Find A Formula For The Nth Term Of Sequence:3,5,9,17

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Understanding how to determine the nth term of a sequence is fundamental in mathematics, especially in algebra and number theory. Sequences appear frequently in various mathematical problems, and finding a general formula allows us to understand the pattern and predict future terms efficiently. In this article, we will explore how to find a formula for the nth term of the sequence 3, 5, 9, 17, and provide a comprehensive step-by-step guide to analyze and derive the formula.

Understanding the Sequence

Before diving into formula derivation, it’s essential to analyze the sequence's pattern carefully. The sequence given is:

3, 5, 9, 17

Let's write down the sequence with their corresponding positions:

n	Term (a_n)
1	3
2	5
3	9
4	17

Our goal is to find a function a(n) that can generate the nth term directly, without needing to compute previous terms.

Initial Observations and Pattern Recognition

To identify the pattern, examine the differences between consecutive terms:

- $5 - 3 = 2$
- $9 - 5 = 4$
- $17 - 9 = 8$

The differences are: 2, 4, 8.

Notice that these differences are doubling each time:

2, 4, 8

This indicates that the sequence might be related to powers of 2 or exponential growth.

Next, examine the second differences:

- $4 - 2 = 2$
- $8 - 4 = 4$

The second differences are not constant; thus, the sequence is not quadratic. The pattern of differences suggests exponential growth, likely involving powers of 2.

Identifying the Pattern: Is it Geometric or Recursive?

Given the differences double each time, consider the possibility that the sequence might be generated by a recursive relation involving powers of 2:

$a_n = a_{n-1} + 2^{n-1}$

Let's verify this with the sequence:

- Starting with $a_1 = 3$
- $a_2 = a_1 + 2^{1} = 3 + 2 = 5$ ✓
- $a_3 = a_2 + 2^{2} = 5 + 4 = 9$ ✓
- $a_4 = a_3 + 2^{3} = 9 + 8 = 17$ ✓

This recursive pattern fits the sequence perfectly.

Thus, the sequence can be defined recursively as:

$a_n = a_{n-1} + 2^{n-1}$ with $a_1 = 3$

While recursive definitions are useful, an explicit formula allows direct calculation without recursion.

Deriving the Explicit Formula

Given the recursive relation:

$a_n = a_{n-1} + 2^{n-1}$

and initial term:

$a_1 = 3$

we can derive the explicit formula by summing the recursive additions:

$a_n = a_1 + \sum_{k=2}^{n} 2^{k-1}$

Note that the sum starts from k=2 because the first term is given, and subsequent terms are generated by adding the powers of 2.

Rewrite the sum:

$a_n = 3 + \sum_{k=2}^{n} 2^{k-1}$

Change the index to make summation clearer:

Let $j = k - 1$

When $k=2$, $j=1$

When $k=n$, $j=n-1$

Thus,

$a_n = 3 + \sum_{j=1}^{n-1} 2^{j}$

Now, the sum of a geometric series:

$\sum_{j=1}^{m} r^{j} = r (r^{m} - 1) / (r - 1)$

For $r=2$:

$\sum_{j=1}^{m} 2^{j} = 2 (2^{m} - 1) / (2 - 1) = 2 (2^{m} - 1)$

Applying this to our sum:

$\sum_{j=1}^{n-1} 2^{j} = 2 (2^{n-1} - 1)$

Therefore, our explicit formula becomes:

$a_n = 3 + 2 (2^{n-1} - 1)$

Simplify:

$a_n = 3 + 2\ 2^{n-1} - 2$

$a_n = (3 - 2) + 2\ 2^{n-1}$

$a_n = 1 + 2^{n}$ (since $2\ 2^{n-1} = 2^{1}\ 2^{n-1} = 2^{n}$)

Final explicit formula:

$a(n) = 1 + 2^{n}$

Verification of the Formula

Let's verify the explicit formula with the known terms:

- For n=1:

$a(1) = 1 + 2^{1} = 1 + 2 = 3$ ✓

- For n=2:

$a(2) = 1 + 2^{2} = 1 + 4 = 5$ ✓

- For n=3:

$a(3) = 1 + 8 = 9$ ✓

- For n=4:

$a(4) = 1 + 16 = 17$ ✓

The formula perfectly matches the sequence.

Summary of the Findings

- The sequence is generated recursively as: $a_n = a_{n-1} + 2^{n-1}$, with $a_1=3$.
- The explicit formula for the nth term is:

$a(n) = 1 + 2^{n}$

This explicit formula allows direct computation of any term in the sequence without recursive calculations.

Understanding the Significance of the Formula

Knowing the explicit formula $a(n) = 1 + 2^{n}$ provides several advantages:

- Efficiency: Calculating any term directly without computing previous terms.
- Pattern Recognition: Recognizing exponential growth in sequences.
- Mathematical Exploration: Applying the formula to analyze sequence behavior for large n.
- Extensions: Using the formula as a basis for exploring related sequences or series.

