Find F: F'(t) = T + 1/t, T>0, F(1) = 6

Find F: F'(t) = T + 1/t, T > 0, F(1) = 6 is a common problem in calculus that involves finding an original function $F(t)$ given its derivative $F'(t)$, a constant T, and an initial condition. This problem combines techniques of integration, the application of initial conditions, and understanding the behavior of functions involving parameters. In this comprehensive guide, we will explore how to approach and solve this problem step-by-step, clarify the concepts involved, and provide insights into similar derivative and integration problems.

Understanding the Problem: Key Components

Before diving into the solution process, it is essential to understand the components involved in the problem.

Derivative Function $F'(t)$

The derivative of the function is given as:

- $F'(t) = T + \frac{1}{t}$
- T is a positive constant ($T > 0$)

This indicates that the rate of change of $F(t)$ with respect to t is composed of a constant term T and a term involving $1/t$.

Initial Condition $F(1) = 6$

The value of the function at $t = 1$ is specified:

- $F(1) = 6$

This initial condition helps determine the constant of integration after integrating $F'(t)$.

Step-by-Step Solution Approach

To find $F(t)$, follow these systematic steps:

Step 1: Write the integral for $F(t)$

Since $F'(t)$ is known, integrate $F'(t)$ with respect to t:

$$
F(t) = \int F'(t) \, dt + C
$$

\]
which becomes:
\[
F(t) = \int \left(T + \frac{1}{t} \right) dt + C
\]

Step 2: Integrate term-by-term

Break down the integral:
\[
F(t) = \int T \, dt + \int \frac{1}{t} \, dt + C
\]
Calculating each integral:

- \(\int T \, dt = T t \), since \(T \) is a constant

- \(\int \frac{1}{t} \, dt = \ln |t| \)

Thus, the general form of \(F(t) \) becomes:
\[
F(t) = T t + \ln |t| + C
\]

Step 3: Apply the initial condition to find \(C \)

Use \(F(1) = 6 \):
\[
6 = T \times 1 + \ln |1| + C
\]
Since \(\ln 1 = 0 \):
\[
6 = T + 0 + C
\]
which simplifies to:
\[
C = 6 - T
\]

Final Expression for \(F(t) \)

Putting it all together, the function \(F(t) \) is:
\[
F(t) = T t + \ln |t| + 6 - T
\]
or equivalently,
\[
\boxed{

$$F(t) = T (t - 1) + \ln |t| + 6$$
}

Understanding the Solution and Its Components

This solution illustrates the importance of integrating derivatives to retrieve original functions, especially when the derivative involves parameters and variable expressions.

Role of the Parameter T

- Since T is positive ($T > 0$), it influences the linear component $T t$ in the solution.
- The term $T (t - 1)$ in the final expression reflects how the initial condition adjusts the function depending on the value of T.

Handling the Absolute Value in $\ln |t|$

- The natural logarithm $\ln |t|$ is defined for all real $t \neq 0$.
- Typically, in calculus problems like this, $t > 0$, especially when initial conditions are given at $t = 1$.
- Therefore, in most practical contexts, we can write:
$$F(t) = T (t - 1) + \ln t + 6$$
assuming $t > 0$.

Applications and Related Problems in Calculus

Understanding how to find a function from its derivative is fundamental in calculus, with applications across physics, engineering, and mathematics.

1. Solving Differential Equations

- The problem is an example of solving a simple first-order differential equation where the derivative involves parameters and variable terms.
- Similar problems involve integrating derivatives with respect to different variables and incorporating initial conditions to find particular solutions.

2. Analyzing Function Behavior

- Once $F(t)$ is known, you can analyze its behavior:

- Increasing or decreasing behavior based on $F'(t)$
- Concavity and convexity via second derivatives

- Limits as $t \to 0^+$ or $t \to \infty$

3. Practice Problems for Mastery

- Find $G(t)$ given $G'(t) = a t + \frac{b}{t}$, with initial conditions.
- Determine $H(t)$ where $H'(t) = c \sin t + d \cos t$, and $H(\pi)$ is known.
- Solve for $K(t)$ given $K'(t) = m e^{n t}$, with $K(0)$ specified.

Summary and Key Takeaways

- To find a function $F(t)$ from its derivative $F'(t)$, integrate term-by-term.
- Apply initial conditions to determine constants of integration.
- When derivatives involve parameters like T, include them in the integration process and interpret their effects on the original function.
- The natural logarithm appears naturally when integrating $1/t$, with domain considerations.

