

Find The Measure Of The Exterior Angle

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Understanding the measure of an exterior angle is fundamental in the study of polygons, a key area within geometry. Whether you're a student working on homework, a teacher preparing lesson plans, or a math enthusiast eager to deepen your knowledge, grasping how to find and understand exterior angles is essential. This article will guide you through the concepts, formulas, and examples related to exterior angles of polygons, ensuring you develop a comprehensive understanding of the topic.

What Is an Exterior Angle?

Before diving into how to find the measure of an exterior angle, it is crucial to understand what an exterior angle is.

Definition of an Exterior Angle

An exterior angle of a polygon is the angle formed between one side of the polygon and the extension of an adjacent side. In simpler terms, if you extend one side of a polygon beyond a vertex, the angle between this extension and the neighboring side is called an exterior angle.

Visual Representation

Imagine a regular hexagon. If you extend one side of the hexagon beyond a vertex, the angle between this extension and the next side of the hexagon is the exterior angle at that vertex.

The Relationship Between Interior and Exterior Angles

Understanding the connection between interior and exterior angles is essential for calculating unknown angles.

Supplementary Angles

At each vertex of a polygon, the interior and exterior angles are supplementary, meaning they add up to 180 degrees:

- Interior angle + Exterior angle = 180°

This relationship holds true for all polygons, regardless of whether they are regular or irregular.

Regular vs. Irregular Polygons

- Regular Polygon: All sides and angles are equal. Each exterior angle is congruent.
- Irregular Polygon: Sides and angles may vary, so exterior angles are not necessarily equal.

How To Find The Measure Of An Exterior Angle

The process of finding an exterior angle depends on whether the polygon is regular or irregular.

For Regular Polygons

In regular polygons, all exterior angles are equal, and the sum of all exterior angles is always 360° . Therefore, the measure of each exterior angle can be found using the formula:

$$\text{Exterior Angle (each)} = 360^\circ / \text{number of sides}$$

Example:

Find the measure of each exterior angle of a regular octagon.

Solution:

Number of sides, $n = 8$

$$\text{Exterior angle} = 360^\circ / 8 = 45^\circ$$

Result: Each exterior angle of a regular octagon measures 45° .

For Irregular Polygons

In irregular polygons, individual exterior angles may vary, but the sum of all exterior angles always totals 360° . To find a specific exterior angle, you need additional information, such as the measure of an interior angle or other angles in the polygon.

Steps to Find an Exterior Angle in an Irregular Polygon:

1. Identify known angles: Gather data on interior angles or other related measurements.
2. Use the interior-exterior relationship: Recall that each interior angle and its adjacent exterior angle are supplementary.
3. Sum of exterior angles: Remember that the total of all exterior angles in any polygon is 360° .
4. Apply algebra: Use given angles and relationships to solve for the unknown exterior angle.

Example:

A pentagon has interior angles measuring 120° , 110° , 130° , 125° , and an unknown angle. Find the measure of the exterior angle at the vertex with the unknown interior angle.

Solution:

- Sum of interior angles of a pentagon: $(n - 2) \times 180^\circ = (5 - 2) \times 180^\circ = 3 \times 180^\circ = 540^\circ$
- Sum of known interior angles: $120^\circ + 110^\circ + 130^\circ + 125^\circ = 485^\circ$
- Unknown interior angle = $540^\circ - 485^\circ = 55^\circ$

Now, the exterior angle at that vertex:

- Exterior angle = $180^\circ - \text{interior angle} = 180^\circ - 55^\circ = 125^\circ$

Result: The exterior angle at that vertex measures 125° .

Key Formulas for Finding Exterior Angles

To streamline calculations, here are the most important formulas related to exterior angles:

- **For regular polygons:** $\text{Exterior Angle} = 360^\circ / n$
- **Sum of all exterior angles of any polygon:** 360°
- **Relationship between interior and exterior angles:** $\text{Interior Angle} + \text{Exterior Angle} = 180^\circ$

Examples and Practice Problems

Practical application solidifies understanding. Here are some examples:

Example 1: Regular Hexagon

Find the measure of each exterior angle.

Solution:

Number of sides, $n = 6$

Exterior angle = $360^\circ / 6 = 60^\circ$

Answer: Each exterior angle measures 60° .

Example 2: Irregular Quadrilateral

Interior angles are 100° , 80° , 95° , and an unknown angle. Find the exterior angle at the vertex with the unknown interior angle.

Solution:

- Sum of interior angles: $(4 - 2) \times 180^\circ = 360^\circ$
- Sum of known interior angles: $100^\circ + 80^\circ + 95^\circ = 275^\circ$
- Unknown interior angle = $360^\circ - 275^\circ = 85^\circ$
- Exterior angle at that vertex = $180^\circ - 85^\circ = 95^\circ$

Answer: The exterior angle at that vertex is 95° .

Common Mistakes to Avoid

- Assuming all exterior angles are equal in irregular polygons: Remember, only regular polygons have equal exterior angles.
- Confusing interior and exterior angles: Always verify which angle you are calculating.
- Forgetting that the sum of exterior angles is 360° : This is a key property that holds for all polygons.

