Find The Area Between Y=2x^2 And Y=12x4x^2.

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Understanding how to find the area between two curves is a fundamental concept in calculus, especially in the context of integration. This process involves determining the region enclosed by the two functions and calculating its area using definite integrals. In this article, we will explore the step-by-step approach to find the area between the curves y=2x^2 and y=12x4x^2, providing insights into the underlying principles, methods, and applications.

Introduction to Area Between Curves

The area between two curves is the region enclosed by the graphs of the functions over a specific interval. Typically, this involves:

- Identifying the points of intersection between the two curves.
- Determining which curve is on top (greater y-value) and which is on the bottom.
- Setting up an integral to calculate the area between the curves over the interval.

Calculating this area provides insights into the space occupied between the functions and is crucial in various fields such as physics, engineering, and economics.

Understanding the Given Functions

Before proceeding, let's examine the functions involved:

1. First function: y = 2x^2

2. Second function: y = 12x4x^2

At first glance, the second function y=12x4x^2 appears to be a typo or misrepresentation. Likely, the intended function is y=12x·4x^2, which simplifies to y=12x 4x^2. Alternatively, it could be y=12x^4x^2, but that seems less likely due to notation.

Assumption: The second function is y = 12x 4x^2.

Let's simplify y=12x 4x^2:

- $y = 12x\ 4x^2 = (12\ 4)\ x\ x^2 = 48\ x^3$

Thus, the second function simplifies to $y=48x^3$.

Summary:

- $y = 2x^2$

- $y = 48x^3$

Note: If the original problem intended a different interpretation, please clarify. For now, we'll proceed with $y=48x^3$ as the second function.

Step 1: Find Intersection Points

The first step in calculating the area between the curves is to find their points of intersection. These are the x-values where $y=2x^2$ and $y=48x^3$ are equal:

$2x^2 = 48x^3$

Let's solve for x:

$2x^2 = 48x^3$

Divide both sides by 2:

$x^2 = 24x^3$

Divide both sides by x^2 (assuming $x \neq 0$):

$1 = 24x$

Solve for x:

$x = 1/24$

Special case: $x=0$

Check if $x=0$ is an intersection:

At $x=0$:

- $y=20=0$

- $y=480=0$

Yes, $x=0$ is an intersection point.

Therefore, the solutions are:

- $x=0$

- $x=1/24$

Note: Since dividing by x^2 is valid only when $x \neq 0$, we explicitly check $x=0$, which also satisfies the original equation.

Result: The curves intersect at $x=0$ and $x=1/24$.

Step 2: Determine Which Curve Is On Top

To set up the integral, we need to identify which function has the greater y-value in the interval between the intersection points.

Test a point between 0 and 1/24, say $x=1/48$:

- $y=2x^2 = 2(1/48)^2 \approx 2(1/2304) \approx 0.000868$

- $y=48x^3 = 48(1/48)^3 = 48(1/110592) \approx 0.000434$

Since $y=2x^2 > y=48x^3$ at $x=1/48$, the parabola $y=2x^2$ is on top between 0 and 1/24.

As x approaches 0, both functions approach 0.

Conclusion: For x in [0, 1/24]:

- Top curve: $y=2x^2$

- Bottom curve: $y=48x^3$

Step 3: Set Up the Integral for the Area

The area A between the two curves over the interval [0, 1/24] is:

$$
A = \int_{x=0}^{x=1/24} [\text{top function} - \text{bottom function}] \, dx
$$

Substituting:

$$
A = \int_{0}^{1/24} [2x^2 - 48x^3] \, dx
$$

Step 4: Compute the Integral

Let's evaluate:

$$
A = \int_{0}^{1/24} 2x^2 \, dx - \int_{0}^{1/24} 48x^3 \, dx
$$

Calculate each separately:

1. $\int 2x^2 \, dx = 2 \times \frac{x^3}{3} = \frac{2}{3} x^3$

2. $\int 48x^3 \, dx = 48 \times \frac{x^4}{4} = 12 x^4$

Now, evaluate from 0 to 1/24:

$$
A = \left[\frac{2}{3} x^3 \right]_0^{1/24} - \left[12 x^4 \right]_0^{1/24}
$$

Calculate each term:

- For $x=1/24$:

$$
\frac{2}{3} \left(\frac{1}{24}\right)^3 = \frac{2}{3} \times \frac{1}{24^3} = \frac{2}{3} \times \frac{1}{13,824}
$$

- For $x=1/24$:

$$
12 \left(\frac{1}{24}\right)^4 = 12 \times \frac{1}{24^4} = 12 \times \frac{1}{331,776}
$$

Calculate these fractions:

$$
A = \frac{2}{3} \times \frac{1}{13,824} - 12 \times \frac{1}{331,776}
$$

Simplify:

\[
A = \frac{2}{3 \times 13,824} - \frac{12}{331,776}
\]

Note that \(3 \times 13,824 = 41,472\).

