Evaluate The Integral.xdx / (7x^2+3)^5

Evaluate The Integral.xdx / (7x^2+3)^5 is a common problem in calculus that involves integrating a function with a quadratic expression in the denominator raised to a power. This type of integral appears frequently in fields such as physics, engineering, and mathematics, particularly in problems involving probability distributions, potential fields, and more. Understanding how to evaluate such integrals is essential for students and professionals working with complex calculus problems.

In this comprehensive guide, we will explore the step-by-step process to evaluate the integral:

$$
\int \frac{x \, dx}{(7x^2 + 3)^5}
$$

by using substitution techniques, integration formulas, and algebraic manipulations. Whether you're a beginner or looking to refine your skills, this article provides detailed explanations to help you master this integral.

Understanding the Integral Structure

Before diving into the solution, it's important to analyze the structure of the integral:

$$
\int \frac{x \, dx}{(7x^2 + 3)^5}
$$

This integral features:

- A numerator involving the linear term x,
- A denominator with a quadratic expression $7x^2 + 3$,
- The quadratic raised to the fifth power.

This structure suggests that substitution techniques, particularly substitution related to the quadratic expression, will be effective.

Step-by-Step Solution Approach

Let's break down the process into clear, manageable steps.

1. Recognize the Suitable Substitution

Since the denominator involves $7x^2 + 3$, and the numerator contains x, a natural substitution is:

$$
u = 7x^2 + 3
$$

This substitution simplifies the integral since its derivative, du, will involve $x dx$.

2. Compute the Differential du

Differentiate both sides with respect to x:

$$
du = 14x \, dx
$$

This relation indicates that:

$$
x \, dx = \frac{du}{14}
$$

Having expressed $x dx$ in terms of du, we are ready to rewrite the integral.

3. Rewrite the Integral in Terms of u

Substitute $u = 7x^2 + 3$ and $x dx = \frac{du}{14}$:

$$
\int \frac{x \, dx}{(7x^2 + 3)^5} = \int \frac{\frac{du}{14}}{u^5} = \frac{1}{14} \int u^{-5} \, du
$$

This simplifies the integral to a basic power rule form.

4. Integrate Using Power Rule

Recall the power rule for integration:

\[

\int u^n \, du = \frac{u^{n+1}}{n+1} + C \quad \text{(for } n \neq -1 \text{)}
\]

Applying this to \(u^{-5}\):

\[
\int u^{-5} \, du = \frac{u^{-4}}{-4} + C
\]

Therefore, the integral becomes:

\[
\frac{1}{14} \times \frac{u^{-4}}{-4} + C = -\frac{1}{56} u^{-4} + C
\]

5. Substitute Back to \(x\)

Recall \(u = 7x^2 + 3\), so:

\[
\boxed{
\int \frac{x \, dx}{(7x^2 + 3)^5} = -\frac{1}{56} \times \frac{1}{(7x^2 + 3)^4} + C
}
\]

This is the evaluated form of the integral.

Final Answer and Interpretation

The integral evaluates to:

\[
\boxed{
\int \frac{x \, dx}{(7x^2 + 3)^5} = -\frac{1}{56 (7x^2 + 3)^4} + C
}
\]

where \(C\) is the constant of integration.

This result indicates that the integral involving the rational function with a quadratic in the denominator simplifies neatly when using substitution. The key step is recognizing the substitution \(u = 7x^2 + 3\) to reduce the integral to a power rule form.

Applications of This Integral

Understanding how to evaluate integrals like $\int \frac{x \, dx}{(7x^2 + 3)^5}$ has practical implications across various scientific disciplines:

- **Physics:** Calculating potential fields and energy distributions where quadratic terms appear in denominators.
- **Engineering:** Signal processing and control systems often involve integrals with quadratic expressions.
- **Probability and Statistics:** Deriving probability density functions that involve quadratic forms in the denominator.
- **Mathematics:** Teaching substitution and power rule techniques for integrating rational functions.

Additional Tips for Evaluating Similar Integrals

When faced with integrals involving quadratic expressions, consider these strategies:

1. Recognize the Derivative Pattern

- If the numerator resembles the derivative of the denominator (or a multiple thereof), substitution is straightforward.

2. Use Substitution Effectively

- Set $u = \text{quadratic expression}$,
- Compute du,
- Express the entire integral in terms of u.

3. Apply Power Rule for Integration

- Integrate powers of u directly once the integral is in the form $\int u^n du$.

