

Evaluate The Indefinite Integral.

$9\sin^4 x \cos(x) dx = +C$

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Understanding how to evaluate indefinite integrals is fundamental in calculus, especially when dealing with trigonometric functions. One such integral that often appears in calculus problems involves the integral of a product of powers of sine and cosine functions. Specifically, the integral:

$$\int 9 \sin^4 x \cos x \, dx = +C$$

is a classic example demonstrating the application of substitution techniques and power-reduction formulas to evaluate integrals involving trigonometric functions. This article provides a comprehensive guide on how to evaluate this indefinite integral step-by-step, along with relevant concepts, techniques, and tips to master similar problems.

Understanding the Integral: $9\sin^4 x \cos(x) dx$

Before diving into solving the integral, it's essential to understand the structure and components involved:

- The integral involves a polynomial power of sine, specifically $\sin^4 x$, multiplied by cosine $\cos x$.
- The constant coefficient 9 can be factored out of the integral.
- The differential element is dx , indicating integration with respect to x .

The goal is to find a function $F(x)$ such that:

$$F'(x) = 9 \sin^4 x \cos x$$

and then express the indefinite integral as:

$$\int 9 \sin^4 x \cos x \, dx = F(x) + C$$

Key Concepts and Techniques for Evaluating the Integral

Several fundamental calculus concepts are utilized to evaluate integrals of this form:

1. Substitution Method

- Recognizes when a part of the integrand is the derivative of another component.
- Particularly useful when the integrand contains a function and its derivative.

2. Power-Reducing Formulas

- Used to simplify powers of sine and cosine functions.
- For example, expressing $\sin^4 x$ in terms of multiple angles or lower powers to facilitate integration.

3. Recognizing Derivatives of Composite Functions

- Noticing that the derivative of $\sin x$ is $\cos x$, which appears in the integrand.

Step-by-Step Solution of the Integral

To evaluate the indefinite integral:

$$I = \int 9 \sin^4 x \cos x \, dx$$

we follow a systematic approach.

Step 1: Factor out the constant coefficient

$$I = 9 \int \sin^4 x \cos x \, dx$$

Step 2: Recognize the substitution opportunity

Notice that:

- $\sin^4 x$ can be written as $(\sin^2 x)^2$.
- The presence of $\cos x \, dx$ suggests substitution involving $\sin x$.

Let's set:

$$u = \sin x$$

then:

$$du = \cos x \, dx$$

$$du = \cos x \, dx$$

This substitution simplifies the integral significantly.

Step 3: Rewrite the integral in terms of (u)

Substituting into the integral:

$$I = 9 \int u^4 \, du$$

since $(\sin^4 x = u^4)$, and $(\cos x \, dx = du)$.

Step 4: Integrate with respect to (u)

The integral becomes:

$$I = 9 \int u^4 \, du$$

which is straightforward:

$$I = 9 \times \frac{u^5}{5} + C$$

or

$$I = \frac{9}{5} u^5 + C$$

Step 5: Substitute back to (x)

Recall that $(u = \sin x)$, so:

$$I = \frac{9}{5} \sin^5 x + C$$

This is the indefinite integral of the original function.

Final Result and Interpretation

The evaluated indefinite integral is:

$$\boxed{\int 9 \sin^4 x \cos x \, dx = \frac{9}{5} \sin^5 x + C}$$

where (C) is an arbitrary constant.

This result demonstrates how substitution simplifies seemingly complex integrals involving powers of sine and cosine functions.

Additional Techniques and Tips for Similar Integrals

While the above solution uses substitution directly, here are some additional techniques and tips for evaluating similar integrals:

1. Power-Reduction Formulas

- When dealing with higher powers of sine and cosine, use identities such as:

$$\sin^2 x = \frac{1 - \cos 2x}{2}$$
$$\cos^2 x = \frac{1 + \cos 2x}{2}$$

- Power-reduction can simplify integrals that are difficult to evaluate directly.

2. Recognizing Patterns for Substitution

- In integrals involving $(\sin^n x \cos x)$, substitution with $(u = \sin x)$ or $(u = \cos x)$ often simplifies the problem.
- For integrals involving $(\sin^m x \cos^n x)$, consider expressing in terms of multiple angles or using identities to reduce powers.

3. Using Differential Identities

- Recall that the derivative of $(\sin^k x)$ or $(\cos^k x)$ can be expressed in terms of lower powers, aiding in substitution.

Common Mistakes to Avoid

- Forgetting the constant of integration: Always include $(+ C)$ in indefinite integrals.
- Misapplication of substitution: Ensure that the substitution variable and differential match correctly.
- Ignoring the domain of functions: Be aware of the domain where the substitution is valid.

