Evaluate The Integral. 0^ 3x^3 Cos(x)dx

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When it comes to calculus, one of the fundamental tasks is evaluating definite integrals, which are essential for understanding areas, accumulated quantities, and many applications across science and engineering. The integral in question, from 0 to 3, of the function $3x^3 \cos(x) \, dx$, presents a classic example of an integral that involves polynomial and trigonometric functions combined. This article provides a comprehensive guide to evaluating this integral, exploring various methods including integration by parts, and offering detailed step-by-step solutions to enhance your calculus skills.

Understanding the Integral $\int_0^3 3x^3 \cos(x) \, dx$

Before diving into calculations, it is important to understand the structure of the integral and the properties of the functions involved.

Components of the Integral

The integrand $3x^3 \cos(x)$ consists of:

- A polynomial part: $3x^3$
- A trigonometric part: $\cos(x)$

The multiplication of these two functions suggests that the integral may require techniques like integration by parts, particularly because neither polynomial nor cosine alone simplifies the other directly.

Importance of Properly Setting Up the Integral

Correctly setting up the integral involves:

- Recognizing the limits of integration: from 0 to 3
- Identifying the integrand's structure
- Deciding on the appropriate method for evaluation

Since the integrand is a product of polynomial and cosine functions, integration by parts is a suitable method.

Methodology for Evaluating $\int_0^3 3x^3 \cos(x) \, dx$

The integral can be tackled using the integration by parts technique, which is based on the formula:

$$
\int u \, dv = uv - \int v \, du
$$

Choosing the right parts u and dv is crucial for simplifying the integral.

Step 1: Selecting u and dv

For the integral $\int 3x^3 \cos(x) \, dx$, a good choice is:

- $u = 3x^3$ (since polynomial reduces upon differentiation)
- $dv = \cos(x) \, dx$

This choice leverages the power of polynomial differentiation to simplify the integral.

Step 2: Computing du and v

Calculations:

- $du = 9x^2 \, dx$
- $v = \int \cos(x) \, dx = \sin(x)$

Step 3: Applying the Integration by Parts Formula

Applying the formula:

$$
\int 3x^3 \cos(x) \, dx = 3x^3 \sin(x) - \int \sin(x) \cdot 9x^2 \, dx
$$

Simplify:

$$
= 3x^3 \sin(x) - 9 \int x^2 \sin(x) \, dx
$$

Now, the problem reduces to evaluating $\int x^2 \sin(x) \, dx$, which again involves polynomial and trigonometric functions.

Evaluating $\int x^2 \sin(x) \, dx$ via Integration by Parts

This integral requires a second application of integration by parts.

Step 1: Choosing u and dv

- $u = x^2$
- $dv = \sin(x) \, dx$

Calculations:

- $du = 2x \, dx$
- $v = -\cos(x)$

Step 2: Applying the formula

$$
\int x^2 \sin(x) \, dx = -x^2 \cos(x) - \int -\cos(x) \cdot 2x \, dx
$$

Simplify:

$$
= -x^2 \cos(x) + 2 \int x \cos(x) \, dx
$$

Now, evaluate $\int x \cos(x) \, dx$ using integration by parts again.

Evaluating \(\int x \cos(x) \, dx \) for Complete Solution

Applying integration by parts:

- \(u = x \)
- \(dv = \cos(x) \, dx \)

Calculations:

- \(du = dx \)
- \(v = \sin(x) \)

Applying the formula:

\[
\int x \cos(x) \, dx = x \sin(x) - \int \sin(x) \, dx
\]

\[
= x \sin(x) + \cos(x) + C
\]

Putting it all together, the integral \(\int x^2 \sin(x) \, dx \) becomes:

\[
= -x^2 \cos(x) + 2 \left(x \sin(x) + \cos(x) \right) + C
\]

Simplify:

\[
= -x^2 \cos(x) + 2x \sin(x) + 2 \cos(x) + C
\]

