Find The Area Of The Region. One Petal Of R=9cos(5)

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Understanding the area enclosed by polar curves is a fundamental aspect of calculus and geometry. When examining the specific curve defined by the polar equation $R = 9\cos(5\theta)$, we are dealing with a type of rose curve, characterized by its petal-like structure. In this comprehensive guide, we will explore how to find the area of one petal of this curve, providing detailed explanations, step-by-step calculations, and useful tips to enhance your understanding of polar area computations.

Introduction to Polar Curves and Area Calculation

Before diving into the specific problem, it is essential to understand the basics of polar equations and how areas are computed within this coordinate system.

What is a Polar Curve?

- A polar curve is a graph of a function $R(\theta)$, where R is the distance from the origin to a point on the curve, and θ is the angle measured from the positive x-axis.
- Examples include circles, roses, lemniscates, and spirals, each characterized by their unique $R(\theta)$ functions.
- The rose curve $R = a \cos(k\theta)$ or $R = a \sin(k\theta)$ produces petal-shaped patterns, with the number of petals depending on the value of k.

Area Enclosed by a Polar Curve

- The area A enclosed by a polar curve $R(\theta)$ between $\theta = \alpha$ and $\theta = \beta$ is given by the integral:

  $$A = \frac{1}{2} \int_{\alpha}^{\beta} [R(\theta)]^2 \, d\theta$$

- This formula accounts for the fact that the area swept out by the radius vector in polar coordinates is proportional to the square of the distance $R(\theta)$.
- For symmetric curves like rose curves, understanding the symmetry can simplify the

calculation by focusing on one petal.

Understanding the Rose Curve R=9cos(5θ)

This specific curve, $R=9\cos(5\theta)$, is a classic example of a rose curve.

Key Features of R=9cos(5θ)

- Number of Petals: Since $k=5$ (an odd integer), the rose has exactly 5 petals.
- Maximum Radius: The maximum value of R occurs when $\cos(5\theta) = 1$, giving $R = 9$.
- Petal Symmetry: The petals are evenly spaced around the origin, with angular separation of $360°/5 = 72°$ or $\pi/2.5$ radians.

Visualizing One Petal

- Each petal is traced out over a specific interval of θ, typically from where $R=0$ on one side to where it returns to zero on the other side.
- By focusing on one petal, we can compute its area and then understand the entire curve's structure.

Determining the Limits of Integration for One Petal

To find the area of a single petal, we need to identify the interval of θ over which that petal extends.

Steps to Find the Petal Limits

1. Set $R = 0$ to find where the petal begins and ends:

$0 = 9\cos(5\theta)$

2. Solve for θ:

   $\cos(5\theta) = 0$

3. Find the solutions for 5θ:

   $5\theta = \pi/2 + n\pi$, where n is an integer

4. Determine the specific interval corresponding to one petal:

   For the first petal, take n=0:

   $5\theta = \pi/2 \rightarrow \theta = \pi/10$

   For the other boundary, n=1:

   $5\theta = 3\pi/2 \rightarrow \theta = 3\pi/10$

5. Thus, the limits for one petal are from $\theta = \pi/10$ to $\theta = 3\pi/10$.

Calculating the Area of One Petal

With the limits established, we can now proceed to compute the area enclosed by one petal.

Step-by-Step Calculation

1. Write the area formula for the petal:

   $$\mathbf{A_{petal} = \frac{1}{2} \int_{\pi/10}^{3\pi/10} [R(\theta)]^2 \, d\theta}$$

2. Substitute $R = 9\cos(5\theta)$:

$$A_{petal} = \frac{1}{2} \int_{\pi/10}^{3\pi/10} [9\cos(5\theta)]^2 \, d\theta$$

3. Simplify the integrand:

$$A_{petal} = \frac{1}{2} \int_{\pi/10}^{3\pi/10} 81\cos^2(5\theta) \, d\theta$$

4. Factor out constants:

$$A_{petal} = (81/2) \int_{\pi/10}^{3\pi/10} \cos^2(5\theta) \, d\theta$$

5. Use the power-reduction formula for $\cos^2(x)$:

$$\cos^2(x) = (1 + \cos(2x)) / 2$$

6. Apply this to the integral:

$$A_{petal} = (81/2) \int_{\pi/10}^{3\pi/10} [(1 + \cos(10\theta)) / 2] \, d\theta$$

7. Simplify the constant:

$$A_{petal} = (81/4) \int_{\pi/10}^{3\pi/10} [1 + \cos(10\theta)] \, d\theta$$

8. Integrate term-by-term:

$$A_{petal} = (81/4) \left[\int_{\pi/10}^{3\pi/10} 1 \, d\theta + \int_{\pi/10}^{3\pi/10} \cos(10\theta) \, d\theta\right]$$

9. Calculate the integrals:

- $\int 1 \, d\theta = \theta$

- $\int \cos(10\theta) \, d\theta = (1/10) \sin(10\theta)$

10. Compute the definite integrals:

$$A\_petal = (81/4) [\theta]_{\pi/10}^{3\pi/10} + (81/4) [(1/10) \sin(10\theta)]_{\pi/10}^{3\pi/10}$$

11. Evaluate the bounds:

- θ from $\pi/10$ to $3\pi/10$
- $\sin(10\theta)$ at $\theta = 3\pi/10$:

$\sin(10\ 3\pi/10) = \sin(3\pi) = 0$
- $\sin(10\theta)$ at $\theta = \pi/10$:

$\sin(10\ \pi/10) = \sin(\pi) = 0$

12. Putting it all together:

$$A\_petal = (81/4) [(3\pi/10 - \pi/10) + (1/10)(0 - 0)] = (81/4) (2\pi/10) = (81/4) (\pi/5)$$

13. Simplify:

$$A\_petal = (81\pi) / (4\ 5) = (81\pi) / 20$$

Final Result and Interpretation

The area of one petal of the curve $R=9\cos(5\theta)$ is:

Area of one petal = $(81\pi) / 20$ square units

Since the entire rose curve has 5 petals, the total area enclosed by the entire curve can be calculated as:

1. Multiply the area of one petal by 5:

$$\text{Total Area} = 5\ (81\pi / 20) = (405\pi) / 20 = (81\pi) / 4$$

This result aligns with the symmetry of the rose curve, where each petal contributes equally to the total enclosed area.

Additional Tips for Calculating Areas of Polar Curves