Applications of the Sequence and Formula

Sequences of this nature appear in various fields, including:

- Computer Science: Analyzing algorithms with exponential time complexity.
- Cryptography: Understanding exponential functions in encryption algorithms.
- Mathematics Education: Teaching recursive and explicit sequence formulas.
- Physics: Modeling phenomena with exponential growth or decay.

Additional Techniques for Sequence Analysis

While in this case, pattern recognition led directly to the explicit formula, other methods are useful for different sequences:

Method 1: Difference Method

- Compute first differences to check if the sequence is arithmetic.
- Compute second differences to check for quadratic sequences.
- Higher-order differences may reveal polynomial degrees.

Method 2: Recursive Relations

- Derive recursive formulas by analyzing how each term relates to previous ones.
- Use initial terms to formulate and verify recursive relations.

Method 3: Generating Functions

- Construct the generating function for the sequence.
- Use algebraic manipulation to derive an explicit formula.

Conclusion

Finding an explicit formula for the nth term of a sequence is a powerful skill in mathematics. For the sequence 3, 5, 9, 17, the process involved analyzing the pattern of differences, recognizing the exponential growth, formulating a recursive relation, and then deriving the explicit formula. The result, a(n) = 1 + 2^{n}, provides a straightforward way to compute any term in the sequence directly.

Mastering these techniques broadens your understanding of sequences and series, enabling you to approach a variety of mathematical problems with confidence. Whether in academic studies, professional applications, or problem-solving exercises, being able to find and verify explicit formulas is an essential mathematical tool.

If you want to explore further, consider applying similar methods to other sequences or extending the current sequence by analyzing its behavior for larger n.

Frequently Asked Questions

What is the pattern in the sequence 3, 5, 9, 17?

The sequence appears to follow a pattern where each term is generated by doubling the previous term and subtracting 1, starting from 3.

How can I find the general formula for the nth term of the sequence 3, 5, 9, 17?

First, observe the pattern or differences between terms; then, derive a recurrence relation or identify a closed-form formula using methods like solving difference equations or recognizing geometric patterns.

Is the sequence 3, 5, 9, 17 arithmetic or geometric?

No, the sequence is neither arithmetic nor geometric. The differences between consecutive terms are 2, 4, and 8, which suggests a different pattern involving powers of 2.

What is the recursive formula for this sequence?

A possible recursive formula is: $a_1 = 3$, and $a_n = 2\ a_{n-1} - 1$ for $n > 1$.

Can I express the nth term of the sequence explicitly?

Yes. Since the sequence doubles and subtracts 1 each time, the explicit formula is $a_n = 2^{(n+1)} - 1$.

How do I verify the formula for the nth term of the sequence?

You can verify by substituting values of n into the formula $a_n = 2^{(n+1)} - 1$ and checking if it matches the sequence terms: for n=1, $a_1=3$; n=2, $a_2=5$; n=3, $a_3=9$; n=4, $a_4=17$.

Additional Resources

Find A Formula For The Nth Term Of Sequence: 3, 5, 9, 17

In the realm of mathematics, sequences serve as fundamental building blocks for understanding patterns, relationships, and structures within numbers. Among these, the task of determining an explicit formula for the n-th term of a sequence is both a classic challenge and a gateway to deeper mathematical insights. In this article, we delve into the sequence 3, 5, 9, 17, exploring methods to find its nth term, investigating its properties, and illustrating the process with comprehensive analysis. This investigation aims to serve as a detailed guide for educators, students, and enthusiasts interested in sequence analysis and pattern recognition.

Understanding the Sequence: Initial Observations

Before attempting to derive a formula, it is essential to understand the sequence's behavior and characteristics.

The sequence provided is:

- Term 1: 3
- Term 2: 5
- Term 3: 9
- Term 4: 17

Let’s denote the sequence as a_n, where:

$$
a_1 = 3, \quad a_2 = 5, \quad a_3 = 9, \quad a_4 = 17
$$

Examining the Differences

One common approach in sequence analysis involves computing the differences between consecutive terms to identify potential patterns.

Term n	a_n	First Difference $d_n = a_{n} - a_{n-1}$
1	3	—
2	5	5 - 3 = 2
3	9	9 - 5 = 4
4	17	17 - 9 = 8

The sequence of differences is: 2, 4, 8

Notice that these differences seem to be doubling each time.

Second Differences

Calculate the differences of the first differences:

d_n	2	4	8
$d_{n+1} - d_n$	—	$4 - 2 = 2$	$8 - 4 = 4$

Second differences: 2, 4

Since the second differences are not constant but are increasing, this suggests the sequence does not follow a simple quadratic pattern. However, the pattern in the first differences indicates exponential growth.

Identifying the Pattern: Recognizing the Sequence Type

Given the observed differences, particularly the doubling pattern, it is plausible that the sequence exhibits exponential or recursive behavior.

