Find K So That 4x+3 Is A Factor Of 20x^3+23x^2-10x+k

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Understanding how to determine the value of k such that a given polynomial has a specific factor is a fundamental skill in algebra. In this article, we will explore the process step-by-step to find the value of k so that the polynomial $20x^3 + 23x^2 - 10x + k$ is divisible by $4x + 3$. This involves concepts like polynomial division, the Remainder Theorem, and factorization techniques, all of which are vital in algebraic problem-solving and polynomial analysis.

Introduction to Polynomial Factors and the Remainder Theorem

Before diving into the problem, it’s essential to understand some foundational concepts:

What Is a Polynomial Factor?

A polynomial $P(x)$ has a factor $(ax + b)$ if and only if there exists another polynomial $Q(x)$ such that:

$$
P(x) = (ax + b) \times Q(x)
$$

When $(ax + b)$ is a factor of $P(x)$, then the polynomial $P(x)$ is divisible by $(ax + b)$ with no remainder.

The Remainder Theorem

The Remainder Theorem states that:
- If a polynomial $P(x)$ is divided by $(x - r)$, the remainder is $P(r)$.
- Conversely, if $P(r) = 0$, then $(x - r)$ is a factor of $P(x)$.

In our problem, the divisor is $4x + 3$. To apply the Remainder Theorem, we need to express this divisor in the form $x - r$.

Transforming the Divisor for Application of the Remainder Theorem

Given the divisor:
$$
4x + 3
$$

To find the corresponding value of r, set the divisor equal to zero:
$$
4x + 3 = 0 \implies 4x = -3 \implies x = -\frac{3}{4}
$$

This means that if $P(x)$ is divisible by $4x + 3$, then:
$$
P\left(-\frac{3}{4}\right) = 0
$$

Our goal, then, is to find the value of k such that:
$$
P\left(-\frac{3}{4}\right) = 0
$$
where
$$
P(x) = 20x^3 + 23x^2 - 10x + k
$$

Calculating $P\left(-\frac{3}{4}\right)$

To evaluate $P\left(-\frac{3}{4}\right)$, substitute $x = -\frac{3}{4}$ into the polynomial:

$$
P\left(-\frac{3}{4}\right) = 20 \left(-\frac{3}{4}\right)^3 + 23 \left(-\frac{3}{4}\right)^2 - 10 \left(-\frac{3}{4}\right) + k
$$

Let’s compute each term step-by-step.

Step 1: Calculate $\left(-\frac{3}{4}\right)^3$

$$
\left(-\frac{3}{4}\right)^3 = - \left(\frac{3}{4}\right)^3 = - \frac{3^3}{4^3} = - \frac{27}{64}
$$

\]

Step 2: Calculate \(\left(-\frac{3}{4}\right)^2 \)

\[
\left(-\frac{3}{4}\right)^2 = \left(\frac{3}{4}\right)^2 = \frac{9}{16}
\]

Step 3: Substitute into the polynomial

\[
P\left(-\frac{3}{4}\right) = 20 \times \left(-\frac{27}{64}\right) + 23 \times \frac{9}{16} + (-10) \times \left(-\frac{3}{4}\right) + k
\]

Now, evaluate each term individually:

- \(20 \times -\frac{27}{64} = -\frac{540}{64} \)
- \(23 \times \frac{9}{16} = \frac{207}{16} \)
- \(-10 \times -\frac{3}{4} = \frac{30}{4} = \frac{15}{2} \)

Expressing all fractions with a common denominator (say, 64) simplifies the addition:

- \(-\frac{540}{64} \) (already over 64)
- \(\frac{207}{16} = \frac{207 \times 4}{64} = \frac{828}{64} \)
- \(\frac{15}{2} = \frac{15 \times 32}{64} = \frac{480}{64} \)

Adding these together:
\[
-\frac{540}{64} + \frac{828}{64} + \frac{480}{64} + k
\]

Combine the fractions:
\[
\frac{-540 + 828 + 480}{64} + k = \frac{(-540 + 828 + 480)}{64} + k
\]

Calculate numerator:
\[
-540 + 828 = 288
\]
\[
288 + 480 = 768
\]

So, the sum becomes:
\[
\frac{768}{64} + k
\]

Since $\frac{768}{64} = 12$, we have:

$$
P\left(-\frac{3}{4}\right) = 12 + k
$$

Determining k for Divisibility

Recall from the Remainder Theorem:
- The polynomial $P(x)$ is divisible by $4x + 3$ if and only if:

$$
P\left(-\frac{3}{4}\right) = 0
$$

Thus:

$$
12 + k = 0
$$

$$
k = -12
$$

Answer: The value of k that makes $4x + 3$ a factor of $20x^3 + 23x^2 - 10x + k$ is -12.

Verifying the Solution

It’s good practice to verify the solution by substituting $k = -12$ back into the polynomial and confirming that $4x + 3$ is indeed a factor.

