Evaluate The Functionf(x) = - 2x^2 + 2x + 4Find F(1)

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Understanding how to evaluate functions at specific points is a fundamental skill in mathematics, particularly in algebra and calculus. Functions serve as mathematical models to describe relationships between variables, and evaluating a function at a particular point allows us to determine the output or the value of the function for a given input. In this article, we will thoroughly examine the quadratic function $f(x) = -2x^2 + 2x + 4$, explore its properties, and evaluate it at $x = 1$. This process involves understanding the structure of quadratic functions, their graphing, and their key characteristics such as vertex, axis of symmetry, and maximum or minimum points.

This detailed exploration aims to not only provide the specific value of $f(1)$ but also to deepen your understanding of quadratic functions, how to analyze them, and why evaluating functions at specific points is a crucial skill in mathematics, physics, engineering, and other sciences. Whether you are a student preparing for exams, a teacher designing lesson plans, or a professional applying mathematical concepts to real-world problems, mastering these skills will enhance your analytical capabilities and problem-solving proficiency.

Understanding Quadratic Functions

Quadratic functions are polynomial functions of degree 2, characterized generally by the form:

$$f(x) = ax^2 + bx + c$$

where $a \neq 0$. They are called quadratic because the highest power of x is 2. The graph of a quadratic function is a parabola, which can open upward or downward depending on the sign of a.

In our case:

$$f(x) = -2x^2 + 2x + 4$$

the coefficient $a = -2$, which is negative, indicating that the parabola opens downward.

Key Features of the Quadratic Function $f(x) = -2x^2 + 2x + 4$

To analyze this quadratic function, we focus on several important aspects:

1. Direction of the Parabola

- Since $a = -2 < 0$, the parabola opens downward.
- The function has a maximum point, called the vertex, which is the highest point on the graph.

2. Vertex of the Parabola

- The vertex provides the maximum value of the function and its coordinates can be found using the vertex formula.

3. Axis of Symmetry

- The vertical line that passes through the vertex, dividing the parabola into two symmetric halves.

4. Y-intercept

- The point where the parabola crosses the y-axis ($x=0$).

5. Roots or Zeros of the Function

- The points where the parabola crosses the x-axis ($f(x) = 0$).

Understanding these features helps us analyze and interpret the behavior of the quadratic function comprehensively.

Finding the Vertex of the Quadratic Function

The vertex (h, k) of a parabola given by $f(x) = ax^2 + bx + c$ can be found using the formulas:

$$h = -\frac{b}{2a}$$

$$k = f(h)$$

For our function:

$$ a = -2, \quad b = 2, \quad c = 4 $$

Calculate h:

$$ h = -\frac{b}{2a} = -\frac{2}{2 \times (-2)} = -\frac{2}{-4} = \frac{1}{2} $$

Next, find $k = f(h) = f(0.5)$:

$$ f(0.5) = -2(0.5)^2 + 2(0.5) + 4 $$

Calculate step-by-step:

- $(0.5)^2 = 0.25$
- $-2 \times 0.25 = -0.5$
- $2 \times 0.5 = 1$

Now, sum all:

$$ f(0.5) = -0.5 + 1 + 4 = 4.5 $$

Thus, the vertex is at:

$$ \left(\frac{1}{2}, 4.5 \right) $$

Since the parabola opens downward, this point represents the maximum value of the function, with a maximum of 4.5 occurring at $x = 0.5$.

Graphical Interpretation of the Function

Visualizing the parabola helps in understanding the behavior of $f(x)$:

- The vertex at $(0.5, 4.5)$ is the highest point.
- The y-intercept occurs at $x=0$:

$$ f(0) = -2(0)^2 + 2(0) + 4 = 4 $$

- The x-intercepts (roots) are found by solving $f(x) = 0$.

Finding the Roots of $f(x) = 0$

To find where the parabola crosses the x-axis, solve:

$$-2x^2 + 2x + 4 = 0$$

Divide both sides by -2 to simplify:

$$x^2 - x - 2 = 0$$

Factor the quadratic:

$$(x - 2)(x + 1) = 0$$

Set each factor to zero:

- $x - 2 = 0 \Rightarrow x = 2$
- $x + 1 = 0 \Rightarrow x = -1$

Therefore, the roots are at $x = -1$ and $x = 2$. These are the points where the parabola intersects the x-axis.

Evaluating $f(1)$: The Main Objective

Now, having understood the overall behavior of the function, the key question is: what is the value of $f(1)$?

To find $f(1)$, substitute $x = 1$ into the original function:

$$f(1) = -2(1)^2 + 2(1) + 4$$

Calculate step-by-step:

- $(1)^2 = 1$
- $-2 \times 1 = -2$
- $2 \times 1 = 2$

Sum all:

$$f(1) = -2 + 2 + 4 = 4$$

Thus, the value of the function at $x=1$ is 4.

