# Find The Length Of The Curve X=etcos(t),y=etsin(t),0t

**Find The Length Of The Curve X=etcos(t), y=etsin(t), 0 ≤ t**

Understanding how to determine the length of a curve is a fundamental aspect of calculus, with applications spanning physics, engineering, computer graphics, and more. In this article, we focus on a specific parametric curve defined by the equations $x = e^t \cos(t)$ and $y = e^t \sin(t)$, with the parameter $t$ ranging from a starting point (often 0) to some upper limit. Our goal is to carefully analyze and compute the arc length of this curve, providing a comprehensive guide that delves into the mathematical steps involved, optimization techniques, and practical applications.

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## Understanding the Parametric Equations

Before calculating the length of the curve, it’s essential to understand the nature of the parametric equations:

- $x(t) = e^t \cos(t)$
- $y(t) = e^t \sin(t)$

These equations describe a curve in the xy-plane, parameterized by $t$, which can be thought of as a "moving" point tracing the path over time or some other variable.

### Key Observations About the Curve

- The equations resemble a spiral pattern due to the exponential term $e^t$ combined with the circular functions $\cos(t)$ and $\sin(t)$.
- The point $(x(t), y(t))$ can be expressed in polar coordinates, providing a different perspective for analysis.

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## Expressing the Curve in Polar Coordinates

Given the parametric equations, notice that:

$$
x(t) = e^t \cos(t), \quad y(t) = e^t \sin(t)
$$

which implies:

$$
r(t) = \sqrt{x(t)^2 + y(t)^2} = \sqrt{(e^t \cos t)^2 + (e^t \sin t)^2} = e^t \sqrt{\cos^2 t + \sin^2 t} = e^t
$$

and

$$
\theta(t) = \arctan\left(\frac{y(t)}{x(t)}\right) = \arctan\left(\frac{e^t \sin t}{e^t \cos t}\right) = \arctan(\tan t) = t
$$

since $\arctan(\tan t) = t$ within principal ranges.

Implication: The curve in polar coordinates is simply $r = e^t$, with the angle $\theta = t$. This makes the curve a spiral where the radius grows exponentially with the angle.

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# Calculating the Arc Length of the Spiral

The general formula for the length $L$ of a curve defined in polar coordinates $r(\theta)$ from $\theta = \alpha$ to $\theta = \beta$ is:

$$
L = \int_{\alpha}^{\beta} \sqrt{r(\theta)^2 + \left( \frac{dr}{d\theta} \right)^2} \, d\theta
$$

In our case, since $r(t) = e^t$ and $\theta = t$, we switch variables from $t$ to $\theta$. The limits of integration are from $t = 0$ to $t = T$, corresponding to $\theta = 0$ to $\theta = T$.

Step-by-step process:

1. Express $r(\theta)$: $r(\theta) = e^\theta$
2. Compute $dr/d\theta$: $\frac{dr}{d\theta} = e^\theta$
3. Substitute into the arc length formula:

$$
L = \int_{0}^{T} \sqrt{e^{2\theta} + (e^{\theta})^2} \, d\theta = \int_{0}^{T} \sqrt{e^{2\theta} + e^{2\theta}} \, d\theta
$$

$$
L = \int_{0}^{T} \sqrt{2 e^{2\theta}} \, d\theta = \int_{0}^{T} \sqrt{2} \, e^{\theta} \, d\theta
$$

4. Simplify the integral:

$$
L = \sqrt{2} \int_{0}^{T} e^{\theta} \, d\theta
$$

5. Perform the integration:

$$
L = \sqrt{2} \left[ e^{\theta} \right]_0^{T} = \sqrt{2} \left( e^{T} - 1 \right)
$$

---

# Final Formula for the Length of the Curve

The length $L$ of the curve from $t = 0$ to $t = T$ is:

$$
\boxed{
L = \sqrt{2} \left( e^{T} - 1 \right)
}
$$

This elegant expression indicates that the length depends exponentially on the upper limit $T$ of the parameter $t$.

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# Interpretation and Applications

Understanding the length of this spiral has numerous practical applications:

- Physics: Describing the trajectory length of particles moving under specific forces.
- Engineering: Calculating the length of spiral springs or coils.
- Computer Graphics: Rendering curves and paths with precise measurements.
- Mathematics Education: Demonstrating the relationship between parametric and polar representations.

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# Optimizing the Calculation for Different Ranges

While we considered $t$ from 0 to $T$, the formula can be adapted for other intervals:

- For $t$ from $a$ to $b$:

$$
L = \sqrt{2} \left( e^{b} - e^{a} \right)
$$

- If the interval is infinite (i.e., $T \to \infty$), the length diverges, indicating an infinitely long spiral.

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# Key Points to Remember

- The parametric equations describe a spiral with exponential growth.
- Converting to polar coordinates simplifies the length calculation.
- The final arc length formula involves an exponential function, illustrating rapid growth.
- The method applies to various ranges and can be adapted to different spirals.