Summary

- The measure of an exterior angle of a regular polygon is calculated by dividing 360° by the number of sides.
- The sum of the exterior angles of any polygon, regular or irregular, always equals 360° .
- Exterior and interior angles are supplementary; they add up to 180° at each vertex.
- To find an unknown exterior angle, use the relationship with the interior angle or leverage the sum of all exterior angles.

Additional Tips for Mastery

- Practice with different types of polygons to become comfortable with various scenarios.
- Use diagrams to visualize the angles and relationships.
- Remember the fundamental properties, such as the sum of exterior angles being 360° .

Conclusion

Finding the measure of an exterior angle is a key skill in geometry that aids in understanding polygon properties and solving related problems. Whether dealing with regular polygons, where calculations are straightforward, or irregular ones, where additional information is needed, the core principles remain consistent. By mastering these concepts, you'll enhance your problem-solving skills and deepen your understanding of geometric figures.

Remember: The key formulas and relationships serve as powerful tools in your mathematical toolkit. Keep practicing with diverse problems to become proficient in calculating exterior angles and applying these principles confidently.

Frequently Asked Questions

What is the formula to find the measure of an exterior angle in a regular polygon?

The measure of each exterior angle in a regular polygon is given by 360° divided by the number of sides, so $\text{Exterior Angle} = 360^\circ / n$.

How can I find the measure of an exterior angle if I know the interior angle of a regular polygon?

The exterior angle is supplementary to the interior angle, so $\text{Exterior Angle} = 180^\circ - \text{Interior Angle}$.

Why do the exterior angles of any convex polygon always sum up to 360° ?

Because as you traverse around the polygon, the exterior angles collectively make a full turn, totaling 360° , regardless of the number of sides.

Can the measure of an exterior angle be different in irregular polygons?

Yes, in irregular polygons, the exterior angles are not necessarily equal, but the sum of all exterior angles still equals 360° .

If a regular hexagon has an exterior angle of 60° , how many sides does the polygon have?

Since each exterior angle in a regular polygon is $360^\circ / n$, set $60^\circ = 360^\circ / n$, which gives $n = 360^\circ / 60^\circ = 6$ sides.

How do you find the measure of an exterior angle in a polygon when only the sum of interior angles is given?

First, find the number of sides using the sum of interior angles formula: $\text{Sum} = (n - 2) \times 180^\circ$. Then, divide 360° by that number to find each exterior angle.

Additional Resources

Exterior Angle

Understanding the measure of an exterior angle is fundamental in the study of geometry, playing a crucial role in understanding the properties of polygons, especially triangles and convex polygons. Whether you're a student trying to master geometric principles or an educator seeking to clarify the concept, grasping how to find and interpret exterior angles is essential. In this comprehensive review, we'll explore the concept from its foundational definitions to practical methods of calculation, including real-world applications, common pitfalls, and tips for mastering the topic.

What Is an Exterior Angle?

Before delving into the methods of calculating the measure of an exterior angle, it's important to establish a clear understanding of what an exterior angle actually is.

Definition and Basic Concept

An exterior angle of a polygon is an angle formed when one side of the polygon is extended outward

beyond a vertex, creating a supplementary angle with the adjacent interior angle at that vertex.

- Interior angle: The angle inside the polygon at a vertex.
- Exterior angle: The angle formed outside the polygon when a side is extended past a vertex.

For example, in a triangle, if you extend one side beyond a vertex, the angle between the extended side and the adjacent side is the exterior angle at that vertex.

Visual Representation

Imagine a simple triangle:

```

'''
A
/\
/\
/\
B-----C
'''
```

- Extend side BC beyond C.
- The angle between the extension of BC and AB is the exterior angle at vertex C.
- Similarly, extend other sides and measure the corresponding exterior angles at vertices A and B.

Properties of Exterior Angles

Understanding the properties of exterior angles reveals why they are so significant in geometry.

Sum of Exterior Angles in a Polygon

One of the most fundamental properties is that the sum of the exterior angles of any convex polygon is always 360 degrees.

- This holds true regardless of the number of sides.
- The exterior angles are measured in the direction around the polygon, typically clockwise or counterclockwise.

Relation with Interior Angles

At each vertex, the interior and exterior angles are supplementary:

$$\text{Interior Angle} + \text{Exterior Angle} = 180^\circ$$

This relationship is crucial because it allows for easy calculation of one if the other is known.

How To Find The Measure Of An Exterior Angle

Calculating the measure of an exterior angle can be straightforward when understanding the relationships involved. The method varies depending on the information available—whether you know the interior angles, the number of sides, or other measurements.

Method 1: Using the Relationship with Interior Angles

Step 1: Determine or find the interior angle(s).

- If you know the interior angles directly, this step is straightforward.
- Often, in regular polygons, all interior angles are equal.

Step 2: Use the supplementary angle relationship.

$$\text{Exterior Angle} = 180^\circ - \text{Interior Angle}$$

Example:

Suppose a regular pentagon has each interior angle measuring 108° :

$$\text{Exterior Angle} = 180^\circ - 108^\circ = 72^\circ$$

Step 3: Verify with the sum of exterior angles.