The sequence of first differences (2, 4, 8) doubles each time, which hints at a geometric progression.

Observation:

$$
d_{n} = 2^{n}
$$

for $n \geq 1$.

Check this hypothesis:

- $d_2 = a_2 - a_1 = 5 - 3 = 2$, and $2^{1} = 2$ (matches)
- $d_3 = 9 - 5 = 4$, $2^{2} = 4$ (matches)
- $d_4 = 17 - 9 = 8$, $2^{3} = 8$ (matches)

Thus, the differences follow:

$$
a_{n} - a_{n-1} = 2^{n-1}
$$

for $n \geq 2$.

Deriving the Explicit Formula

Having identified the recursive relation:

$$
a_{n} = a_{n-1} + 2^{n-1}
$$

and knowing the initial term $a_1 = 3$, we can express the sequence explicitly.

Step-by-step derivation:

$$
a_{n} = a_1 + \sum_{k=2}^{n} (a_k - a_{k-1}) = 3 + \sum_{k=2}^{n} 2^{k-1}
$$

Rearranged:

$$
a_{n} = 3 + \sum_{k=1}^{n-1} 2^{k}
$$

since substituting $k = k' + 1$, where $k' = 1, 2, \dots, n-1$.

The sum of the geometric series:

$$
\sum_{k=1}^{m} 2^{k} = 2^{m+1} - 2
$$

for $m \geq 1$.

Applying this to our sum:

$$
a_{n} = 3 + (2^{n} - 2) = 2^{n} + 1
$$

Final Explicit Formula:

\[
\boxed{
a_{n} = 2^{n} + 1

}
\]

Verification of the Formula

Let's verify the explicit formula with initial terms:

- For $n=1$:

$$
a_1 = 2^{1} + 1 = 2 + 1 = 3 \quad \checkmark
$$

- For $n=2$:

$$
a_2 = 2^{2} + 1 = 4 + 1 = 5 \quad \checkmark
$$

- For $n=3$:

$$
a_3 = 2^{3} + 1 = 8 + 1 = 9 \quad \checkmark
$$

- For $n=4$:

$$
a_4 = 2^{4} + 1 = 16 + 1 = 17 \quad \checkmark
$$

The formula matches the sequence perfectly.

Properties of the Sequence

Understanding the sequence further enhances comprehension.

Growth Rate

The sequence $a_{n} = 2^{n} + 1$ exhibits exponential growth, doubling with each successive term, which aligns with the observed differences.

Recurrence Relation

The explicit formula can be derived from the recurrence relation:

$$
a_{n} = a_{n-1} + 2^{n-1}
$$

with the initial condition $a_1 = 3$.

This recursive form is useful for computational purposes, especially when generating terms sequentially.

Closed-Form Expression

The explicit formula:

$$
a_{n} = 2^{n} + 1
$$

provides a direct computation method, eliminating the need for recursive calculations.

Alternative Approaches and Generalizations

While the above derivation is straightforward, alternative methods can also be employed to analyze such sequences.

Method 1: Polynomial Fitting

Given the sequence's initial terms, one might attempt polynomial interpolation to find a polynomial $P(n)$ such that:

$$
P(1) = 3, \quad P(2) = 5, \quad P(3) = 9, \quad P(4) = 17
$$

However, since the differences suggest exponential behavior, polynomial fitting would likely lead to higher-degree polynomials rather than a simple closed-form.

Method 2: Recognizing Patterns in Differences

As previously identified, the sequence's first differences are powers of two:

$$
a_{n} - a_{n-1} = 2^{n-1}
$$

This insight directly leads to the sum of a geometric series, simplifying the derivation of the explicit formula.

Generalizations

Sequences exhibiting exponential difference patterns can often be expressed as:

$$
a_{n} = A \cdot r^{n} + B
$$

for constants A, B, r. In our case, $r=2$, $A=1$, and $B=1$, leading to the explicit formula.

Conclusion: Significance and Applications

The process of finding a formula for the n-th term of the sequence: 3, 5, 9, 17 exemplifies fundamental techniques in sequence analysis, emphasizing the importance of difference tables and recognizing geometric patterns.

The explicit formula:

$$
a_{n} = 2^{n} + 1
$$

not only simplifies computation but also provides insights into the sequence's growth behavior and recursive structure. Recognizing such patterns is crucial in various fields, including computer science (algorithm analysis), physics (modeling exponential phenomena), and finance (compound interest calculations).

Furthermore, this investigation underscores the importance of pattern recognition and mathematical deduction in deriving closed-form expressions, skills that are vital for advanced mathematical research and problem-solving.

In summary, by analyzing the differences, identifying the geometric pattern, and summing the series, we successfully derived the explicit formula for the sequence:

$$
\boxed{
a_{n} = 2^{n} + 1
}
$$

This formula encapsulates the sequence's exponential growth and serves as a foundational example of sequence analysis techniques.

References

- Stewart, J.

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