Applications of Indefinite Integrals in Real-World Contexts

The techniques discussed are not just theoretical—they have practical applications in various fields:

- Physics: Calculating work, energy, and other quantities involving trigonometric functions.
- Engineering: Signal processing and analyzing waveforms.
- Mathematics: Solving differential equations and modeling periodic phenomena.

Understanding how to evaluate integrals like $\int \sin^4 x \cos x \, dx$ equips students and professionals with tools to handle complex problems involving trigonometric functions.

Conclusion

Evaluating the indefinite integral:

$$\int \sin^4 x \cos x \, dx = \frac{1}{5} \sin^5 x + C$$

serves as an excellent example of practical integration techniques involving substitution and power identities. Recognizing patterns within the integrand allows for straightforward substitution, transforming complex expressions into simple polynomial integrals. Mastering these techniques enhances problem-solving skills in calculus and prepares individuals to tackle more advanced integrals involving trigonometric functions.

By understanding the underlying concepts, practicing a variety of similar problems, and applying substitution and identities judiciously, students and professionals can confidently evaluate indefinite integrals involving powers of sine and cosine functions, thereby strengthening their calculus foundation and analytical capabilities.

Frequently Asked Questions

How do you approach evaluating the indefinite integral of $9\sin^4(x)\cos(x) \, dx$?

You can approach this integral by recognizing the substitution $u = \sin(x)$, which simplifies the integral since $du = \cos(x) \, dx$. This transforms the integral into $9u^4 \, du$, making it straightforward to evaluate.

What substitution is most effective for integrating $9\sin^4(x)\cos(x) \, dx$?

The most effective substitution is $u = \sin(x)$, because the differential $du = \cos(x) \, dx$ allows the integral to be written purely in terms of u .

What is the indefinite integral of $9\sin^4(x)\cos(x) \, dx$ after substitution?

After substituting $u = \sin(x)$, the integral becomes $9u^4 \, du$, which integrates to $(9/5) u^5 + C$, or equivalently $(9/5) \sin^5(x) + C$.

How do you verify the indefinite integral result for $9\sin^4(x)\cos(x) \, dx$?

You differentiate the result $(9/5) \sin^5(x) + C$ with respect to x , applying the chain rule. The derivative should simplify back to $9\sin^4(x)\cos(x)$, confirming the integral's correctness.

Can this integral be solved without substitution? If yes, how?

While substitution is the most straightforward method, one could attempt to expand $\sin^4(x)$ using power-reduction formulas and then integrate term-by-term. However, substitution is typically simpler and more efficient.

Why is the integral expressed as '+ C' at the end?

The '+ C' indicates the constant of integration, which accounts for all possible vertical shifts in the antiderivative, reflecting the family of functions differing by a constant.

Additional Resources

Evaluating the Indefinite Integral of $9\sin^4 x \cos x \, dx$: A Comprehensive Analysis

When delving into the realm of calculus, particularly the evaluation of indefinite integrals, familiarity with various techniques such as substitution, identities, and algebraic manipulation becomes essential. One such integral that often appears in trigonometric calculus is:

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\[
\int 9 \sin^4 x \cos x \, dx
\]
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This integral, while seemingly straightforward at first glance, offers a rich opportunity to explore multiple strategies for evaluation, including substitution methods and the use of trigonometric identities. In this detailed review, we will examine the integral step-by-step, analyze the underlying techniques, and discuss the broader implications for calculus learners and practitioners.

Understanding the Integral and Its Components

Before jumping into the evaluation, it's crucial to understand the structure of the integral:

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\[
\int 9 \sin^4 x \cos x \, dx
\]
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- The function within the integral involves a power of sine, specifically $(\sin^4 x)$, multiplied by $(\cos x)$.
- The constant multiplier 9 simply scales the integral and can be factored out for simplicity.
- The differential (dx) indicates integration with respect to (x) .

This integral is a classic example where substitution methods are effective because of the presence of $(\sin^4 x)$ and $(\cos x)$, which are derivatives of each other in some sense.

Approach to Evaluating the Integral

The integral can be approached through different methods, but the most effective in this context is substitution, particularly because of the presence of $(\sin x)$ and $(\cos x)$. The steps generally involve:

1. Factoring out constants
2. Identifying substitution variables
3. Expressing powers of sine in terms of multiple angles or using identities
4. Applying substitution to simplify the integral

Let's analyze these in detail.

Step-by-Step Solution

Step 1: Factor Out the Constant

The integral contains a constant coefficient of 9, so we can write:

$$\int 9 \sin^4 x \cos x \, dx$$

This simplifies calculations since constants can be moved outside the integral.