Verification Process:

- Substitute $k = -12$:

$$
P(x) = 20x^3 + 23x^2 - 10x - 12
$$

- Check whether $P\left(-\frac{3}{4}\right) = 0$, which we already calculated:

$$
P\left(-\frac{3}{4}\right) = 12 + (-12) = 0
$$

- Since the Remainder Theorem confirms that the polynomial evaluates to zero at $x = -\frac{3}{4}$, $4x + 3$ is a factor.

Conclusion and Summary

In this comprehensive guide, we've demonstrated a methodical approach to find the value of \(k \) such that the polynomial \(20x^3 + 23x^2 - 10x + k \) is divisible by \(4x + 3 \). The key steps involved:

- Recognizing the relation between the divisor and the Remainder Theorem.
- Converting the divisor \(4x + 3 \) into the form \(x - r \) to find the specific value of \(r \).
- Substituting \(r = -\frac{3}{4} \) into the polynomial.
- Simplifying and solving the resulting equation for \(k \).

The final value, \(\boxed{k = -12} \), ensures that \(4x + 3 \) is a factor of the polynomial, making the polynomial divisible with no remainder.

This problem underscores the power of the Remainder Theorem in algebraic factorization problems and highlights a practical technique for solving similar problems involving polynomial divisibility, factors, and unknown coefficients.

Additional Tips for Polynomial Factorization Problems

- Always convert the divisor into the form \(x - r \) to apply the Remainder Theorem effectively.
- When dealing with polynomials with unknown coefficients, setting the polynomial equal to zero at specific points is a straightforward way to find those coefficients.
- Remember that if a polynomial has a linear factor \(ax + b \), then the root corresponding to this factor is \(x = -\frac{b}{a

Frequently Asked Questions

How do I determine the value of K so that 4x + 3 is a factor of 20x^3 + 23x^2 - 10x + K?

You can find K by using the Factor Theorem: since 4x + 3 is a factor, then when x = -3/4, the polynomial must equal zero. Substitute x = -3/4 into the polynomial and solve for K.

What is the step-by-step process to find K in the polynomial 20x^3 + 23x^2 - 10x + K given that 4x + 3 is a factor?

First, set 4x + 3 = 0, which gives x = -3/4. Next, substitute x = -3/4 into the polynomial:

$20(-3/4)^3 + 23(-3/4)^2 - 10(-3/4) + K = 0$, then solve for K.

How do I verify that 4x + 3 is a factor of the given polynomial after finding K?

After finding the value of K, substitute $x = -3/4$ into the polynomial with that K. If the expression equals zero, then $4x + 3$ is indeed a factor.

Can synthetic division be used to find K for the polynomial 20x^3 + 23x^2 - 10x + K?

Synthetic division is more suitable for dividing polynomials; here, using the Factor Theorem by substitution is more straightforward to find K when $4x + 3$ is a factor.

What is the value of K if 4x + 3 is a factor of 20x^3 + 23x^2 - 10x + K?

By substituting $x = -3/4$ into the polynomial and solving for K, the value of K is 3/4.

Is it necessary to perform polynomial division to find K in this problem?

No, polynomial division isn't necessary. Using the Factor Theorem and substitution of $x = -3/4$ to solve for K is sufficient.

How can I check my answer for K after calculating it for the polynomial 20x^3 + 23x^2 - 10x + K?

After calculating K, substitute $x = -3/4$ back into the polynomial with that K. If the result is zero, your K value is correct and confirms that $4x + 3$ is a factor.

Additional Resources

Find K So That 4x+3 Is A Factor Of 20x^3+23x^2-10x+k

When tackling polynomial factorization problems, especially those involving finding specific constants that make a given binomial a factor, the problem "Find K So That 4x+3 Is A Factor Of 20x^3+23x^2-10x+k" presents an interesting challenge. This type of problem combines the concepts of polynomial division, the Factor Theorem, and algebraic manipulation. Understanding how to approach such questions not only enhances algebraic skills but also deepens comprehension of polynomial properties, factoring techniques, and the role of parameters within polynomial expressions. This article offers a comprehensive exploration of the problem, breaking down the process step-by-step, discussing relevant mathematical principles, and providing insights into solving similar problems.

Understanding the Problem

What Does It Mean for a Polynomial to Have a Factor?

A polynomial $P(x)$ has a factor $(ax + b)$ if and only if $P(x)$ becomes zero when x is a root of $(ax + b)$. Specifically, if $(ax + b)$ is a factor, then:

$$
P\left(-\frac{b}{a}\right) = 0
$$

In this problem, the factor is $4x + 3$. To determine whether $4x + 3$ is a factor of the cubic polynomial $20x^3 + 23x^2 - 10x + k$, we can apply the Factor Theorem, which states that:

$$
\text{If } (ax + b) \text{ is a factor of } P(x), \text{ then } P\left(-\frac{b}{a}\right) = 0
$$

Given $4x + 3$, the root is:

$$
x = -\frac{3}{4}
$$

Thus, the problem reduces to evaluating:

$$
P\left(-\frac{3}{4}\right) = 0
$$

and solving for k.

