Find The Derivative.f(X) = (2e^3x + 2e^-2x)^4

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Understanding how to differentiate complex functions is a fundamental aspect of calculus. In this article, we will explore the step-by-step process to find the derivative of the function $f(x) = (2e^{3x} + 2e^{-2x})^4$. This involves applying the chain rule, recognizing the structure of the composition of functions, and simplifying the result effectively. Whether you're a student preparing for exams or a professional refreshing your calculus skills, this comprehensive guide will help you master the differentiation process for this type of function.

Understanding the Function Structure

Before diving into the differentiation process, it’s essential to understand the structure of the function:

$$
f(x) = (2e^{3x} + 2e^{-2x})^4
$$

This function is a composite function where the outer function is a power function, and the inner function is a sum involving exponential functions.

Components of the Function

- Inner function $g(x)$:

$$
g(x) = 2e^{3x} + 2e^{-2x}
$$

- Outer function $f(g)$:

$$
f(g) = g^4
$$

The overall function can be viewed as $f(x) = (g(x))^4$. To differentiate $f(x)$, we will use the chain rule, which states:

$$
f'(x) = f'(g(x)) \times g'(x)
$$

Step-by-Step Differentiation Process

Applying the chain rule involves two main steps:

1. Differentiating the outer function $f(g) = g^4$ with respect to g.
2. Differentiating the inner function $g(x)$ with respect to x.

Let's proceed systematically.

Step 1: Differentiate the Outer Function

The outer function is a simple power function:

$$
f(g) = g^4
$$

Differentiating with respect to g:

$$
f'(g) = 4g^3
$$

Step 2: Differentiate the Inner Function $g(x)$

Recall:

$$
g(x) = 2e^{3x} + 2e^{-2x}
$$

Differentiate each term separately:

- Derivative of $2e^{3x}$:

$$
\frac{d}{dx} [2e^{3x}] = 2 \times \frac{d}{dx} [e^{3x}] = 2 \times e^{3x} \times 3 = 6e^{3x}
$$

- Derivative of $2e^{-2x}$:

$$
\frac{d}{dx} [2e^{-2x}] = 2 \times \frac{d}{dx} [e^{-2x}] = 2 \times e^{-2x} \times (-2) = -4e^{-2x}
$$

Adding these derivatives:

$$
g'(x) = 6e^{3x} - 4e^{-2x}
$$

Combining the Results to Find $f'(x)$

With the derivatives of the outer and inner functions, apply the chain rule:

$$
f'(x) = f'(g(x)) \times g'(x) = 4g^3 \times g'(x)
$$

Substitute $g(x)$ and $g'(x)$:

$$
f'(x) = 4 \left(2e^{3x} + 2e^{-2x} \right)^3 \times \left(6e^{3x} - 4e^{-2x} \right)
$$

Final expression for the derivative:

$$
\boxed{
f'(x) = 4 \left(2e^{3x} + 2e^{-2x} \right)^3 \times \left(6e^{3x} - 4e^{-2x} \right)
}
$$

Simplification of the Derivative Expression

While the derivative expression above is correct, further simplification can make it more manageable, especially for applications or graphing.

Step 1: Factor common terms if possible

Observe the second term:

$$
6e^{3x} - 4e^{-2x}
$$

Factor out 2:

$$

2 (3e^{3x} - 2e^{-2x})
\]

Rewrite the derivative as:

\[
f'(x) = 4 \left(2e^{3x} + 2e^{-2x} \right)^3 \times 2 \left(3e^{3x} - 2e^{-2x} \right)
\]

Combine the coefficients:

\[
f'(x) = 8 \left(2e^{3x} + 2e^{-2x} \right)^3 \times \left(3e^{3x} - 2e^{-2x} \right)
\]

Step 2: Factor further if needed

Notice the common factor of 2 in the first term inside the cube:

\[
2e^{3x} + 2e^{-2x} = 2 (e^{3x} + e^{-2x})
\]