Step 2: Recognize Suitable Substitution

Observe that:

- The integrand contains $\sin^4 x$ and $\cos x$.
- The derivative of $\sin x$ is $\cos x$, which suggests setting $u = \sin x$.

Thus, let:

$$\begin{aligned} u &= \sin x \\ du &= \cos x \, dx \end{aligned}$$

This substitution is promising because:

- $\sin^4 x = u^4$
- $\cos x \, dx = du$

The integral becomes:

$$\int 9 u^4 \, du$$

Step 3: Rewrite the Integral in Terms of u

Using the substitution:

$$\int 9 \sin^4 x \cos x \, dx = 9 \int u^4 \, du$$

This is a straightforward polynomial integral:

$$= 9 \times \frac{u^5}{5} + C$$

where C is the constant of integration.

Step 4: Back-Substitute to $\sin x$

Recall that $u = \sin x$, so:

$$\int \sin^5 x \, dx = \frac{9}{5} \sin^5 x + C$$

This is the simplified indefinite integral.

Final Result and Interpretation

The evaluated indefinite integral is:

$$\boxed{\int \sin^4 x \cos x \, dx = \frac{9}{5} \sin^5 x + C}$$

This result encapsulates the effectiveness of substitution and the importance of recognizing derivative relationships between $\sin x$ and $\cos x$.

Alternative Methods and Considerations

While the substitution method described above is the most straightforward, other techniques can also be explored:

Using Power-Reducing Identities

- Express $\sin^4 x$ in terms of multiple angles using power-reduction formulas:

$$\sin^4 x = \left(\sin^2 x \right)^2 = \left(\frac{1 - \cos 2x}{2} \right)^2$$

- Expand and simplify, then integrate term by term.

- This method, however, tends to be more cumbersome for this particular integral, especially given the presence of $\cos x$.

Why the substitution method is preferable here

- The substitution $u = \sin x$ directly simplifies the integral to a

polynomial form.

- It exploits the derivative relationship between $\sin x$ and $\cos x$ for efficient calculation.

Potential pitfalls to avoid

- Ignoring the constant in substitution: Always remember to include the constant of integration at the end.
- Incorrect substitution of limits (for definite integrals): When working with definite integrals, substituting limits accordingly is essential.
- Overcomplicating the integral: Recognize when substitution simplifies the integral rather than unnecessarily complicating it with identities.

Broader Implications and Learning Points

This integral exemplifies several fundamental concepts in calculus:

- Substitution Technique: Recognizing derivatives within the integrand simplifies complex integrals.
- Trigonometric Identities: While power-reduction identities are useful, sometimes recognizing derivative relationships is more efficient.
- Constant Multipliers: Always factor constants outside the integral for clarity and simplicity.
- Indefinite vs. Definite Integrals: The indefinite integral includes a constant of integration, emphasizing the general solution.

Key Takeaways:

- The integral $\int \sin^4 x \cos x \, dx$ is a classic example demonstrating the power of substitution.
- Recognizing the derivative relationship between $\sin x$ and $\cos x$ can streamline solution methods.
- Alternative methods, such as using identities, can be valuable but may introduce unnecessary complexity.

Extensions and Related Topics

Students and practitioners can extend their understanding through:

- Evaluating integrals involving higher powers of sine and cosine, employing power-reduction formulas.
- Exploring integrals involving products of different trigonometric functions, such as $\int \sin^m x \cos^n x \, dx$.
- Applying substitution techniques to definite integrals, paying attention to changing limits.
- Using computer algebra systems (CAS) to verify solutions and handle more complex integrals.

Conclusion

The evaluation of the indefinite integral:

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\[
\boxed{
\int 9 \sin^4 x \cos x \, dx = \frac{9}{5} \sin^5 x + C
}
\]
```

demonstrates the effectiveness of substitution methods rooted in recognizing derivative relationships within the integrand. By carefully analyzing the integrand's structure, choosing an appropriate substitution, and simplifying the integral accordingly, we arrive at a concise and elegant solution.

This process underscores the importance of strategic thinking in calculus, where understanding the interplay between algebra and trigonometry can significantly simplify problem-solving. Mastery of such techniques is vital for students and professionals working with complex integrals, differential equations, and advanced mathematical modeling.

In summary, evaluating integrals like $\int 9 \sin^4 x \cos x \, dx$ not only enhances your computational skills but also deepens your understanding of the interconnectedness of calculus concepts. This integral serves as a valuable example of how recognizing substitution opportunities rooted in familiar derivatives can lead to straightforward solutions, reinforcing core calculus principles.

[Evaluate The Indefinite Integral \$9\sin 4x\cos X \, Dx \, C\$](#)

Evaluate The Indefinite Integral $9\sin 4x\cos X \, Dx \, C$

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