Conclusion

The problem **Find F: F'(t) = T + 1/t, T > 0, F(1) = 6** exemplifies fundamental calculus techniques that are essential for solving differential equations and understanding the relationship between a function and its derivative. By carefully integrating and applying initial conditions, you can reconstruct functions accurately and analyze their behavior in various contexts. Mastery of these methods is crucial for students and professionals working with mathematical models, physics problems, and engineering systems.

For further practice, try modifying the parameters, initial conditions, or derivatives to develop a deeper understanding of integration and differential equations.

Frequently Asked Questions

How do you find the function F(t) given its derivative F'(t) = T + 1/t with T > 0?

To find F(t), integrate F'(t) with respect to t: $F(t) = \int(T + 1/t)\, dt = Tt + \ln|t| + C$, where C is the constant of integration.

What is the value of the constant C in the function F(t) = Tt + ln|t| + C if F(1) = 6?

Substitute $t = 1$ and $F(1) = 6$ into the function: $6 = T 1 + \ln(1) + C$. Since $\ln(1) = 0$, we get $C = 6 - T$.

Given $F'(t) = T + 1/t$ and $F(1) = 6$, what is the explicit form of $F(t)$?

$F(t) = Tt + \ln|t| + (6 - T)$.

How does the parameter T affect the shape of the function $F(t)$?

Since $F(t) = Tt + \ln|t| + (6 - T)$, increasing T makes the linear part Tt more dominant, affecting the slope. The overall shape combines linear growth and logarithmic behavior, with T controlling the slope of the linear component.

Can $F(t)$ be defined for $t < 0$ given the derivative $F'(t) = T + 1/t$?

No, because $1/t$ is undefined at $t = 0$ and is negative for $t < 0$, so the function's domain is typically $t > 0$ to keep the logarithm and derivative well-defined.

What is the significance of the initial condition $F(1) = 6$ in solving for $F(t)$?

It allows us to determine the constant of integration C, ensuring the specific solution matches the given initial value.

If $T = 2$, what is the explicit form of $F(t)$ given $F(1) = 6$?

Substitute $T = 2$ into $F(t)$: $F(t) = 2t + \ln|t| + (6 - 2) = 2t + \ln|t| + 4$.

Additional Resources

In-Depth Analysis of the Function Find F: $F'(t) = T + 1/t$, with $T > 0$, and $F(1) = 6$

Understanding the behavior and properties of functions defined through derivatives is fundamental in calculus. The problem at hand involves a function $F(t)$ with a given derivative, $F'(t) = T + \frac{1}{t}$, where $T > 0$, and an initial condition $F(1) = 6$. This setup provides a rich ground for exploring integration, constant determination, and the function’s qualitative features. Let us delve into each aspect systematically.

Overview of the Problem and Its Significance

The task is to determine the explicit form of the function $F(t)$, given its derivative and an initial condition. This is a classic problem in differential calculus: reconstructing a function from its derivative. Such problems appear widely in physics, engineering, and applied mathematics, where the derivative often models a rate of change (velocity, growth rate, etc.), and the original function signifies position, accumulated quantity, or state.

Specifically, the derivative $F'(t) = T + \frac{1}{t}$ combines a constant term T with a reciprocal term $\frac{1}{t}$. The constant T is positive, influencing the overall trend of $F(t)$, while the $\frac{1}{t}$ component introduces a logarithmic behavior, especially as $t \to 0^+$ or $t \to \infty$. The initial condition $F(1) = 6$ anchors the function, allowing us to determine the integration constant.

Step-by-Step Derivation of the Function $F(t)$

Integration of the Derivative

Since $F'(t) = T + \frac{1}{t}$, integrating both sides with respect to t yields:

$$
F(t) = \int F'(t) \, dt = \int \left(T + \frac{1}{t} \right) dt
$$

Breaking this into simpler integrals:

$$
F(t) = \int T \, dt + \int \frac{1}{t} \, dt
$$

which evaluates to:

$$
F(t) = T t + \ln |t| + C
$$

where C is the constant of integration.

Note: Since the problem context involves $t > 0$ (implied by the initial condition at $t=1$, and the natural logarithm's domain), we can simplify $\ln |t|$ to $\ln t$.