So,

\[
f'(x) = 8 \left[2 (e^{3x} + e^{-2x}) \right]^3 \times \left(3e^{3x} - 2e^{-2x} \right)
\]

Expressing as:

\[
f'(x) = 8 \times 2^3 \times (e^{3x} + e^{-2x})^3 \times (3e^{3x} - 2e^{-2x})
\]

Calculate \(8 \times 2^3\):

\[
8 \times 8 = 64
\]

Thus, the derivative simplifies to:

\[
\boxed{
f'(x) = 64 \times (e^{3x} + e^{-2x})^3 \times (3e^{3x} - 2e^{-2x})
}
\]

Applications of the Derivative in Real-World Contexts

Understanding the derivative of such functions has practical significance across various fields.

1. Physics: Exponential Decay and Growth

Functions involving exponentials are common in modeling natural phenomena such as radioactive decay, population growth, and cooling. The derivative indicates the rate at which these processes change over time.

2. Economics: Compound Interest and Investment Growth

Exponential functions model compound interest, and derivatives help determine marginal rates of return, informing investment decisions.

3. Engineering: Signal Processing

Analyzing the rate of change of signals modeled by exponential functions is critical for designing filters and understanding system dynamics.

4. Biology: Population Dynamics

Modeling the growth or decline of populations with exponential components allows for predicting future trends and implementing control measures.

Additional Tips for Differentiating Complex Functions

Differentiating functions like $f(x) = (2e^{3x} + 2e^{-2x})^4$ may seem challenging initially. Here are tips to handle such problems effectively:

- Identify the inner and outer functions to determine the correct application of the chain rule.
- Simplify expressions where possible before differentiating to make calculations more manageable.
- Use factoring to extract common factors, which can simplify the derivative.
- Remember exponential derivatives:

$$
\frac{d}{dx} e^{ax} = a e^{ax}
$$

- Double-check your work by verifying derivatives with computational tools or alternative methods.

Conclusion

Differentiating the function $f(x) = (2e^{3x} + 2e^{-2x})^4$ involves understanding the composite nature of the function and applying the chain rule meticulously. The key steps include differentiating the outer power function and the inner exponential sum, then combining these results. Simplification leads to a concise form:

$$
\boxed{
f'(x) = 64 (e^{3x} + e^{-2x})^3 (3e^{3x} - 2e^{-2x})
}
$$

This derivative provides insight into the rate of change of the original function across different values of x. Mastery of such differentiation techniques is vital for solving complex calculus problems and applying these concepts in science, engineering, economics, and beyond.

Meta Description: Learn how to find the derivative of $f(x) = (2e^{3x} + 2e^{-2x})^4$ with step-by-step instructions, simplifications, and practical applications in calculus.

Frequently Asked Questions

What is the derivative of $f(x) = (2e^{3x} + 2e^{-2x})^4$?

Using the chain rule, the derivative is $f'(x) = 4(2e^{3x} + 2e^{-2x})^3 (6e^{3x} - 4e^{-2x})$.

How do I differentiate a composite function like $(2e^{3x} + 2e^{-2x})^4$?

You apply the chain rule: differentiate the outer function raised to the 4th power, then multiply by the derivative of the inner function $(2e^{3x} + 2e^{-2x})$.

What is the derivative of the inner function $2e^{3x} + 2e^{-2x}$?

The derivative of $2e^{3x}$ is $6e^{3x}$, and the derivative of $2e^{-2x}$ is $-4e^{-2x}$, so the total is $6e^{3x} - 4e^{-2x}$.

Can you show step-by-step how to find the derivative of $f(x) = (2e^{3x} + 2e^{-2x})^4$?

