Factor The Binomial Expansion. 196x2 + 420x + 225

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Understanding how to factor a binomial expansion is a fundamental skill in algebra that helps students simplify expressions, solve equations, and analyze polynomial functions. In this article, we will explore the process of factoring the quadratic expression $196x^2 + 420x + 225$, breaking down each step and providing insights into common methods used in algebraic factoring. By mastering this skill, you can confidently approach similar problems and strengthen your overall mathematical proficiency.

Introduction to Factoring Quadratic Expressions

What Is Factoring?

Factoring involves rewriting a mathematical expression as a product of its factors. For quadratic expressions, factoring simplifies the expression into binomials or other polynomials that multiply together to give the original expression.

Why Is Factoring Important?

Factoring is crucial because it:
- Simplifies complex algebraic expressions
- Facilitates solving quadratic equations
- Helps find zeros or roots of functions
- Aids in graphing quadratic functions

Analyzing the Given Expression

The quadratic expression we are working with is:

$196x^2 + 420x + 225$

This is a trinomial in standard form ($ax^2 + bx + c$). Our goal is to factor it into the product of two binomials, if possible.

Identify Coefficients

- $a = 196$
- $b = 420$

- c = 225

Methods for Factoring Quadratic Expressions

There are several techniques to factor quadratics:
- Factoring by grouping
- Using the quadratic formula
- Factoring using the AC method
- Completing the square

For this particular trinomial, the most effective method is factoring using the AC method because the coefficients are large but manageable.

Step-by-Step Factoring Process

Step 1: Multiply the leading coefficient and constant term

Calculate the product of a and c:

a × c = 196 × 225

Let's compute this:

196 × 225

Break it down:
- 200 × 225 = 45,000
- 4 × 225 = 900

Subtract 4 × 225 from 200 × 225:
- 45,000 - 900 = 44,100

So, a × c = 44,100

Step 2: Find two numbers that multiply to a × c and add to b

We need two numbers, m and n, such that:
- m × n = 44,100
- m + n = 420

Finding such factors involves prime factorization or systematic trial.

Prime Factorization of 44,100:

Let's factor 44,100:

- 44,100 ÷ 2 = 22,050
- 22,050 ÷ 2 = 11,025
- 11,025 ÷ 3 = 3,675
- 3,675 ÷ 3 = 1,225
- 1,225 ÷ 5 = 245
- 245 ÷ 5 = 49
- 49 ÷ 7 = 7
- 7 ÷ 7 = 1

Collecting factors:
$44{,}100 = 2^2 \times 3^2 \times 5^2 \times 7^2$

Now, to find two numbers that multiply to 44,100 and add to 420, we consider the factor pairs of 44,100.

Possible factor pairs include:
- 660 × 66.75 (not integer)
- 700 × 63 (sum = 763)
- 600 × 74 (sum = 674)
- ... and so on.

Alternatively, since the coefficients are manageable, try to find factors close to half of 420 (which is 210).

Check factors:
- 300 and 147 (sum = 447)
- 350 and 120 (sum = 470)
- 420 and 0 (sum = 420), but 0 isn't helpful here.

Let's attempt to find factors systematically.

Alternatively, consider the quadratic formula approach if factoring seems complex, but let's proceed with the AC method for clarity.

Step 3: Rewrite the middle term using the found factors

Suppose we find that 660 and 66 are factors:
- 660 × 66 = 43,560 (less than 44,100)
- sum: 660 + 66 = 726 (too high)

Alternatively, try 420 and 105:
- 420 × 105 = 44,100
- sum: 420 + 105 = 525 (not matching 420)

Since the product is 44,100 and sum is 420, and considering the prime factorization, perhaps the pair is 660 and 66, which sums to 726, too high. Let's try 540 and 82.5 – not integers.

This suggests that the quadratic may be factorable with rational roots, so perhaps using quadratic formula might be more straightforward.