Final Assembly of the Integral \(\int 3x^3 \cos(x) \, dx \)

Recall the earlier expression:

\[
\int 3x^3 \cos(x) \, dx = 3x^3 \sin(x) - 9 \int x^2 \sin(x) \, dx
\]

Substitute the evaluated $\int x^2 \sin(x) \, dx$:

$$
= 3x^3 \sin(x) - 9 \left(-x^2 \cos(x) + 2x \sin(x) + 2 \cos(x) \right) + C
$$

Distribute:

$$
= 3x^3 \sin(x) + 9x^2 \cos(x) - 18x \sin(x) - 18 \cos(x) + C
$$

This is the indefinite integral. To evaluate the definite integral from 0 to 3, substitute the upper and lower limits and subtract.

Evaluating the Definite Integral from 0 to 3

Calculate at $x = 3$:

$$
I(3) = 3(3)^3 \sin(3) + 9(3)^2 \cos(3) - 18(3) \sin(3) - 18 \cos(3)
$$

Calculate at $x = 0$:

$$
I(0) = 3(0)^3 \sin(0) + 9(0)^2 \cos(0) - 18(0) \sin(0) - 18 \cos(0)
$$

Simplify:

- $I(0) = 0 + 0 - 0 - 18 \times 1 = -18$

Now, substitute $x = 3$:

- $3 \times 27 \times \sin(3) = 81 \sin(3)$
- $9 \times 9 \times \cos(3) = 81 \cos(3)$
- $-18 \times 3 \times \sin(3) = -54 \sin(3)$
- $-18 \times \cos(3) = -18 \cos(3)$

Adding these:

$$
I(3) = 81 \sin(3) + 81 \cos(3) - 54 \sin(3) - 18 \cos(3)
$$

Combine like terms:

$$
= (81 \sin(3) - 54 \sin(3)) + (81 \cos(3) - 18 \cos(3))
$$

$$
= 27 \sin(3) + 63 \cos(3)
$$

Finally, the definite integral:

$$
\boxed{
\int_0^3 3x^3 \cos(x) \, dx = I(3) - I(0) = (27 \sin(3) + 63 \cos(3)) - (-18)
= 27 \sin(3) + 63 \cos(3) + 18
}
$$

Conclusion and Summary

Evaluating the integral $\int_0^3 3x^3 \cos(x) \, dx$ involves multiple steps of integration by parts, applied recursively to handle the polynomial and trigonometric components. Here is a summarized process:

- Use integration by parts to reduce the polynomial degree.
- Continue applying integration by parts until the polynomial term is fully differentiated.
- Carefully substitute limits to evaluate the definite integral.
- Combine like terms to arrive at the final value, expressed in terms of sine and cosine functions at the upper limit, along with a constant.

This process exemplifies the power of integration techniques in calculus, especially when dealing with complex polynomial-trigonometric integrals.

Additional Tips for Evaluating Similar Integrals

Frequently Asked Questions

How do I set up the integral of 0 to 3x^3 cos(x) dx?

To evaluate the integral, you need to specify the limits of integration. If the integral is from 0 to some value a, it would be written as $\int_0$^a 3x^3 cos(x) dx. If you mean the indefinite integral, it is ∫ 3x^3 cos(x) dx.

What is the best method to evaluate ∫ 3x^3 cos(x) dx?

The best method is to use integration by parts repeatedly or apply the tabular integration technique because of the polynomial times cosine function.

Can I use substitution for ∫ 3x^3 cos(x) dx?

Substitution alone is not straightforward here because the integrand involves a polynomial multiplied by a trigonometric function. Integration by parts is more suitable.

What is the first step in integrating ∫ 3x^3 cos(x) dx by parts?

Choose u = 3x^3 (to differentiate) and dv =

cos(x) dx (to integrate). Then, compute du and v accordingly.

How many times do I need to apply integration by parts for ∫ 3x^3 cos(x) dx?