Yes. First, identify the outer function u^4 and inner function $u = 2e^{3x} + 2e^{-2x}$. Then, differentiate outer: $4u^3$, and multiply by derivative of inner: $6e^{3x} - 4e^{-2x}$. So, $f'(x) =$

$4(2e^{3x} + 2e^{-2x})^3 (6e^{3x} - 4e^{-2x})$.

What is the simplified form of the derivative for $f(x) = (2e^{3x} + 2e^{-2x})^4$?

The derivative simplifies to $f'(x) = 4(2e^{3x} + 2e^{-2x})^3 (6e^{3x} - 4e^{-2x})$.

Why do we use the chain rule in differentiating this function?

Because the function is a composition of an outer function raised to the 4th power and an inner function $(2e^{3x} + 2e^{-2x})$, requiring the chain rule to differentiate properly.

How does the exponential function's derivative help in finding $f'(x)$?

The derivatives of exponential functions like e^{ax} are straightforward: $d/dx\, e^{ax} = a\, e^{ax}$. This property simplifies differentiating the inner functions within the composition.

What is the significance of the exponent 4 in the function $f(x)$?

The exponent 4 indicates that the inner sum is raised to the 4th power, which affects the derivative via the chain rule, introducing a factor of 4 and a cubed term of the inner function.

Additional Resources

Find The Derivative.$f(x) = (2e^{3x} + 2e^{-2x})^4$

Introduction

In the realm of calculus, the process of differentiation is fundamental, serving as the backbone for analyzing rates of change, slopes of curves, and various applied mathematics problems. Among the myriad functions encountered, composite functions involving exponential expressions often pose interesting challenges and opportunities for applying differentiation rules. The function under investigation, Find The Derivative.$f(x) = (2e^{3x} + 2e^{-2x})^4$, exemplifies such a scenario, requiring an adept application of the chain rule coupled with exponential differentiation techniques.

This article aims to thoroughly analyze and compute the derivative of the given function, exploring the underlying calculus principles, step-by-step procedures, and potential implications of the result. We will begin by dissecting the structure of the function, then proceed to explicit differentiation, and finally discuss the significance of the derivative in various contexts.

Understanding the Function

The given function is:

$$ f(x) = \left(2e^{3x} + 2e^{-2x} \right)^4 $$

This is a composite function where the outer function is a fourth power, and the inner function is a sum of exponential expressions scaled by constants.

Structural Breakdown

- Outer function: $g(u) = u^4$
- Inner function: $u(x) = 2e^{3x} + 2e^{-2x}$

Thus, $f(x) = (u(x))^4$.

Understanding this structure allows us to employ the chain rule effectively, differentiating $g(u)$ with respect to u, and $u(x)$ with respect to x.

Differentiation Strategy

To find $f'(x)$, we rely on the chain rule:

$$
f'(x) = g'(u) \cdot u'(x)
$$

where:

- $g'(u) = 4u^3$
- $u'(x) = \frac{d}{dx}[2e^{3x} + 2e^{-2x}]$

The process involves two main steps:

1. Differentiate the outer function $g(u)$ with respect to u.
2. Differentiate the inner function $u(x)$ with respect to x.

Step-by-Step Derivation

Step 1: Differentiate the outer function

Given:

$$
g(u) = u^4
$$
$$
g'(u) = 4u^3
$$

This is straightforward, following the power rule.

Step 2: Differentiate the inner function

Recall:

$$
u(x) = 2e^{3x} + 2e^{-2x}
$$

Differentiating term-by-term:

- $\frac{d}{dx} [2e^{3x}] = 2 \cdot \frac{d}{dx} e^{3x}$

Since:

$$
\frac{d}{dx} e^{ax} = a e^{ax}
$$

we have:

$$
\frac{d}{dx} [2e^{3x}] = 2 \cdot 3 e^{3x} = 6 e^{3x}
$$

Similarly,

$$
\frac{d}{dx} [2e^{-2x}] = 2 \cdot (-2) e^{-2x} = -4 e^{-2x}
$$

Thus,

$$
u'(x) = 6 e^{3x} - 4 e^{-2x}
$$

Final Expression for $f'(x)$

Applying the chain rule:

$$
f'(x) = g'(u) \cdot u'(x) = 4 \left(2e^{3x} + 2e^{-2x} \right)^3 \cdot \left(6 e^{3x} - 4 e^{-2x} \right)
$$

This expression can be left as is for many applications, but further algebraic manipulation may be performed for clarity or specific evaluations.