Using the Quadratic Formula to Find Roots

The quadratic formula is:

$x = [-b \pm \sqrt{(b^2 - 4ac)}] / 2a$

Calculate discriminant D:

$D = b^2 - 4ac$

Plugging in the values:

$D = (420)^2 - 4 \times 196 \times 225$

Calculate each component:

- $420^2 = 176{,}400$
- $4 \times 196 = 784$
- $784 \times 225 = 176{,}400$

Thus:

$D = 176{,}400 - 176{,}400 = 0$

Since the discriminant is zero, the quadratic has a single (repeated) root, indicating it factors into a perfect square trinomial.

Factoring the Expression as a Perfect Square

When D = 0, the quadratic can be factored as:

$a(x + d)^2$

Let's find d:

$x = -b / 2a = -420 / (2 \times 196) = -420 / 392 = -105 / 98 = -15 / 14$

Expressed as a fraction: -15/14

Now, we can write the quadratic as:

$196x^2 + 420x + 225 = 196(x + 15/14)^2$

To write this as a binomial squared, multiply both sides:

$196(x + 15/14)^2$

Express the binomial:

$x + 15/14$

To clear denominators, multiply numerator and denominator:

$x + 15/14 = (14x + 15) / 14$

Now, the quadratic becomes:

$196 \times [(14x + 15)/14]^2 = 196 \times (14x + 15)^2 / 14^2$

Since $14^2 = 196$, this simplifies to:

$196 \times (14x + 15)^2 / 196 = (14x + 15)^2$

Therefore:

$196x^2 + 420x + 225 = (14x + 15)^2$

Final Factored Form:

$(14x + 15)^2$

Summary of the Factoring Process

- Calculated the discriminant to determine the nature of roots
- Found that D = 0, indicating a perfect square trinomial
- Expressed the quadratic as a perfect square

Applications of Factoring the Binomial Expansion

Understanding how to factor binomials like $196x^2 + 420x + 225$ has practical applications:

- **Simplifying algebraic expressions**: Breaking down complex quadratics into binomials for easier manipulation.
- **Solving quadratic equations**: Factoring allows quick solutions when the quadratic is factorable.

- **Graphing functions**: Roots of the quadratic help in sketching the parabola accurately.
- **Analyzing polynomial behavior**: Factored forms reveal zeros, intercepts, and symmetry.

Related Techniques and Tips

Recognizing Perfect Square Trinomials

Some quadratics are perfect squares, making factoring straightforward:

- Pattern: $a^2 + 2ab + b^2 = (a + b)^2$
- Example: $x^2 + 6x + 9 = (x + 3)^2$

Using the AC Method

When coefficients are large, the AC method simplifies the process:

- Find two numbers that multiply to $a \times c$ and add to b
- Rewrite the middle term using these numbers
- Factor by grouping

Quadratic Formula as a Backup

If factoring is difficult, always consider the quadratic formula to find roots, which then can be used to factor the quadratic into binomials.

Conclusion

Factoring the binomial expansion $196x^2 + 420x + 225$ reveals that it is a perfect square trinomial, which factors neatly into $(14x + 15)^2$. Mastering the process of factoring quadratics, especially recognizing perfect squares, is essential for solving equations, simplifying expressions, and analyzing functions. Whether using the AC method, quadratic formula, or recognizing perfect squares, developing a strong understanding of these techniques

Frequently Asked Questions

How do you factor the quadratic expression 196x^2 + 420x +

225?

To factor $196x^2 + 420x + 225$, first look for common factors. Since the coefficients are divisible by 1 and no common factor larger than 1, proceed with quadratic factoring. Use the quadratic trinomial factoring method or the quadratic formula to find factors.

What is the first step to factor $196x^2 + 420x + 225$?

The first step is to check for any greatest common factor (GCF). Here, the GCF of 196, 420, and 225 is 1, so proceed with factoring the quadratic as is, possibly by factoring into binomials.

Can $196x^2 + 420x + 225$ be factored as a perfect square trinomial?