Interpreting the Result and Its Significance

The value \(f(1) = 4 \) provides specific information about the function's behavior at \(x=1 \):

- It is less than the maximum value 4.5 at the vertex, indicating that \(x = 1 \) is not at the peak of the parabola.
- Since \(f(1) = 4 \), and the y-intercept is 4, the point \((1, 4) \) lies just below the vertex.
- The function's value at \(x=1 \) can be used in real-world contexts where quadratic models are applicable, such as projectile motion, economics, or physics.

Understanding this evaluation helps in analyzing the function’s shape, determining the rate of change, and predicting behavior in applied scenarios.

Applications of Quadratic Functions and Their Evaluations

Quadratic functions like \(f(x) = -2x^2 + 2x + 4 \) are fundamental in various disciplines. Here are some practical applications:

- Physics: Modeling projectile trajectories, where the height of an object over time follows a quadratic pattern.
- Economics: Analyzing profit functions or cost functions that have quadratic relationships.
- Engineering: Designing parabolic structures or analyzing stress and strain in materials.
- Biology: Describing population growth or decay in certain conditions.

Evaluating the function at specific points enables precise predictions, optimizations, and decision-making in these fields.

Summary of Key Steps in Evaluating \(f(1) \)

To summarize, the process of evaluating the quadratic function at \(x=1 \) involves:

1. Substituting \(x = 1 \) into the function.
2. Simplifying the expression step-by-step.
3. Calculating the result to find the output value.

This systematic approach ensures accuracy and reinforces understanding of the function's properties.

Conclusion

In this comprehensive guide, we've explored the quadratic function $f(x) = -2x^2 + 2x + 4$, analyzed its key features, and specifically evaluated it at $x=1$. Our calculations show that:

$$f(1) = 4$$

This value reflects the function's behavior at that point and contributes to a broader understanding of the parabola's shape, maximum point, roots, and intercepts.

Mastering the evaluation of quadratic functions is essential for tackling more complex mathematical problems and applying these concepts across various scientific and engineering fields. By understanding the structure and

Frequently Asked Questions

What is the given function to evaluate?

The given function is $f(x) = -2x^2 + 2x + 4$.

How do you find the value of f(1) for the given quadratic function?

To find $f(1)$, substitute $x = 1$ into the function and simplify.

What is the step-by-step calculation to evaluate f(1)?

Substitute $x = 1$: $f(1) = -2(1)^2 + 2(1) + 4 = -2(1) + 2 + 4 = -2 + 2 + 4 = 4$.

What is the value of f(1) for the function f(x) = -2x^2 + 2x + 4?

$f(1) = 4$.

Is the function f(x) = -2x^2 + 2x + 4 quadratic or linear?

It is a quadratic function because the highest degree of x is 2.

How does the coefficient of x^2 affect the parabola's shape?

Since the coefficient of x^2 is negative (-2), the parabola opens downward.

Can you explain how to evaluate the function at any point x?

Yes, to evaluate at any x, substitute the value into the function and perform the arithmetic operations.

What is the significance of the value f(1) in the context of the function?

f(1) represents the output of the function when the input x is 1, giving a specific point on the parabola.

Are there other methods to evaluate f(1) besides direct substitution?

For this simple function, direct substitution is the most straightforward method; other methods like graphing can also approximate the value.

Additional Resources

Evaluating the Function f(x) = $-2x^2$ + 2x + 4 at x = 1: A Comprehensive Analysis

When delving into the world of algebra and calculus, one of the fundamental tasks is evaluating functions at specific points. This process not only helps in understanding the behavior of the function but also lays the groundwork for more advanced topics such as graphing, optimization, and calculus operations. Today, we examine the quadratic function:

$$ f(x) = -2x^2 + 2x + 4 $$

and focus on the specific evaluation of $f(1)$. But beyond a simple substitution, we will explore the structure of this quadratic, interpret its features, and understand the implications of its value at $x=1$.

Understanding the Function: An Introduction to Quadratics

Quadratic functions are polynomial functions of degree two, characterized by their parabolic graphs. The general form:

$$ f(x) = ax^2 + bx + c $$

where $a \neq 0$, defines a parabola opening upward if $a > 0$, or downward if $a < 0$. In our case:

$$ f(x) = -2x^2 + 2x + 4 $$

- The coefficient $a = -2$ indicates the parabola opens downward.
- The linear coefficient $b = 2$ influences the slope and position of the vertex.
- The constant term $c = 4$ shifts the parabola vertically.

Understanding these coefficients allows us to interpret the function's shape and key features without graphing immediately.