You need to apply integration by parts four times to completely reduce the polynomial degree, or use tabular integration for efficiency.

What is the final form of the indefinite integral ∫ 3x^3 cos(x) dx?

The final answer involves a combination of polynomial terms multiplied by sine and cosine functions, plus the constant of integration. It can be expressed as a sum of terms generated through repeated integration by parts.

How do I evaluate the definite integral of 0 to 3 of 3x^3 cos(x) dx?

First, find the indefinite integral using integration by parts, then substitute the limits 0 and 3 into the antiderivative and compute the difference.

Are there any online tools to help evaluate ∫ 3x^3 cos(x) dx?

Yes, online symbolic calculators such as WolframAlpha or Symbolab can evaluate this integral once you input the function and specify the limits or indicate indefinite integration.

Additional Resources

Evaluate The Integral 0 to 3x^3 Cos(x) dx is a mathematical challenge that involves integrating a function with variable limits and a product of polynomial and trigonometric expressions. This type of integral is common in calculus, especially when dealing with functions that combine polynomial growth with oscillatory behavior. In this comprehensive review, we will explore the methods to evaluate this integral, analyze its complexities, and discuss the significance of understanding such integrals in broader mathematical and applied contexts.

Understanding the Integral

The integral in question is:

$$
\int_{0}^{3x^3} \cos(t) \, dt
$$

where the upper limit is not a constant but a function of x, specifically $3x^3$. This setup introduces an additional layer of complexity because the variable of integration, t, is bounded by a function of x. To evaluate this integral with respect to x, we need to consider both the integral's evaluation for a fixed x and the differentiation or further integration involving the upper limit.

Significance of Variable Limits

Variable limits in integrals are common in applications such as physics, engineering, and economics, where the domain of a function depends on another variable. Understanding how to evaluate and manipulate such integrals is fundamental for solving real-world problems involving changing boundaries or dynamic systems.

Methods for Evaluating the Integral

Direct Integration of the Inner Integral

Since the integrand is $\cos(t)$, the indefinite integral is straightforward:

$$
\int \cos(t) \, dt = \sin(t) + C
$$

Applying the Fundamental Theorem of Calculus, the definite integral from 0 to $3x^3$:

$$
\int_{0}^{3x^3} \cos(t) \, dt = \sin(3x^3) - \sin(0) = \sin(3x^3)
$$

because $\sin(0) = 0$.

This simplifies our problem considerably, as the integral reduces to a function of x:

$$
F(x) = \sin(3x^3)
$$

Now, if the goal is to evaluate or analyze this integral in terms of x, the problem becomes the differentiation or further integration of $F(x)$.

Differentiating the Result: Leibniz Rule

Suppose we need to differentiate the integral with respect to x, which is common in applications involving variable limits. The Leibniz rule states:

$$
\frac{d}{dx} \int_{a(x)}^{b(x)} f(t) \, dt = f(b(x)) \cdot b'(x) - f(a(x)) \cdot a'(x)
$$

In our case, $a(x) = 0$ (constant) and $b(x) = 3x^3$. Since $a'(x) = 0$, the derivative simplifies to:

$$
\frac{d}{dx} \int_{0}^{3x^3} \cos(t) \, dt = \cos(3x^3) \cdot \frac{d}{dx}(3x^3) = \cos(3x^3) \cdot 9x^2
$$

Thus, the derivative of the integral with respect to x is:

$$
F'(x) = 9x^2 \cos(3x^3)
$$

This demonstrates how the integral's behavior relates to the polynomial term and the cosine function, illustrating the interplay between the polynomial growth and oscillations.

Evaluating the Integral as a Function of x

From the above, the integral as a function of x is:

$$
I(x) = \sin(3x^3)
$$

This expression can be used in further calculations, such as integrating with respect to x, finding maxima and minima, or analyzing the function's behavior.