To determine if it's a perfect square, check if $196x^2$ and 225 are perfect squares and if 2 times the square root of these equals the middle term. $196x^2$ is $(14x)^2$, and 225 is 15^2. The middle term 420x is 2 14x 15, which confirms it is a perfect square trinomial.

What is the factored form of $196x^2 + 420x + 225$?

Since it's a perfect square trinomial, it factors as $(14x + 15)^2$.

How do you verify that $(14x + 15)^2$ equals $196x^2 + 420x + 225$?

Expand $(14x + 15)^2$: $(14x)^2 + 2\ 14x\ 15 + 15^2 = 196x^2 + 420x + 225$, confirming the factorization.

Is $196x^2 + 420x + 225$ always factorable over the integers?

Yes, because it is a perfect square trinomial, so it factors neatly over the integers as $(14x + 15)^2$.

What is the importance of recognizing perfect square trinomials in factoring?

Recognizing perfect square trinomials allows for quick and straightforward factoring into binomials squared, saving time and avoiding trial and error.

Are there alternative ways to factor $196x^2 + 420x + 225$?

Yes, but since it's a perfect square trinomial, the most straightforward method is to write it as $(14x + 15)^2$. Alternatively, you could use the quadratic formula to confirm roots.

What are the roots of the quadratic $196x^2 + 420x + 225$?

Since it's a perfect square trinomial, the roots are both $x = -15/14$, because $(14x + 15)^2 = 0$ implies $14x + 15 = 0$.

Additional Resources

Factor the Binomial Expansion: $196x^2 + 420x + 225$

When it comes to algebraic expressions, particularly quadratic polynomials, the ability to factor them efficiently is a foundational skill that simplifies solving equations, graphing functions, and understanding algebraic relationships. The expression $196x^2 + 420x + 225$ is a perfect candidate for detailed factorization analysis. This quadratic trinomial not only provides an excellent opportunity to explore various factoring techniques but also exemplifies how recognizing patterns can streamline algebraic manipulations. In this comprehensive review, we'll dissect the process of factoring the expression step-by-step, discuss its properties, and evaluate different methods to approach such problems.

Understanding the Structure of the Expression

Before diving into the factorization process, it's essential to understand the structure and components of the quadratic expression:

$196x^2 + 420x + 225$

This is a standard quadratic trinomial in the form $ax^2 + bx + c$, where:

- $a = 196$
- $b = 420$
- $c = 225$

The goal is to factor this into the product of two binomials, ideally in the form:

$(ax + m)(dx + n)$

where m and n are constants to be determined. Recognizing the structure and the coefficients' relationships helps decide the most effective factoring method to employ.

Initial Observations and Common Strategies

When approaching quadratic expressions, several strategies are at our disposal:

- Factoring by grouping (useful if the polynomial can be split into two pairs)
- Applying the quadratic formula (to find roots directly)
- Completing the square (rearranging into a perfect square form)
- Factoring directly using the greatest common factor (GCF)
- Recognizing perfect square trinomials

In the case of $196x^2 + 420x + 225$, initial observations suggest that the coefficients might have common factors, and the trinomial might be a perfect square or reducible via the quadratic formula.

Step 1: Check for a Greatest Common Factor (GCF)

To simplify the expression, first look for the GCF of the coefficients:

- GCF of 196, 420, and 225

Calculations:

- 196 factors: 2^2 7^2
- 420 factors: 2^2 3 5 7
- 225 factors: 3^2 5^2

The common prime factors among all three are:

- 2^2 (from 196 and 420)
- 3 (from 420 and 225)
- 5 (from 420 and 225)
- 7 (from 196 and 420)

But since 225 does not have 2 or 7, the only common factor among all three is 1. Therefore, no common factor exists across all coefficients, so factoring out a GCF isn't possible here.