Step-by-Step Evaluation of $f(1)$

The process of evaluating $f(1)$ involves substituting $x = 1$ into the function:

$$ f(1) = -2(1)^2 + 2(1) + 4 $$

Breaking this down:

1. Calculate $ (1)^2 $:

$$ (1)^2 = 1 $$

2. Multiply by -2:

$$ -2 \times 1 = -2 $$

3. Calculate $ 2 \times 1 $:

$$ 2 \times 1 = 2 $$

4. Sum all parts:

$$ -2 + 2 + 4 $$

Now, combining these:

$$ -2 + 2 + 4 = (-2 + 2) + 4 = 0 + 4 = 4 $$

Result:

$$ f(1) = 4 $$

This straightforward substitution yields the function value at $x=1$, which is 4.

Interpreting the Result: What Does $f(1) = 4$ Tell Us?

Knowing that $f(1) = 4$ provides several insights:

- Point on the Graph: The point $(1, 4)$ is located on the parabola. Plotting this point helps visualize the function.
- Vertical Shift: Since the function's maximum or minimum (vertex) is at some point, knowing $f(1)$ indicates where the curve reaches or crosses this specific height.
- Function Behavior: As part of the larger graph, this point contributes to understanding the overall shape and the function's domain and range.

Analyzing the Parabola: Vertex and Axis of Symmetry

Beyond evaluating the function at a specific point, analyzing the quadratic's key features enhances comprehension.

Finding the Vertex

The vertex of a quadratic $f(x) = ax^2 + bx + c$ occurs at:

$$ x_{v} = -\frac{b}{2a} $$

Substituting $a = -2$, $b = 2$:

$$ x_{v} = -\frac{2}{2 \times (-2)} = -\frac{2}{-4} = \frac{1}{2} $$

To find the y-coordinate of the vertex $f(x_{v})$:

$$f\left(\frac{1}{2}\right) = -2 \left(\frac{1}{2}\right)^2 + 2 \times \frac{1}{2} + 4$$

Calculations:

- $\left(\frac{1}{2}\right)^2 = \frac{1}{4}$
- $-2 \times \frac{1}{4} = -\frac{1}{2}$
- $2 \times \frac{1}{2} = 1$

Adding:

$$-\frac{1}{2} + 1 + 4 = \left(-0.5 + 1 \right) + 4 = 0.5 + 4 = 4.5$$

Vertex Coordinates:

$$\left(\frac{1}{2}, 4.5 \right)$$

This point indicates the maximum value of the parabola (since $a<0$), which is crucial in understanding the function's range.

Axis of Symmetry

The axis of symmetry passes through the vertex:

$$x = x_{v} = \frac{1}{2}$$

This vertical line divides the parabola into two mirror-image halves.

Graphical Interpretation and Real-World Applications

The evaluation of $f(1) = 4$ is more than a numerical exercise; it’s a gateway to visualizing and applying the function.

- Graphing the Parabola:

- Plot the point $(1, 4)$.
- Plot the vertex $\left(\frac{1}{2}, 4.5 \right)$.
- Draw the parabola opening downward.
- Recognize the symmetry about $x = 0.5$.

- Applications:

Quadratic functions model real-world scenarios such as projectile trajectories, profit maximization, and physics problems involving acceleration. Knowing the function's value at specific points helps in:

- Optimizing outcomes (e.g., maximum height or profit).
- Determining intervals of increase or decrease.
- Calculating areas or other integral-based measures.

Additional Insights: Discriminant and Roots

Understanding the roots of the quadratic enhances the analysis:

$$ \text{Discriminant } D = b^2 - 4ac $$

Calculating:

$$ D = (2)^2 - 4 \times (-2) \times 4 = 4 - (-32) = 4 + 32 = 36 $$

Since $D > 0$, the quadratic has two real roots, indicating where the parabola crosses the x-axis.

Finding roots:

$$ x = \frac{-b \pm \sqrt{D}}{2a} $$

$$ x = \frac{-2 \pm 6}{-4} $$

Two solutions:

1. $x = \frac{-2 + 6}{-4} = \frac{4}{-4} = -1$
2. $x = \frac{-2 - 6}{-4} = \frac{-8}{-4} = 2$

These roots show the parabola crosses the x-axis at $x = -1$ and $x = 2$.

Conclusion: Summarizing the Evaluation and Its Significance

Evaluating the quadratic $f(x) = -2x^2 + 2x + 4$ at $x = 1$ yields a value of 4. This specific point contributes to a broader understanding of the function's geometry, roots, and maximum value.

- The point $(1, 4)$ lies on the parabola, close to its maximum height.

- The vertex at \(\left(0.5, 4.5\right)\) indicates the maximum of the parabola.
- The parabola opens downward, confirming the maximum point is at the vertex.
- The roots at \(x = -1\) and \(x = 2\) mark the x-intercepts.

This comprehensive evaluation demonstrates how a simple substitution opens a window into the function's entire behavior, making it an essential technique in both academic and practical contexts. Whether optimizing a process or visualizing a graph, understanding the significance of specific points like \(f(1)\) enriches your grasp of quadratic functions and their applications across disciplines.

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