Find The General Solution To Y+12y+36y=0.

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Understanding how to find the general solution of differential equations is fundamental in mathematics, particularly in fields such as physics, engineering, and applied sciences. The given differential equation appears to involve derivatives of a function y, but as it stands, the notation seems to be missing some components. Based on common forms of differential equations, it is likely that the equation was intended to be:

$$
\frac{d^2 y}{dx^2} + 12 \frac{dy}{dx} + 36 y = 0
$$

This is a second-order linear homogeneous differential equation with constant coefficients, which is a classic problem in differential equations. In this article, we will explore how to find its general solution step by step, including the characteristic equation method, roots analysis, and the formulation of the general solution.

Understanding the Differential Equation

What is the Equation?

The differential equation:

$$
\frac{d^2 y}{dx^2} + 12 \frac{dy}{dx} + 36 y = 0
$$

is a second-order linear homogeneous differential equation with constant coefficients. Such equations are common when modeling physical systems like harmonic oscillators, electrical circuits, and mechanical vibrations.

Why is it Important?

Solving this differential equation provides the general form of solutions that describe the behavior of the system under study. The general solution encompasses all possible solutions, including particular solutions for specific initial conditions.

Methodology for Solving the Differential Equation

Step 1: Write the Characteristic Equation

For linear differential equations with constant coefficients, the standard approach involves assuming solutions of the form:

$$
y = e^{rx}
$$

Substituting into the differential equation yields the characteristic (or auxiliary) equation:

$$
r^2 + 12 r + 36 = 0
$$

Step 2: Solve the Characteristic Equation

This quadratic equation can be solved using factoring, completing the square, or the quadratic formula.

Quadratic Formula:

$$
r = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}
$$

where $a=1$, $b=12$, and $c=36$.

Calculating discriminant:

$$
\Delta = b^2 - 4ac = 12^2 - 4 \times 1 \times 36 = 144 - 144 = 0
$$

Since the discriminant is zero, the roots are real and equal.

Calculating roots:

$$
r = \frac{-12 \pm \sqrt{0}}{2} = \frac{-12}{2} = -6
$$

Thus, the characteristic equation has a repeated root:

$$
r = -6
$$

Step 3: Write the General Solution

For a second-order linear differential equation with constant coefficients, the solution depends on the nature of the roots:

- Distinct real roots: \(y = C_1 e^{r_1 x} + C_2 e^{r_2 x} \)

- Repeated roots: \(y = (A + Bx) e^{r x} \)

- Complex roots: \(y = e^{\alpha x} (C_1 \cos \beta x + C_2 \sin \beta x) \)

In our case, since there is a repeated real root \(r = -6 \), the general solution takes the form:

\[
\boxed{
y(x) = (A + Bx) e^{-6x}
}
\]

where \(A \) and \(B \) are arbitrary constants determined by initial conditions.

Summary of the Solution

The process to find the general solution involves recognizing the form of the differential equation, deriving the characteristic equation, solving for roots, and then constructing the solution based on the root types. For the given differential equation, the roots are repeated and real, leading to the general solution:

\[
\boxed{
\boxed{
y(x) = (A + Bx) e^{-6x}
}
}
\]

where \(A \) and \(B \) are arbitrary constants.

Additional Concepts and Clarifications

Initial Conditions and Particular Solutions

The general solution contains arbitrary constants A and B, which can be determined if initial or boundary conditions are provided. For example, if $y(0)$ and $y'(0)$ are known, then:

$$
\begin{cases}
y(0) = (A + 0) e^{0} = A \\
y'(x) = \frac{d}{dx} [(A + Bx) e^{-6x}]
\end{cases}
$$

Using these, you can solve for A and B.

Significance of Repeated Roots

Repeated roots indicate solutions where the exponential solution overlaps, requiring the introduction of a multiplicative x term to find the second linearly independent solution. This is a standard procedure in solving linear differential equations.

Visualizing the Solution

The solution $y(x) = (A + Bx) e^{-6x}$ describes a damped exponential behavior, which is typical in systems experiencing damping proportional to velocity, such as a mass-spring system with damping.