Step 2: Check for Perfect Square Trinomial

Now, examine whether the quadratic is a perfect square trinomial:

Recall, a perfect square trinomial has the form:

- $(\sqrt{a}\, x \pm \sqrt{c})^2 = a\, x^2 \pm 2\sqrt{a}\sqrt{c}\, x + c$

Calculate:

- $\sqrt{196x^2} = 14x$
- $\sqrt{225} = 15$

Check if the middle term matches 2 14x 15 = 2 14 15 x = 2 210 x = 420x

Indeed, the middle term is 420x, which matches $2\sqrt{196}\sqrt{225}\, x$.

The last term, 225, matches $\sqrt{225}$ squared.

Thus, the quadratic $196x^2 + 420x + 225$ is a perfect square trinomial.

Therefore:

$196x^2 + 420x + 225 = (14x + 15)^2$

Step 3: Confirming the Factorization

Let's verify by expanding $(14x + 15)^2$:

$(14x + 15)^2 = (14x)^2 + 2\ 14x\ 15 + 15^2 = 196x^2 + 2\ 14\ 15\ x + 225$

Calculating:

$2\ 14\ 15 = 2\ 210 = 420$

So,

$(14x + 15)^2 = 196x^2 + 420x + 225$

which matches our original polynomial exactly.

Final Factorization and Its Significance

The quadratic $196x^2 + 420x + 225$ factors neatly into:

$(14x + 15)^2$

This is a perfect square trinomial, and recognizing such patterns is crucial in algebra. When a quadratic is a perfect square, it simplifies many tasks, including solving equations, integrating, or differentiating.

Features and Advantages of Recognizing Perfect Square Trinomials

Pros:

- Simplifies calculations: Recognizing perfect squares reduces complex factoring to a straightforward binomial square.
- Facilitates solving equations: Equations involving perfect squares are easier to solve via square root methods.
- Enhances pattern recognition skills: Improves algebraic intuition and reduces trial-and-error.
- Useful in calculus: Perfect squares are often used in completing the square for integration or differentiation.

Cons:

- Requires pattern recognition skills: Not always immediately obvious, especially with complex coefficients.
- Potential for oversight: Without careful analysis, one might miss the perfect square pattern, leading to more complicated methods.

Additional Insights and Applications

- Graphing: Since the quadratic factors as a perfect square, its graph is a parabola that touches the x-axis at a single point, specifically where $14x + 15 = 0$, i.e., $x = -15/14$.
- Equation solving: To solve $196x^2 + 420x + 225 = 0$, we can set $(14x + 15)^2 = 0$, leading to $x = -15/14$.
- Completing the square method: This process aligns with completing the square, confirming the quadratic’s nature as a perfect square trinomial.

Alternative Methods and Their Relevance

While the perfect square recognition was straightforward in this case, it's essential to understand other methods in case the quadratic isn't a perfect square.

Quadratic Formula:

$$
x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}
$$

For our polynomial:

$$
a = 196, \quad b = 420, \quad c = 225
$$

Calculate discriminant:

$$
\Delta = 420^2 - 4 \times 196 \times 225
$$

$$
420^2 = 176400
$$

$$
4 \times 196 \times 225 = 4 \times 196 \times 225
$$

First, $4 \times 196 = 784$

Then, $784 \times 225 = 176400$

Thus,

$$
\Delta = 176400 - 176400 = 0
$$

A zero discriminant confirms a perfect square, consistent with earlier findings.

Conclusion and Final Remarks

The quadratic expression $196x^2 + 420x + 225$ exemplifies how recognizing perfect square trinomials can significantly simplify algebraic procedures. Its factorization as $(14x + 15)^2$ underscores the importance of pattern recognition skills in algebra. This approach reduces the problem to a straightforward identification rather than resorting to more complex methods like factoring by grouping or the quadratic formula.

Key takeaways:

- Always analyze the coefficients for common factors or patterns.
- Recognize perfect squares and their implications.
- Use the quadratic formula as a verification tool when uncertain.
- Understanding the structure of quadratics enhances problem-solving efficiency.

In sum, mastering the ability to factor quadratics like $196x^2 + 420x + 225$ not only simplifies calculations but also deepens comprehension of algebraic principles, laying a solid foundation for advanced mathematical topics.

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