Broader Context and Applications

Mathematical Significance

Evaluating integrals with variable upper limits is a foundational skill in calculus, enabling solutions to problems involving accumulated quantities, areas under curves, and system responses over time. The specific integral $\int_{0}^{3x^3} \cos(t) dt$ is a classic example illustrating the application of the

Fundamental Theorem of Calculus and Leibniz rule.

Practical Applications

- Physics: Computing work done or energy transferred where the limits depend on spatial or temporal variables.
- Engineering: Analyzing systems with boundaries that change over time, such as expanding materials or dynamic signal limits.
- Economics: Modeling accumulated profit, cost, or resource consumption where the limits depend on external variables.

Pros and Cons of the Approach

Pros:

- The integral simplifies nicely using fundamental calculus principles.
- Clear application of the Leibniz rule provides insight into the relationship between the integral and its upper limit.
- The method is generalizable to other integrals with variable limits and different integrands.

Cons:

- If the integrand were more complex, such as involving products or compositions of functions, the evaluation would require more advanced techniques like substitution, parts, or numerical methods.
- The approach assumes the integrand is integrable and the functions involved are differentiable, which may not always be the case.

Extending the Analysis

Suppose we now want to evaluate the integral of $F(x) = \sin(3x^3)$ over some interval or find its maximum/minimum points. This involves analyzing the derivative $F'(x) = 9x^2 \cos(3x^3)$.

Critical Points and Behavior

The critical points occur where $F'(x) = 0$:

$$
9x^2 \cos(3x^3) = 0
$$

which implies:

- $x = 0$, or

- $\cos(3x^3) = 0$

The cosine function equals zero at:

$$
3x^3 = \frac{\pi}{2} + n\pi, \quad n \in \mathbb{Z}
$$

leading to:

$$
x = \left(\frac{\pi/2 + n\pi}{3}\right)^{1/3}
$$

These points mark potential maxima, minima, or points of inflection, depending on the second derivative or further analysis.

Further Integrations

If the goal is to integrate $F(x)$ over a specific interval, say from $x = a$ to $x = b$, the integral becomes:

$$
\int_{a}^{b} \sin(3x^3) \, dx
$$

which generally requires substitution because of the x^3 inside the sine argument. Substituting:

$$
u = 3x^3 \Rightarrow du = 9x^2 dx
$$

but since du involves $x^2 dx$, which appears in the derivative, the substitution can be complex and may necessitate numerical methods or special functions for exact evaluation.

Summary and Final Thoughts

Evaluating the integral $\int_{0}^{3x^3} \cos(t) dt$ illustrates core concepts in calculus, including the fundamental theorem, differentiation under the integral sign, and variable substitution. The integral simplifies neatly to $\sin(3x^3)$, providing a clear link between the polynomial growth of the limit and the oscillatory nature of the cosine function.

Understanding such integrals is crucial for mathematical modeling and real-world problem-solving, especially in fields where dynamic boundaries or parameters are involved. The approach highlights the importance of foundational calculus techniques and their power to simplify seemingly complex problems.

Key Takeaways:

- The integral of $\cos(t)$ from 0 to $3x^3$ is $\sin(3x^3)$.
- Differentiating this with respect to x uses the Leibniz rule, revealing the integral's sensitivity to the boundary's change.
- The behavior of the resulting function $\sin(3x^3)$ can be analyzed for critical points, maxima, and minima.
- Techniques used here are applicable to a wide range of problems involving variable limits and integrand compositions.

Final Remarks

Mastering the evaluation of integrals with variable limits and understanding their derivatives provides a powerful toolkit for tackling advanced mathematical problems and practical applications. Whether in physics, engineering, or economics, these techniques enable the modeling of complex systems with changing parameters, leading to deeper insights and more accurate predictions. The integral $\int_{0}^{3x^3} \cos(t) dt$ serves as a simple yet profound example of the elegance and utility of calculus in analyzing the interplay between polynomial and trigonometric functions.

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