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# Conclusion

Calculating the length of a curve like $x = e^t \cos t$, $y = e^t \sin t$ reveals the beauty of calculus and its ability to handle complex curves with elegant formulas. By understanding the underlying geometry, transforming to polar coordinates, and applying the arc length integral, we derive a concise expression that captures the essence of this exponential spiral’s length. Whether for academic purposes, engineering designs, or computer graphics, mastering these techniques enables precise measurements and deeper insights into the fascinating world of curves.

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Remember: The key to solving such problems lies in recognizing the nature of the parametric equations, leveraging coordinate transformations, and applying integral calculus thoughtfully.

# Frequently Asked Questions

## How do I find the length of the curve given by $x = e^t \cos(t)$ and $y = e^t \sin(t)$ for $0 \leq t$?

To find the length, compute the derivatives dx/dt and dy/dt, then use the arc length formula: $L = \int$ from 0 to t of $\sqrt{(dx/dt)^2 + (dy/dt)^2}\, dt$. Simplify the integrand and evaluate the integral accordingly.

## What is the derivative of $x = e^t \cos(t)$ with respect to t?

Using the product rule, $dx/dt = e^t \cos(t) - e^t \sin(t) = e^t (\cos(t) - \sin(t))$.

## What is the derivative of $y = e^t \sin(t)$ with respect to t?

Applying the product rule, $dy/dt = e^t \sin(t) + e^t \cos(t) = e^t (\sin(t) + \cos(t))$.

## How do I set up the integral for the length of the given parametric curve?

The arc length $L = \int_0^T \sqrt{(dx/dt)^2 + (dy/dt)^2}\, dt$, where T is the upper limit of t. Substitute the derivatives to get the integrand and evaluate the integral.

## Is there a simplification for the integrand $\sqrt{(dx/dt)^2 + (dy/dt)^2}$ in this case?

Yes, plugging in the derivatives, the integrand simplifies to $\sqrt{e^{2t}(\cos(t) - \sin(t))^2 + e^{2t}(\sin(t) + \cos(t))^2}$, which can be further simplified.

## What is the simplified form of the integrand for the curve length calculation?

Simplifying inside the square root yields $e^{2t} [ (\cos(t) - \sin(t))^2 + (\sin(t) + \cos(t))^2 ]$. Expanding and combining terms, this simplifies to $2 e^{2t}$ (since the sum of the squares simplifies to 2).

## What is the final integral expression for the length of the curve from t=0 to t=π?

The length $L = \int_0^\pi \sqrt{2 e^{2t}}\, dt = \int_0^\pi e^{t} \sqrt{2}\, dt = \sqrt{2} \int_0^\pi e^{t}\, dt$.

## How do I evaluate the integral to find the actual length of the curve?

Integrate $e^{t}$ from 0 to π, which is $e^{t}$ evaluated from 0 to π: $e^{\pi} - e^{0} = e^{\pi} - 1$. Multiply by $\sqrt{2}$ to get the total length: $L = \sqrt{2} (e^{\pi} - 1)$.

## What is the final answer for the length of the curve between t=0 and t=π?

The length of the curve from t=0 to t=π is $L = \sqrt{2} (e^{\pi} - 1)$.

# Additional Resources

Find the Length of the Curve \( x = e^{t} \cos(t), \quad y = e^{t} \sin(t), \quad 0 \leq t \leq 2\pi \)

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Introduction

Calculating the length of a curve defined parametrically is a fundamental problem in calculus, with applications spanning physics, engineering, and computer graphics. Here, we focus on a specific parametric curve given by:

$$
x(t) = e^{t} \cos(t), \quad y(t) = e^{t} \sin(t), \quad t \in [0, 2\pi].
$$

The goal is to find the total arc length of this curve over the given interval. This problem not only serves as an excellent exercise in applying the arc length formula but also provides insight into the geometric nature of the curve.

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Understanding the Parametric Equations

Before diving into the computation, it’s essential to understand the behavior and nature of the parametric equations.

1. Formulation of the Curve

- The equations resemble the polar-to-Cartesian conversion, where:

$$
x = r \cos(t), \quad y = r \sin(t),
$$

with $r = e^{t}$.

- This indicates that the curve is a spiral, expanding outward as $t$ increases because $r = e^{t}$ grows exponentially.

2. Geometric Interpretation

- The parametric equations describe a spiral that winds outward as $t$ increases, with the radius $r$ increasing exponentially.

- When $t = 0$, the point is at:

$$
x(0) = e^{0} \cos(0) = 1, \quad y(0) = e^{0} \sin(0) = 0,
$$

so it starts at $(1,0)$.

- At $t = 2\pi$:

$$
x(2\pi) = e^{2\pi} \cos(2\pi) = e^{2\pi} \times 1 = e^{2\pi},
$$

$$
y(2\pi) = e^{2\pi} \sin(2\pi) = 0,
$$

which shows the endpoint is at $(e^{2\pi}, 0)$.

- This indicates the curve makes about one and a half revolutions around the origin, expanding exponentially outward.