- Since the polygon is regular, all exterior angles are equal.
- Sum of all exterior angles = 360° .
- Total exterior angles = number of sides $\times$ measure of each exterior angle.
- For a pentagon:

$$\begin{aligned} & \backslash[\\ & 5 \times 72^\circ = 360^\circ \\ & \backslash] \end{aligned}$$

which confirms the calculation.

Method 2: Using the Sum of Exterior Angles in a Polygon

This method is particularly useful for irregular polygons where interior angles are unknown but the number of sides is known.

Step 1: Count the number of sides, n .

Step 2: Recall that the sum of all exterior angles is 360° .

Step 3: For regular polygons, divide 360° by n to find each exterior angle:

$$\begin{aligned} & \backslash[\\ & \text{Exterior Angle} = \frac{360^\circ}{n} \\ & \backslash] \end{aligned}$$

Example:

A hexagon (6 sides):

$$\begin{aligned} & \backslash[\\ & \text{Exterior Angle} = \frac{360^\circ}{6} = 60^\circ \\ & \backslash] \end{aligned}$$

Note: For irregular polygons, individual exterior angles may vary; in such cases, additional information or measurements are needed.

Method 3: Using Interior Angles and Polygon Type

When the interior angles are known, but the polygon isn't regular:

Step 1: Sum all interior angles using the formula:

$$\text{Sum of Interior Angles} = (n - 2) \times 180^\circ$$

Step 2: Calculate each interior angle (if they are equal):

$$\text{Interior Angle} = \frac{(n - 2) \times 180^\circ}{n}$$

Step 3: Find the exterior angle at each vertex:

$$\text{Exterior Angle} = 180^\circ - \text{Interior Angle}$$

Example:

Suppose a quadrilateral has interior angles measuring 85° , 95° , 80° , and 100° . The exterior angles at each vertex are:

- At vertex with 85° : $(180^\circ - 85^\circ = 95^\circ)$
- At vertex with 95° : $(180^\circ - 95^\circ = 85^\circ)$
- At vertex with 80° : $(180^\circ - 80^\circ = 100^\circ)$
- At vertex with 100° : $(180^\circ - 100^\circ = 80^\circ)$

Sum check:

$$95^\circ + 85^\circ + 100^\circ + 80^\circ = 360^\circ$$

which confirms the property.

Special Cases: Regular and Irregular Polygons

Different approaches apply depending on whether the polygon is regular (all sides and angles equal) or irregular.

Regular Polygons

- All interior angles are equal.
- All exterior angles are equal.
- Measure of each exterior angle:

$$\boxed{\text{Exterior Angle} = \frac{360^\circ}{n}}$$

where n is the number of sides.

Example: For a regular octagon:

$$\frac{360^\circ}{8} = 45^\circ$$

- Interior angle:

$$180^\circ - 45^\circ = 135^\circ$$

Irregular Polygons

- Exterior angles may vary at each vertex.
- The sum of all exterior angles still equals 360° .
- To find a specific exterior angle, measure or use known interior angles at that vertex.

Practical Applications of Exterior Angle Calculations

Understanding how to find exterior angles isn't merely an academic exercise; it has real-world applications.

Architecture and Engineering

Designers and architects often use exterior angles to determine the slopes of roofs, angles of walls, or the layout of complex structures. Precise angle measurement ensures structural integrity and aesthetic appeal.

Navigation and Robotics

In navigation, exterior angles aid in plotting courses and understanding turning angles. Robotics algorithms utilize exterior angles in path planning and movement precision.

Art and Design

Artists and designers leverage geometric principles, including exterior angles, to create patterns, tessellations, and symmetrical designs.

Educational Tools

Teaching geometry with practical angle measurement promotes spatial reasoning and problem-solving skills.

Common Challenges and Tips for Mastery

Despite the straightforwardness of the concepts, learners often encounter obstacles.

Challenges

- Confusing interior and exterior angles.
- Misidentifying the extension side of the polygon.
- Forgetting the sum of exterior angles always equals 360° .
- Difficulties with irregular polygons where angles vary.

Tips for Success

- Always specify whether you're measuring an interior or exterior angle.
- Use diagrams to visualize the problem.
- Remember the fundamental property: sum of exterior angles = 360° .
- For regular polygons, divide 360° by the number of sides.
- Practice with diverse polygon types to build intuition.
- Verify calculations by summing all exterior angles to ensure they total 360° .

Conclusion

The measure of an exterior angle is a vital concept in geometry, bridging simple properties with complex polygonal structures. Whether through the relationship with interior angles, the total sum properties, or direct measurement, mastering how to find exterior angles enhances problem-solving skills and deepens geometric understanding. As we've explored, the key lies in recognizing the relationships, applying the correct formulas, and visualizing the angles accurately. With practice and application, calculating exterior angles becomes an intuitive part of geometric reasoning, empowering learners and professionals alike to analyze and design with precision.

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