Conclusion

The process of solving the differential equation $\frac{d^2 y}{dx^2} + 12 \frac{dy}{dx} + 36 y = 0$ demonstrates the power of characteristic equations and roots analysis in understanding the behavior of systems modeled by differential equations. Recognizing the form of the roots—whether real, repeated, or complex—guides the construction of the general solution. In the case of repeated roots, as with our example, the solution involves an exponential multiplied by a polynomial of degree one, leading to:

$$
\boxed{
y(x) = (A + Bx) e^{-6x}
}
$$

This comprehensive approach underscores the essential techniques in solving second-order linear homogeneous differential equations with constant coefficients, equipping students and professionals with the tools to analyze a wide range of dynamic systems.

Frequently Asked Questions

What is the differential equation given in the problem?

The differential equation is $Y'' + 12Y' + 36Y = 0$.

How do you identify the type of differential equation?

It is a second-order linear homogeneous differential equation with constant coefficients.

What is the characteristic equation associated with the differential equation?

The characteristic equation is $r^2 + 12r + 36 = 0$.

How do you solve the characteristic equation to find the general solution?

Factor or use the quadratic formula: $r^2 + 12r + 36 = (r + 6)^2 = 0$, so $r = -6$ (a repeated root).

What is the general solution to the differential equation?

Since $r = -6$ is a repeated root, the general solution is $Y(t) = (C1 + C2\ t) e^{-6t}$, where C1 and C2 are arbitrary constants.

Additional Resources

Find The General Solution To Y + 12y + 36y = 0

Mathematics often presents us with differential equations that challenge our understanding and problem-solving skills. Among these, linear differential equations with constant coefficients are fundamental, serving as building blocks for modeling various physical phenomena such as vibrations, heat conduction, and electrical circuits. Today, we delve into a specific example: solving the differential equation $Y + 12y + 36y = 0$. While at first glance this may seem intimidating, a systematic approach reveals its structure and solution.

In this article, we will explore how to find the general solution to this differential equation, breaking down each step to make the process transparent and accessible. We will discuss the nature of the equation, the methods used for solving such equations, and interpret the solutions in context.

Understanding the Differential Equation

At its core, the equation Y + 12y + 36y = 0 appears to be a second-order linear differential equation with constant coefficients. However, the notation as presented is somewhat unconventional. Usually, differential equations involving derivatives are expressed with derivatives explicitly indicated, such as:

$$ y'' + ay' + by = 0 $$

where y'' is the second derivative of y with respect to the independent variable (often t or x), and a, b are constants.

Clarification of Notation:

Given the equation:

$$ Y + 12y + 36y = 0 $$

It’s reasonable to interpret Y as the second derivative of y, i.e.,

$$ y'' + 12 y' + 36 y = 0 $$

This interpretation aligns with standard notation in differential equations, where:

- y'' denotes the second derivative $\frac{d^2 y}{dt^2}$,
- y' denotes the first derivative $\frac{dy}{dt}$,
- y is the original function.

Assumption:

Based on the typical form, the differential equation is:

$$
\boxed{
y'' + 12 y' + 36 y = 0
}
$$

This is a homogeneous linear differential equation with constant coefficients, a well-studied class of equations.

The Nature of Homogeneous Linear Differential Equations

Characteristics:

- Homogeneity: The right-hand side is zero, indicating no external forcing or input.
- Linearity: The equation involves the sum of derivatives multiplied by constants.
- Constant Coefficients: The coefficients (1, 12, 36) are constants, simplifying the solution process.

These features allow us to apply a standard approach involving the characteristic equation and its roots to find the general solution.

The Method of Solving: Characteristic Equation

Step 1: Set up the characteristic equation

For a differential equation of the form:

$$
y'' + a y' + b y = 0
$$

the solution involves finding roots of the characteristic polynomial:

$$
r^2 + a r + b = 0
$$

In our case:

$$
r^2 + 12 r + 36 = 0
$$

Step 2: Solve the quadratic

Applying the quadratic formula:

$$
r = \frac{-a \pm \sqrt{a^2 - 4b}}{2}
$$

Plugging in $a = 12$ and $b = 36$:

$$
r = \frac{-12 \pm \sqrt{(12)^2 - 4 \times 36}}{2}
$$

Calculating the discriminant:

$$
(12)^2 - 4 \times 36 = 144 - 144 = 0
$$

Since the discriminant is zero, the roots are real and equal:

$$
r = \frac{-12 \pm 0}{2} = -6
$$

Result:

The characteristic roots are:

$$
r = -6, -6
$$

This indicates a repeated root.

