to 0$.

Applying this process to the function $y = (x-1)(x+3)$ at $x=2$ helps to understand the instantaneous rate of change at that specific point.

Step-by-Step Calculation of the Derivative at X=2 Using First Principles

1. Write the function explicitly

The given function is:

$$
y = (x - 1)(x + 3)
$$

Expanding this:

$$
y = x^2 + 3x - x - 3 = x^2 + 2x - 3
$$

While expanding makes the calculation straightforward, for the purpose of applying the first principle, it’s often better to work with the factored form directly or use the original form to understand the process.

2. Set up the difference quotient

To find the derivative at $x = 2$ using the first principle:

$$
f'(2) = \lim_{h \to 0} \frac{f(2+h) - f(2)}{h}
$$

Calculate $f(2+h)$:

$$
f(2+h) = ((2+h) - 1)((2+h) + 3)
$$

Simplify:

$$
f(2+h) = (1 + h)(5 + h)
$$

Similarly, compute $f(2)$:

$$
f(2) = (2 - 1)(2 + 3) = (1)(5) = 5
$$

3. Calculate $f(2+h) - f(2)$

$$
f(2+h) - f(2) = (1 + h)(5 + h) - 5
$$

Expand $(1 + h)(5 + h)$:

$$
(1)(5) + (1)(h) + h(5) + h^2 = 5 + h + 5h + h^2
$$

Combine like terms:

$$
5 + 6h + h^2
$$

Subtract 5:

$$
(5 + 6h + h^2) - 5 = 6h + h^2
$$

4. Form the difference quotient and take the limit

Divide by h:

$$
\frac{6h + h^2}{h} = 6 + h
$$

Now, take the limit as $h \to 0$:

$$
\lim_{h \to 0} (6 + h) = 6
$$

Result:

$$
\boxed{
\frac{dy}{dx} \bigg|_{x=2} = 6
}
$$

This means the slope of the tangent to the curve at $x=2$ is 6, indicating a fairly steep positive slope at that point.

Alternative Method: Using the Power Rule for Derivatives

While the first principle provides a foundational understanding, calculating derivatives using shortcut rules is often more efficient once you understand the concept.

1. Differentiate the expanded form $y = x^2 + 2x - 3$

Applying the power rule:

$$
\frac{dy}{dx} = 2x + 2
$$

Evaluate at $x=2$:

$$
\frac{dy}{dx} \bigg|_{x=2} = 2(2) + 2 = 4 + 2 = 6
$$

This confirms the result obtained via the first principle.

Importance of the First Principle in Calculus

The first principle is essential because:

- It provides a rigorous foundation for the derivative concept.
- It helps in understanding the geometric interpretation of the derivative as the slope of the tangent line at a point.
- It enables the differentiation of functions that might not follow standard rules directly.

Mastering the first principle also enhances problem-solving skills in calculus, especially when dealing with complex functions or when learning advanced topics such as limits and continuity.

Applications of Derivatives in Real Life

Derivatives are not just mathematical abstractions—they have practical applications across various fields:

1. Physics

- Calculating velocity as the derivative of displacement with respect to time.
- Determining acceleration as the derivative of velocity.

2. Economics

- Finding marginal cost and revenue by differentiating total cost and revenue functions.
- Analyzing rate of change in stock prices.

3. Engineering

- Optimizing systems by analyzing the rate of change of different variables.
- Designing control systems for maximum efficiency.

Summary and Key Takeaways

- The function $y = (x - 1)(x + 3)$ can be expanded to $y = x^2 + 2x - 3$.
- Using first principles, the derivative at $x = 2$ is computed as 6.
- The process involves calculating the difference quotient and taking the limit as $h \to 0$.
- Shortcut rules like the power rule confirm the derivative quickly once the concept is understood.
- Mastery of the first principle provides a solid foundation for understanding calculus and its applications.

Final Thoughts

Calculating derivatives through the first principle is a critical skill in calculus that deepens your understanding of how functions behave locally. For the function $y = (x-1)(x+3)$, evaluating the derivative at $x=2$ reveals an instantaneous rate of change of 6, indicating the steepness of the tangent line at that point. Whether you're a student learning calculus or a professional applying these concepts in real-world scenarios, grasping the first principle equips you with a robust tool for analyzing and understanding dynamic systems.

Remember: Practice is key—try applying the first principle to different functions to strengthen your understanding of derivatives and their significance in mathematics and beyond.

Frequently Asked Questions

What is the function given in the problem?

The function is Dydx = (x - 1)(x + 3).

What does 'At X=2' signify in this context?

It indicates that we are evaluating the derivative of the function at x = 2 using the first principles method.

What is the first principle (definition) of a derivative?

The first principle of a derivative is defined as the limit of the difference quotient as h approaches zero: f'(x) = lim_{h→0} [f(x+h) - f(x)] / h.

How do you set up the difference quotient for this function at x=2?

You compute [f(2+h) - f(2)] / h, where f(x) = (x - 1)(x + 3).

What is the value of f(2) for the given function?

f(2) = (2 - 1)(2 + 3) = (1)(5) = 5.

How do you compute f(2+h) for the function?

f(2+h) = [(2+h) - 1] [(2+h) + 3] = (1 + h) (5 + h).

What is the calculated derivative at x=2 using the first principles?

By simplifying the difference quotient and taking the limit as h approaches zero, the derivative at x=2 is 7.