4. Always Substitute Back

- After integration, revert to the original variable to present the final answer in terms of x.

Conclusion

Evaluating integrals such as $\int \frac{x \, dx}{(7x^2 + 3)^5}$ demonstrates the power of substitution techniques in calculus. By identifying the quadratic component and its derivative, the integral simplifies to a basic power rule problem. Remembering these steps and strategies can help you confidently tackle similar integrals involving quadratic denominators raised to powers.

Mastering such techniques enhances your calculus toolkit, which is essential for solving complex mathematical problems in science and engineering. Practice with various integrals will further solidify your understanding and ability to evaluate challenging integrals with confidence.

Frequently Asked Questions

How can I evaluate the integral of x dx over $(7x^2 + 3)^5$?

You can evaluate the integral by substitution. Let $u = 7x^2 + 3$, then $du = 14x \, dx$, so $x \, dx = du / 14$. Replacing in the integral, it becomes $(1/14) \int du / u^5$, which simplifies to $(1/14) (-1/4 u^4) + C = -1/56 (7x^2 + 3)^4 + C$.

What substitution is best for solving $\int x \, dx / (7x^2 + 3)^5$?

The best substitution is $u = 7x^2 + 3$, because its derivative $du = 14x \, dx$ directly relates to the numerator $x \, dx$, simplifying the integral significantly.

Is the integral $\int x \, dx / (7x^2 + 3)^5$ solvable using standard substitution methods?

Yes, by substituting $u = 7x^2 + 3$, the integral reduces to a power of u, making it straightforward to evaluate.

How do I handle the integral involving a quadratic expression raised to a power in the denominator?

Use substitution to simplify the quadratic expression, then rewrite the integral in terms of the new variable, often resulting in a power integral that can be directly integrated.

Can this integral be expressed in terms of elementary functions?

Yes, after substitution and simplification, the integral evaluates to a simple algebraic expression involving $(7x^2 + 3)^4$, which are elementary functions.

What is the final answer to the integral $\int x \, dx / (7x^2 + 3)^5$?

The evaluated integral is $-1/56 \, (7x^2 + 3)^{(-4)} + C$.

Are there alternative methods to evaluate this integral besides substitution?

While substitution is the most straightforward method, integration by parts is less practical here. The substitution method is preferred due to its simplicity.

How does understanding the chain rule help in solving this integral?

The chain rule guides the substitution process by identifying the inner function's derivative, which helps simplify the integral into a basic power form.

What are common pitfalls when evaluating integrals like $\int x \, dx / (7x^2 + 3)^5$?

Common pitfalls include forgetting to adjust the differential after substitution, miscalculating the derivative of the inner function, or mishandling the algebra during simplification.

Additional Resources

Evaluate the Integral: $\displaystyle \int \frac{x\,dx}{(7x^2 + 3)^5}$

Introduction

In the realm of integral calculus, integrals involving rational functions combined with polynomial expressions are ubiquitous. They often demand a combination of substitution techniques, algebraic manipulation, and familiarity with standard integral forms. The integral under scrutiny here,

$$
\int \frac{x\,dx}{(7x^2 + 3)^5},
$$

serves as a classic example of such problems, challenging the mathematician to select the most effective method for evaluation. This article aims to dissect this integral rigorously, exploring various approaches, and ultimately arriving at a comprehensive, step-by-step solution suitable for inclusion in advanced calculus review materials or scholarly publications.

Understanding the Integral

The integral features a rational function where the numerator is linear in x, and the denominator is a high power of a quadratic expression. The presence of x in the numerator suggests that substitution methods involving the derivative of the quadratic in the denominator could be fruitful. The structure hints at the potential for a substitution related to the quadratic's inner function.

Let's restate the integral explicitly:

$$
I = \int \frac{x\,dx}{(7x^2 + 3)^5}.
$$

Our goal is to evaluate I in terms of elementary functions or standard integral forms, along with an explicit expression for the indefinite integral.

Preliminary Analysis and Strategy

Recognizing the Structure

- The numerator $x\,dx$ is suggestive of a substitution involving $u = 7x^2 + 3$, since the derivative of $7x^2 + 3$ is $14x$, which closely resembles the numerator $x\,dx$ but differs by a constant factor.

- The denominator is $(7x^2 + 3)^5$, a high power, indicating that substitution could reduce the integral to a rational function in u.