Applying the Factor Theorem

Substituting $x = -\frac{3}{4}$ into the Polynomial

Given:

$$
P(x) = 20x^3 + 23x^2 - 10x + k
$$

we substitute $x = -\frac{3}{4}$:

$$
P\left(-\frac{3}{4}\right) = 20\left(-\frac{3}{4}\right)^3 + 23\left(-\frac{3}{4}\right)^2 - 10\left(-\frac{3}{4}\right) + k
$$

Calculating each term carefully:

- Cube term:

$$
\left(-\frac{3}{4}\right)^3 = -\frac{27}{64}
$$
$$
20 \times -\frac{27}{64} = -\frac{540}{64} = -\frac{135}{16}
$$

- Square term:

$$
\left(-\frac{3}{4}\right)^2 = \frac{9}{16}
$$
$$
23 \times \frac{9}{16} = \frac{207}{16}
$$

- Linear term:

$$
-10 \times -\frac{3}{4} = \frac{30}{4} = \frac{15}{2}
$$

Now, putting everything together:

$$
P\left(-\frac{3}{4}\right) = -\frac{135}{16} + \frac{207}{16} + \frac{15}{2} + k
$$

Express all terms with denominator 16 for easier addition:

$$
\frac{15}{2} = \frac{120}{16}
$$

So,

$$
P\left(-\frac{3}{4}\right) = \left(-\frac{135}{16} + \frac{207}{16} + \frac{120}{16}\right) + k
$$

Combine the fractions:

$$
\frac{-135 + 207 + 120}{16} = \frac{192}{16} = 12
$$

Therefore,

$$
P\left(-\frac{3}{4}\right) = 12 + k
$$

Solving for K

According to the Factor Theorem, if $4x + 3$ is a factor, then:

$$
P\left(-\frac{3}{4}\right) = 0
$$

which implies:

$$
12 + k = 0
$$

Solving for k:

$$
k = -12
$$

Result: The value of K that makes $4x + 3$ a factor of the polynomial $20x^3 + 23x^2 - 10x + k$ is -12.

Verification and Validation

Why Verification Is Important

Verifying the solution involves substituting $k = -12$ back into the polynomial and confirming that $4x + 3$ is indeed a factor. This process ensures that no algebraic errors have occurred and that the solution is consistent.

Verification Process

Substitute $k = -12$ into the polynomial:

$$
P(x) = 20x^3 + 23x^2 - 10x - 12
$$

Now, evaluate $P(-\frac{3}{4})$:

$$
20 \times -\frac{27}{64} + 23 \times \frac{9}{16} - 10 \times -\frac{3}{4} - 12
$$

Calculations:

- $20 \times -\frac{27}{64} = -\frac{540}{64} = -\frac{135}{16}$
- $23 \times \frac{9}{16} = \frac{207}{16}$
- $-10 \times -\frac{3}{4} = \frac{30}{4} = \frac{15}{2} = \frac{120}{16}$

Adding these together:

$$
-\frac{135}{16} + \frac{207}{16} + \frac{120}{16} - 12
$$

Express -12 as $-\frac{192}{16}$:

$$
\frac{-135 + 207 + 120 - 192}{16} = \frac{0}{16} = 0
$$

The evaluation confirms that $P(-\frac{3}{4}) = 0$, verifying that $4x + 3$ is a factor when $k = -12$.

Broader Context and Related Topics

Factor Theorem and Polynomial Roots

The Factor Theorem is central to polynomial algebra, providing a straightforward method to test whether a binomial is a factor of a polynomial. It links the roots of the polynomial directly to its factors, enabling algebraic factorization without long division in many cases.

Features:

- Efficient way to test factors
- Connects roots with factors
- Simplifies polynomial division processes

Limitations:

- Only applicable when the root is known or easily testable
- Does not directly factor the polynomial; additional steps may be necessary

Polynomial Division and Synthetic Division

While the Factor Theorem allows for quick evaluation, polynomial division methods (long division or synthetic division) are integral to fully factorize polynomials once roots are known. Synthetic division, in particular, simplifies computations when dividing by linear factors like $4x + 3$.

Features:

- Useful for dividing polynomials for factorization
- Synthetic division is faster and less error-prone

Limitations:

- Requires familiarity with division algorithms
- Not always straightforward with higher-degree polynomials or complex roots

Parameter Identification in Polynomials

The problem's parameter k exemplifies how coefficients in polynomials can depend on conditions like divisibility or factorization. Setting parameters to satisfy algebraic conditions is common in polynomial theory, especially in optimization, algebraic geometry, and solving equations with constraints.

Practical Implications and Applications

Understanding how to find parameters like k ensures students and mathematicians can manipulate polynomials efficiently in various contexts, including:

- Solving equations in calculus and algebra

- Analyzing polynomial functions in modeling real-world systems
- Designing algorithms for algebraic computations in software

This problem also emphasizes the importance of systematic approaches — substituting roots, simplifying expressions, and checking solutions — skills that are transferable across mathematical disciplines.

Conclusion

The problem "Find K So That 4x+3 Is A Factor Of 20x^3+23x^2-10x+k" offers a rich exploration of polynomial properties, the Factor Theorem, and algebraic manipulation. By understanding the process—identifying the root, substituting

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