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### Deriving the Arc Length Formula

The general formula for the length $L$ of a parametric curve between $t = a$ and $t = b$ is:

$$
L = \int_{a}^{b} \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2} \, dt.
$$

Applying this to our specific functions:

$$
L = \int_{0}^{2\pi} \sqrt{\left(\frac{d}{dt} e^{t} \cos(t)\right)^2 + \left(\frac{d}{dt} e^{t} \sin(t)\right)^2} \, dt.
$$

---

### Computing $\frac{dx}{dt}$ and $\frac{dy}{dt}$

Let's compute the derivatives:

1. Derivative of $x(t) = e^{t} \cos(t)$:

Using the product rule:

$$
\frac{dx}{dt} = \frac{d}{dt} (e^{t} \cos(t)) = e^{t} \cos(t) + e^{t} (-\sin(t)) = e^{t} (\cos(t) - \sin(t)).
$$

2. Derivative of $y(t) = e^{t} \sin(t)$:

Similarly:

$$
\frac{dy}{dt} = e^{t} \sin(t) + e^{t} \cos(t) = e^{t} (\sin(t) + \cos(t)).
$$

---

Simplifying the Integrand

Substitute the derivatives into the arc length integral:

$$
L = \int_{0}^{2\pi} \sqrt{ \left[ e^{t} (\cos(t) - \sin(t)) \right]^2 + \left[ e^{t} (\sin(t) + \cos(t)) \right]^2 } \, dt.
$$

Factor out $e^{2t}$:

$$
L = \int_{0}^{2\pi} \sqrt{ e^{2t} (\cos(t) - \sin(t))^2 + e^{2t} (\sin(t) + \cos(t))^2 } \, dt,
$$

which simplifies to:

$$
L = \int_{0}^{2\pi} e^{t} \sqrt{ (\cos(t) - \sin(t))^2 + (\sin(t) + \cos(t))^2 } \, dt.
$$

Now, focus on simplifying the expression under the square root:

$$
(\cos(t) - \sin(t))^2 + (\sin(t) + \cos(t))^2.
$$

---

Simplifying the Radicand

Compute each term separately:

1. $(\cos(t) - \sin(t))^2 = \cos^2(t) - 2 \sin(t) \cos(t) + \sin^2(t)$.

2. $(\sin(t) + \cos(t))^2 = \sin^2(t) + 2 \sin(t) \cos(t) + \cos^2(t)$.

Adding these:

$$
\cos^2(t) - 2 \sin(t) \cos(t) + \sin^2(t) + \sin^2(t) + 2 \sin(t) \cos(t) + \cos^2(t).
$$

Notice that the $-2 \sin(t) \cos(t)$ and $+2 \sin(t) \cos(t)$ cancel each other out. The sum simplifies to:

$$
\cos^2(t) + \sin^2(t) + \sin^2(t) + \cos^2(t) = 2 (\sin^2(t) + \cos^2(t)) = 2 \times 1 = 2,
$$

since $\sin^2(t) + \cos^2(t) = 1$.

Thus, the integrand simplifies dramatically:

$$
L = \int_{0}^{2\pi} e^{t} \sqrt{2} \, dt = \sqrt{2} \int_{0}^{2\pi} e^{t} \, dt.
$$

---

Final Integration and Calculation

The integral becomes straightforward:

$$
L = \sqrt{2} \left[ e^{t} \right]_0^{2\pi} = \sqrt{2} \left( e^{2\pi} - e^{0} \right) = \sqrt{2} \left( e^{2\pi} - 1 \right).
$$

This elegant closed-form expression gives the total length of the spiral from $t = 0$ to $t = 2\pi$.

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Geometric and Analytical Insights

1. Nature of the Curve

- The spiral expands exponentially, with the radius increasing as $r = e^{t}$.

- Over one full period $0 \leq t \leq 2\pi$, the spiral makes approximately one revolution.

- The length depends on the exponential growth, which is captured succinctly in the formula.

2. Behavior of the Length

- The length grows exponentially with the upper limit $2\pi$ because of the $e^{2\pi}$ term.

- Since $e^{2\pi} \approx 535.491$, the total length is approximately:

$$
L \approx \sqrt{2} (535.491 - 1) \approx 1.414 \times 534.491 \approx 757.314.
$$

This indicates a very long spiral despite the limited angular sweep because of exponential expansion.

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Broader Context and Applications

Calculating the length of such a curve has extensive applications:

- Physics: Determining the length of a path taken by a particle moving under exponential growth conditions.

- Engineering: Designing spiral-shaped structures or pathways where length and expansion are critical considerations.

- Mathematics: Understanding the geometric properties of exponential spirals, which are special cases of logarithmic spirals.

- Computer Graphics: Rendering spirals and calculating their lengths for animations or structural modeling.

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Extensions and Generalizations

1. Varying the Interval

- The process can be extended to any interval $[a, b]$, making the formula:

$$
L = \sqrt{2} (e^{b} - e^{a}).
$$

- For example, for $t \in [0, T]$, the length is:

$$
L(T) = \sqrt{2} (e^{T} - 1).
$$

2. Different Spirals

- The method applies to other spirals where $r = e^{k t}$ for some constant $k$, with adjustments in derivatives.

- For \( r

## Find The Length Of The Curve X Etcos T Y Etsin T 0t

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