Additional Resources

Derivative Calculation: An In-Depth Analysis of \(D_{y}x = (x-1)(x+3) \) at \(x=2 \) Using First Principles

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Introduction

When exploring the fundamental concepts of calculus, the derivative stands out as a core idea that provides insight into the rate at which functions change. Understanding how to compute derivatives using first principles — also known as the definition of the derivative — is not only essential for grasping the theoretical underpinnings of calculus but also invaluable for practical problem-solving.

This article takes an in-depth look at calculating the derivative of the function \(y = (x-1)(x+3) \) at the specific point \(x = 2 \), employing the method of first principles. We will dissect each step, explain the underlying principles, and provide a comprehensive understanding suitable for learners and enthusiasts alike.

Understanding the Function: \(y = (x - 1)(x + 3) \)

Before diving into the derivative calculation, it’s crucial to understand the nature of the function itself.

The Function Structure

- Type of Function: It’s a quadratic function, as expanding the product yields a second-degree polynomial.
- Expanded Form:
$$
y = (x - 1)(x + 3) = x^2 + 3x - x - 3 = x^2 + 2x - 3
$$
- Graphical Representation: This is a parabola opening upwards with its vertex at a point that can be found by completing the square or using calculus methods.

Significance

Knowing the explicit form simplifies the process of calculating derivatives and analyzing the behavior of the function around specific points. While the first principles method does not require expansion, having the explicit form offers validation and cross-verification.

The Concept of the First Principles Derivative

The derivative of a function at a specific point measures the instantaneous rate of change or the slope of the tangent line at that point. The first principles approach defines this as a limit:

$$
f'(a) = \lim_{h \to 0} \frac{f(a + h) - f(a)}{h}
$$

Where:
- $f(a)$ is the function value at point a,
- h is a small change in x,
- The limit as $h \to 0$ ensures the change is infinitesimally small, capturing the true slope at a.

Applying this to our specific function at $x = 2$:

$$
f'(2) = \lim_{h \to 0} \frac{f(2 + h) - f(2)}{h}
$$

Step-by-Step Derivation at $x=2$

1. Calculate $f(2)$

Using the explicit form of the function:

\[

$$
f(2) = (2 - 1)(2 + 3) = (1)(5) = 5
$$

This is the function value at $x=2$.

2. Formulate $f(2 + h)$

Plug in $x = 2 + h$:

$$
f(2 + h) = [(2 + h) - 1][(2 + h) + 3] = (1 + h)(5 + h)
$$

Expanding this:

$$
f(2 + h) = (1 + h)(5 + h) = 1 \times 5 + 1 \times h + h \times 5 + h \times h = 5 + h + 5h + h^2
$$

Simplify:

$$
f(2 + h) = 5 + 6h + h^2
$$

---

3. Set up the difference quotient

Now, evaluate:

$$
\frac{f(2 + h) - f(2)}{h} = \frac{(5 + 6h + h^2) - 5}{h} = \frac{6h + h^2}{h}
$$

Factor numerator:

$$
\frac{h(6 + h)}{h}
$$

Since $h \neq 0$ in the limit process, cancel h:

$$
6 + h
$$

---

4. Take the limit as $h \to 0$

$$
f'(2) = \lim_{h \to 0} (6 + h) = 6 + 0 = 6
$$

Result: The derivative of $y = (x-1)(x+3)$ at $x=2$ is 6.

Interpretation and Significance

The value 6 indicates that at $x=2$, the function has a tangent line with a slope of 6. In practical terms, this means for a very small change in x around 2, the function y increases approximately 6 times that change.

This outcome aligns with the derivative of the expanded quadratic form:

$$
f'(x) = 2x + 2
$$

At $x=2$:

$$
f'(2) = 2(2) + 2 = 4 + 2 = 6
$$

This confirms the correctness of the first principles calculation.

Broader Insights and Applications

Advantages of the First Principles Approach

- Foundational Understanding: It provides a clear insight into what the derivative represents.
- Applicability: It can be used for functions where rules like power rule or product rule are not immediately applicable.
- Conceptual Clarity: Understanding the limit process deepens comprehension of how calculus captures instantaneous change.

Limitations

- Complexity: For more complicated functions, the algebra can become unwieldy.
- Efficiency: Using derivative rules is faster once the function's form is known, but first principles is invaluable for learning and proofs.

Practical Uses and Contexts

The derivative at a point like $x=2$ has numerous applications:

- Physics: Calculating velocity as the rate of change of position.
- Economics: Marginal analysis to determine how a small change in input affects output.
- Engineering: Sensitivity analysis in control systems.
- Data Science: Gradient calculations for optimization algorithms.

Summary of Key Takeaways

Aspect	Explanation
Function	$y = (x-1)(x+3)$
Expanded form	$y = x^2 + 2x - 3$
Derivative at $x=2$	6
Method used	First principles (limit definition)
Significance	Measures the instantaneous rate of change at a specific point

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Final Thoughts

Calculating derivatives via first principles is more than an academic exercise; it embodies the core philosophy of calculus — understanding the nature of change at an infinitesimal level. The process illustrated here for $y = (x-1)(x+3)$ at $x=2$ exemplifies how foundational concepts translate into precise, meaningful results. Whether you're a student, educator, or professional, mastering this approach enriches your analytical toolkit and deepens your appreciation of the mathematical world.

In conclusion, the derivative of $y = (x-1)(x+3)$ at $x=2$ is 6, a value that encapsulates the function’s local behavior and exemplifies the power of the first principles method. Embracing this technique fosters a profound understanding of calculus’s core principles and prepares you for more advanced analysis across various scientific disciplines.

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