Potential Substitution: $u = 7x^2 + 3$

- Differentiating:

$$
du = 14x\,dx \quad \Rightarrow \quad x\,dx = \frac{du}{14}.
$$

- Substituting into the integral:

$$
I = \int \frac{x\,dx}{u^5} = \int \frac{1}{u^5} \cdot \frac{du}{14} = \frac{1}{14} \int u^{-5} du.
$$

This approach simplifies the integral significantly.

Step-by-Step Solution

Step 1: Substitution

Set

$$
u = 7x^2 + 3,
$$

then

$$
du = 14x\,dx \quad \Rightarrow \quad x\,dx = \frac{du}{14}.
$$

Therefore,

$$
I = \int \frac{x\,dx}{u^5} = \frac{1}{14} \int u^{-5} du.
$$

Step 2: Integrate in terms of u

The integral:

$$
\int u^{-5} du
$$

is straightforward:

$$
\int u^{-5} du = \frac{u^{-4}}{-4} + C,
$$

since

$$
\int u^{n} du = \frac{u^{n+1}}{n+1} + C, \quad n \neq -1.
$$

Applying this:

$$
\int u^{-5} du = -\frac{1}{4} u^{-4} + C.
$$

Step 3: Back-substitute to x

Putting it all together:

$$
I = \frac{1}{14} \times \left(-\frac{1}{4} u^{-4}\right) + C = -\frac{1}{56} u^{-4} + C.
$$

Recall that $u = 7x^2 + 3$, so:

$$
I = -\frac{1}{56} (7x^2 + 3)^{-4} + C.
$$

Final Result

$$
\boxed{
\int \frac{x\,dx}{(7x^2 + 3)^5} = -\frac{1}{56} \left(7x^2 + 3\right)^{-4} + C.
}
$$

This explicit form demonstrates that the integral reduces elegantly via substitution, highlighting the importance of recognizing derivative relationships within the integrand.

Additional Perspectives

Alternative Approaches

While the substitution method outlined above is straightforward and efficient, other techniques could be considered, such as:

- Partial Fraction Decomposition:
In this case, partial fractions are less applicable due to the high power of a quadratic polynomial and the absence of simpler factors.

- Integration by Parts:
Not particularly advantageous here, because the integral reduces more naturally via substitution.

- Trigonometric Substitutions:
Typically employed for integrals involving $\sqrt{a^2 - x^2}$ or similar forms, but unnecessary given the algebraic substitution's simplicity.

Consideration of Definite Integrals

If the integral is definite, say from $x=a$ to $x=b$, the evaluation involves substituting the limits into the antiderivative:

$$
I_{[a,b]} = -\frac{1}{56} \left(7b^2 + 3\right)^{-4} + \frac{1}{56} \left(7a^2 + 3\right)^{-4}.
$$

Practical Implications and Applications

This integral's evaluation is more than an academic exercise; it appears in various fields such as physics, engineering, and applied mathematics, where integrals of rational functions with quadratic denominators arise naturally, especially in problems involving potential energy, probability distributions, and signal processing.

The explicit form is particularly useful for:

- Computational Purposes:
Facilitating quick numerical evaluation.

- Analytical Solutions:
Enabling further analytical development, such as in solving differential equations where such integrals appear as part of the solution process.

- Teaching and Pedagogy:
Demonstrating the effectiveness of substitution methods in integral calculus.

Conclusion

The integral

$$
\int \frac{x\,dx}{(7x^2 + 3)^5}
$$

can be evaluated efficiently through a well-chosen substitution:

$$
u = 7x^2 + 3,
$$

which simplifies the integral to a power function in u. The resulting explicit expression,

$$
-\frac{1}{56} (7x^2 + 3)^{-4} + C,
$$

illustrates how recognizing derivative relationships simplifies seemingly complex integrals. This process underscores the importance of strategic substitution in integral calculus and provides a valuable example for both students and professionals engaged in analytical problem solving.

References

- Stewart, J. (2015). Calculus: Early Transcendentals (8th Edition). Cengage Learning.
- Apostol, T. M. (1967). Calculus, Volume 1. Wiley.
- Spivak, M. (2008). Calculus. Publish or Perish Press.

Note: This detailed investigation confirms that strategic substitution centered around the derivative of the quadratic in the denominator is the most effective approach for evaluating the integral.

[Evaluate The Integral Xdx 7x 2 3 5]

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