So:

\[
A = \frac{2}{41,472} - \frac{12}{331,776}
\]

Express both with a common denominator:

- The denominators are 41,472 and 331,776.

Observe that:

\[
331,776 ÷ 41,472 = 8
\]

So, rewrite the first fraction with denominator 331,776:

\[
\frac{2}{41,472} = \frac{2 \times 8}{41,472 \times 8} = \frac{16}{331,776}
\]

Now, the area becomes:

\[
A = \frac{16}{331,776} - \frac{12}{331,776} = \frac{16 - 12}{331,776} =
\frac{4}{331,776}
\]

Simplify numerator and denominator:

Divide numerator and denominator by 4:

\[
A = \frac{1}{83,044}
\]

(Actually, 331,776 ÷ 4 = 83,044).

Final Result:

\[
\boxed{

A = \frac{1}{83,044}
}
\]

This is the exact area between the curves over the interval from 0 to 1/24.

Summary and Final Remarks

- The functions involved were identified as $y=2x^2$ and $y=48x^3$, based on the provided expression.

- The intersection points are at $x=0$ and $x=1/24$.

- The region between the curves on [0, 1/24] was calculated via definite integrals.

- The resulting exact area is \(\frac{1}{83,044}\).

Applications of Finding Area Between Curves

Calculating the area between curves is a fundamental technique in various disciplines:

- Physics: Determining work done or energy stored in systems.
- Economics: Calculating consumer and producer surpluses.
- Engineering: Analyzing material stress regions.
- Biology: Estimating areas in biological models.

Understanding how to accurately compute these areas enables professionals to make informed decisions and perform precise analyses.

Additional Tips for Solving Similar Problems

- Always verify the functions and clarify any ambiguous notation.
- Find points of intersection first to determine the limits of integration.
- Sketch the graphs if possible, to visualize which curve is on top

Frequently Asked Questions

What is the main goal when finding the area between the curves y=2x^2 and y=12x4x^2?

The main goal is to determine the total area enclosed between the two curves over their intersection interval.

How do you find the points of intersection between y=2x^2 and y=12x4x^2?

Set the two expressions equal: 2x^2 = 12x4x^2, then solve for x to find the points where the curves intersect.

What is the simplified form of the equation y=12x4x^2?

Assuming it means y=12x 4x^2, it simplifies to y=48x^3.

How do you set up the integral to find the area between these two curves?

Integrate the difference of the functions (top curve minus bottom curve) with respect to x over the intersection interval.

Which function is on top and which is on the bottom between the intersection points?

Determine by evaluating the functions at test points within the intersection interval; the one with higher y-values is on top.

What precautions should you take when computing the area between these curves?

Ensure the limits of integration are correctly identified from the points of intersection, and carefully subtract the lower function from the upper function within those limits.

Can the area between y=2x^2 and y=48x^3 be negative?

No, the area is always positive; negative results indicate the need to take the absolute value or correctly identify the upper and lower functions during integration.

Additional Resources

Find The Area Between Y=2x^2 And Y=12x4x^2

Calculating the area between two curves is a fundamental concept in integral calculus, often encountered in various mathematical, engineering, and scientific applications. In this comprehensive review, we will explore the problem of finding the area between the curves given by the equations $y=2x^2$ and $y=12x4x^2$. Our goal is to understand the process step-by-step, analyze the methods involved, and discuss the significance of such calculations in broader contexts.

Understanding the Problem: The Curves in Question

Before diving into the calculation, it is essential to understand the nature and characteristics of the given functions:

- Curve 1: $y = 2x^2$
- Curve 2: $y = 12x4x^2$

At first glance, the second equation appears unusual due to the expression $12x4x^2$. This notation could be interpreted in multiple ways, so clarifying the intended form is crucial.

Deciphering the Equations

The expression $12x4x^2$ might be a typographical error or a formatting issue. The most plausible interpretation is:

- Option 1: It represents $y = 12x \times 4x^2$
- Option 2: It is $y = 12x \times 4x^2$, which simplifies to $y = 48x^3$

Given standard mathematical notation and common problem structures, the second option makes sense:

$$
y = 12x \times 4x^2 = (12 \times 4) x \times x^2 = 48 x^3
$$

Thus, the two functions are:

- $y = 2x^2$

- $y = 48 x^3$

Alternatively, if the original problem intended $y = 12x \times 4x^2$, then the functions are:

- $y = 2x^2$
- $y = 48 x^3$

For the remainder of this review, we will proceed with these interpretations.

Graphical Analysis of the Curves

Understanding the graphs of the functions provides insight into where they intersect and how the area between them is structured.