Determining the Constant C

Applying the initial condition $F(1) = 6$:

$$
F(1) = T \cdot 1 + \ln 1 + C = T + 0 + C = 6

\]

Thus:

$$
C = 6 - T
$$

Final explicit form of $F(t)$:

$$
\boxed{
F(t) = T t + \ln t + 6 - T
}
$$

This expression explicitly defines $F(t)$ in terms of the parameter T.

Analyzing the Behavior of $F(t)$

Understanding the qualitative behavior of $F(t)$ is essential for applications and theoretical insights.

Monotonicity and Critical Points

- Since $F'(t) = T + \frac{1}{t}$, and $T > 0$, it follows that:

$$
F'(t) > 0 \quad \text{for all } t > 0
$$

- Implication: $F(t)$ is strictly increasing on $(0, \infty)$.

- Conclusion: The function has no local maxima or minima for $t > 0$; it is monotonically increasing.

Behavior at the Boundaries

- As $t \to 0^+$:

$$
F(t) \sim T \cdot 0 + \ln t + 6 - T \to -\infty
$$

because $\ln t \to -\infty$. So, $F(t) \to -\infty$ near zero.

- As $t \to \infty$:

$$
F(t) \sim T t + \ln t + 6 - T \to +\infty
$$

since the linear term dominates.

Graphical Interpretation

- The function starts from $-\infty$ as $t \to 0^+$.

- It increases monotonically, crossing $F(1) = 6$, with the slope $F'(t)$ remaining positive.

- As $t \to \infty$, $F(t) \to \infty$.

Parameter T: Its Role and Implications

The parameter $T > 0$ significantly influences the shape and growth rate of $F(t)$.

Impact on Growth Rate

- Larger T values increase the linear component $T t$, resulting in a steeper ascent of $F(t)$.

- Smaller T values (closer to zero but still positive) make the logarithmic term $\ln t$ more prominent near $t = 1$.

Special Cases and Limits

- As $T \to 0^+$:

$$
F(t) \to \ln t + 6
$$

- As $T \to \infty$:

$$
F(t) \to T t + \text{(smaller terms)} \quad \text{dominates}
$$

These limits highlight how the linear term's dominance varies with T.

Applications and Contexts of T

- In physical models, T could represent a constant rate of change or a baseline trend, with the $1/t$ term capturing time-dependent effects such as decay or growth influenced by inverse proportionality.

- In economic or biological models, tuning T adjusts the overall trend, while the logarithmic component models multiplicative or proportional effects.

Deeper Mathematical Insights: Derivatives and Integrability

Higher-Order Derivatives

- The second derivative:

$$
F''(t) = - \frac{1}{t^2}
$$

which is negative for $t > 0$.

Implication: $F(t)$ is concave downward across its domain, indicating that the slope $F'(t)$ is decreasing, although remaining positive.

Concavity and Inflection Points

- Since $F''(t) < 0$, the function is strictly concave downward; no points of inflection exist in $(0, \infty)$.

Integrability and Series Expansions

- The explicit form facilitates series expansion for small or large t.

- Near $t=1$, the function behaves smoothly, allowing Taylor series expansion:

\[

$$
F(t) = F(1) + F'(1)(t - 1) + \frac{F''(1)}{2}(t - 1)^2 + \dots
$$

with known derivatives:

$$
F'(t) = T + \frac{1}{t} \Rightarrow F'(1) = T + 1
$$

$$
F''(t) = - \frac{1}{t^2} \Rightarrow F''(1) = -1
$$

Thus:

$$
F(t) \approx 6 + (T + 1)(t - 1) - \frac{1}{2} (t - 1)^2 + \dots
$$

which can be useful for local approximations.

Applications and Broader Contexts

The form of $F(t)$ and its derivative appear in various scientific and engineering contexts.

Physics: Kinematics and Motion

- If $F(t)$ represents position, then $F'(t)$ is velocity.

- The derivative $F'(t) = T + \frac{1}{t}$ suggests a velocity that starts high (near $t=0$) and approaches T as $t \to \infty$, with a logarithmic correction.

- The negative second derivative indicates the acceleration is negative, implying deceleration or diminishing velocity increases over time.

Economics and Growth Models

- $F(t)$ could model accumulated wealth, with $F'(t)$ representing the growth rate.

- The T term could be a baseline growth rate, while $1/t$ models diminishing returns or effects that diminish over time.

Biological Systems

- In population dynamics, the derivative could represent growth rate influenced by resource availability

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