Formulating the General Solution

Step 3: Write the general solution based on roots

For a second-order linear differential equation with constant coefficients:

- Distinct real roots: r_1 and r_2

$$
y(t) = C_1 e^{r_1 t} + C_2 e^{r_2 t}
$$

- Repeated real roots: r

$$
y(t) = (A + B t) e^{r t}
$$

Since our roots are repeated ($r = -6$), the general solution is:

$$
\boxed{
y(t) = (A + B t) e^{-6 t}
}
$$

where A and B are arbitrary constants determined by initial conditions.

Interpreting the Solution

The general solution describes the entire family of solutions to the differential equation. The term $e^{-6 t}$ indicates exponential decay, which is common in systems experiencing damping or resistance. The presence of the $B t e^{-6 t}$ term reflects the multiplicity of the root, which introduces a polynomial factor in the solution, affecting the behavior over time.

Physical and Practical Significance:

- In mechanical systems, such an equation might model a damped harmonic oscillator with no driving force.
- The exponential decay suggests the system's oscillations diminish over time, settling to equilibrium.
- The constants A and B are determined by initial displacement and velocity, respectively.

Extending the Solution: Initial Conditions

While the general solution contains arbitrary constants, specific problems often specify initial conditions such as:

$$

$$
y(0) = y_0, \quad y'(0) = y_1
$$

Using these, we can solve for A and B:

1. Plug $t = 0$:

$$
y(0) = (A + B \times 0) e^{0} = A
$$

so,

$$
A = y_0
$$

2. Compute $y'(t)$:

$$
y'(t) = \frac{d}{dt} [(A + B t) e^{-6 t}]
$$

Applying the product rule:

$$
y'(t) = (B) e^{-6 t} + (A + B t)(-6) e^{-6 t} = e^{-6 t} \left[B - 6 (A + B t) \right]
$$

At $t=0$:

$$
y'(0) = e^{0} [B - 6 A] = B - 6 A
$$

Given $y'(0) = y_1$, we have:

$$
B - 6 y_0 = y_1 \Rightarrow B = y_1 + 6 y_0
$$

Thus, the particular solution:

$$
y(t) = \left(y_0 + (y_1 + 6 y_0)t \right) e^{-6 t}
$$

Summary of the Solution Process

To recap, the steps to solve the differential equation $y'' + 12 y' + 36 y = 0$ are:

1. Recognize the equation as a second-order linear differential equation with constant coefficients.
2. Formulate and solve the characteristic equation:
\[

$$
r^2 + 12 r + 36 = 0
$$

3. Find roots:
- Roots are repeated: $r = -6$
4. Write the general solution:

$$
y(t) = (A + B t) e^{-6 t}
$$

5. Use initial conditions to determine constants A and B as needed.

Broader Context and Applications

Understanding such differential equations is essential across engineering, physics, and applied mathematics. For example:

- Mechanical Vibrations: Damped oscillators often follow equations similar to this, with exponential decay representing damping forces.
- Electrical Circuits: RLC circuits' voltage and current behavior are modeled by second-order differential equations.
- Population Models: Certain models of populations with delayed effects lead to similar equations.

The ability to identify roots, understand their multiplicity, and construct the general solution is a vital skill, forming the foundation for more complex differential equations.

Final Thoughts

The process of solving $y'' + 12 y' + 36 y = 0$ exemplifies the power of systematic mathematical methods. Recognizing the structure of the equation, formulating the characteristic polynomial, solving for roots, and translating these roots into a complete family of solutions allows us to understand the potential behaviors of the system modeled by the differential equation.

Mathematics continues to serve as a language that describes the natural and engineered world, and mastering these techniques opens doors to analyzing an astonishing array of phenomena. Whether you're a student beginning your journey or a professional applying these principles, understanding how to derive the general solution is a fundamental step toward deeper insights into dynamic systems.

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