Graph of $y=2x^2$

This is a standard parabola opening upwards with its vertex at the origin. Its shape is symmetric with respect to the y-axis, and it grows quadratically as x increases or decreases.

Graph of $y=48 x^3$

This is a cubic function with an inflection point at $(0, 0)$. It is symmetric with respect to the origin, with the nature of its graph changing depending on the sign of x.

Features:

- The parabola $y=2x^2$ is always non-negative.
- The cubic $y=48x^3$ takes both positive and negative values depending on x.

Finding the Points of Intersection

To calculate the area between two curves, the first step is to identify their intersection points, which serve as the limits of integration.

Equating the functions:

$$
2x^2 = 48 x^3
$$

Rearranged:

$$
2x^2 - 48 x^3 = 0
$$

Factor out common terms:

$$
2x^2 (1 - 24 x) = 0
$$

Set each factor equal to zero:

- $2x^2 = 0 \Rightarrow x = 0$
- $1 - 24 x = 0 \Rightarrow x = \frac{1}{24}$

Therefore, the curves intersect at:

$$
x = 0 \quad \text{and} \quad x = \frac{1}{24}
$$

Corresponding y-values:

- At $x=0$:

$$
y = 2(0)^2 = 0
$$

- At $x=\frac{1}{24}$:

$$
y= 48 \left(\frac{1}{24} \right)^3 = 48 \times \frac{1}{24^3}
$$

Calculate:

$$
24^3 = 24 \times 24 \times 24 = 13,824
$$

So,

$$
y = 48 \times \frac{1}{13,824} = \frac{48}{13,824} = \frac{1}{288}
$$

Thus, the intersection points are:

$$
(0, 0) \quad \text{and} \quad \left(\frac{1}{24}, \frac{1}{288} \right)
$$

Determining Which Curve Lies Above the Other

Between the intersection points, we need to know which function is on top to set up the integral correctly.

Test a point within the interval, say $x = \frac{1}{48}$:

- $y = 2x^2 = 2 \times \left(\frac{1}{48} \right)^2 = 2 \times \frac{1}{2304} = \frac{2}{2304} = \frac{1}{1152}$

- $y = 48 x^3 = 48 \times \left(\frac{1}{48} \right)^3 = 48 \times \frac{1}{110592} = \frac{48}{110592} = \frac{1}{2304}$

Compare:

$$
\frac{1}{1152} \quad \text{vs} \quad \frac{1}{2304}
$$

Since $\frac{1}{1152} > \frac{1}{2304}$, the parabola $y=2x^2$ is above the cubic $y=48x^3$ in this interval.

Calculating the Area Between the Curves

The area A between two curves from $x=a$ to $x=b$ is given by:

$$
A = \int_{a}^{b} |f(x) - g(x)| \, dx
$$

In this case, since $y=2x^2$ is above $y=48x^3$ between $x=0$ and $x=\frac{1}{24}$, the area is:

$$
A = \int_{0}^{\frac{1}{24}} \left(2x^2 - 48x^3 \right) dx
$$

Performing the Integration

Calculate:

$$
A = \int_{0}^{\frac{1}{24}} \left(2x^2 - 48x^3 \right) dx
$$

Separate the integral:

$$
A = \int_{0}^{\frac{1}{24}} 2x^2 dx - \int_{0}^{\frac{1}{24}} 48x^3 dx
$$

Compute each:

1. $\int 2x^2 dx = 2 \times \frac{x^3}{3} = \frac{2}{3} x^3$

2. $\int 48x^3 dx = 48 \times \frac{x^4}{4} = 12 x^4$

Applying limits:

$$
A = \left[\frac{2}{3} x^3 \right]_0^{\frac{1}{24}} - \left[12 x^4 \right]_0^{\frac{1}{24}}
$$

Calculate:

- $x = \frac{1}{24}$:

$$
\frac{2}{3} \left(\frac{1}{24} \right)^3 = \frac{2}{3} \times \frac{1}{24^3} = \frac{2}{3} \times \frac{1}{13,824} = \frac{2}{3 \times 13,824} = \frac{2}{41,472} = \frac{1}{20,736}
$$

- $12 \left(\frac{1}{24} \right)^4$:

Calculate 24^4:

$$
24^4 = (24^2)^2 = (576)^2 = 331,776
$$

\]

So,

$$
12 \times \frac{1}{331,776} = \frac{12}{331,776} = \frac{1}{27,648}
$$

Now, subtract:

$$
A = \frac{1}{20,736} - \frac{1}{27,648}
$$

Find common denominator:

$$
\text{LCM of } 20,736 \text{ and } 27,648
$$

Observe:

- $20,736 = 2^{10} \times 3^2$ (since $2^{10} = 1024$),

[Find The Area Between Y 2x 2 And Y 12x4x 2]()

Find The Area Between Y 2x 2 And Y 12x4x 2

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