Simplification and Alternative Forms

Factoring Out Common Terms

Observe that in the second factor, both terms have a common factor:

$$
6 e^{3x} - 4 e^{-2x} = 2 \left(3 e^{3x} - 2 e^{-2x} \right)
$$

Thus, the derivative can be expressed as:

$$
f'(x) = 4 \left(2e^{3x} + 2e^{-2x} \right)^3 \cdot 2 \left(3 e^{3x} - 2 e^{-2x} \right) = 8 \left(2e^{3x} + 2e^{-2x} \right)^3 \left(3 e^{3x} - 2 e^{-2x} \right)
$$

Alternatively, factor out constants inside the inner parentheses:

$$
f'(x) = 8 \left(2 (e^{3x} + e^{-2x}) \right)^3 \left(3 e^{3x} - 2 e^{-2x} \right)
$$

which simplifies as:

$$
f'(x) = 8 \cdot 8 (e^{3x} + e^{-2x})^3 \left(3 e^{3x} - 2 e^{-2x} \right) = 64 (e^{3x} + e^{-2x})^3 \left(3 e^{3x} - 2 e^{-2x} \right)
$$

This form emphasizes the structure and may simplify further evaluation or algebraic manipulation.

Applications and Significance

Understanding the derivative of such exponential functions is crucial across various scientific and engineering disciplines, including:

- Physics: Analyzing exponential growth or decay scenarios, such as radioactive decay, capacitor discharge, or population models.
- Economics: Modeling compound interest or growth rates over time.
- Biology: Studying enzyme kinetics or population dynamics involving exponential functions.
- Mathematics: Solving differential equations involving exponential terms.

Moreover, the process exemplifies the power of the chain rule, especially when dealing with composite functions involving exponential components. Recognizing the structure facilitates efficient differentiation, enabling analysts and researchers to derive meaningful insights from complex functions.

Broader Implications and Advanced Considerations

While the derivative computed here is explicit, in many practical contexts, the importance extends beyond the algebraic form. For example:

- Graphical Interpretation: The derivative indicates the slope of the tangent at any point x, revealing increasing or decreasing behavior.
- Critical Points: Setting $f'(x) = 0$ allows identification of local maxima, minima, or inflection points.
- Asymptotic Behavior: Analyzing the limit of $f'(x)$ as $x \to \pm \infty$ informs about the function's long-term tendencies.

In advanced calculus, such derivatives are foundational for optimization problems, curve sketching, and differential equation solutions.

Summary

In conclusion, the derivative of the function $f(x) = (2e^{3x} + 2e^{-2x})^4$ can be thoroughly derived using the chain rule:

$$
\boxed{
f'(x) = 64 (e^{3x} + e^{-2x})^3 \left(3 e^{3x} - 2 e^{-2x} \right)
}
$$

This expression captures the rate of change of the function at any point x, encapsulating the interplay of exponential growth and decay components. Mastery of such differentiation techniques not only enhances mathematical fluency but also empowers analysts to model and interpret complex real-world phenomena accurately.

Final Remarks

The exploration of Find The Derivative.f(x) = (2e^{3x} + 2e^{-2x})^4 underscores the elegance and utility of calculus. By systematically applying the chain rule, factoring, and recognizing exponential derivatives, we arrive at a comprehensive understanding of the function's behavior. Such analytical skills are indispensable across disciplines, fostering deeper insights into the dynamic systems that shape our world.

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